

Monitorando o COVID-19 no Brasil: uma aplicação de funções de crescimento para avaliação de desempenho dos Estados

RESUMO

Uma proposta de monitoramento e avaliação do desempenho dos Estados Brasileiros frente ao COVID-19 é apresentada por meio de um conjunto de funções não-lineares de crescimento. Isto é necessário dado o seu ineditismo em face às políticas públicas conhecidas pela sociedade brasileira do século XXI. Os dados diários sobre os casos acumulados dos 27 Estados brasileiros foram coletados no site do Ministério da Saúde. Em 14-novembro-2020, as funções de Richards, Weibull, Gompertz e Logística são as que melhor explicam a evolução do COVID-19 nos Estados brasileiros, nessa ordem. Além disso, identificou-se, via sistema de quadrantes, os Estados que estão sendo mais/menos eficientes no curto/longo prazo. Assim, é esperado que este quadro de monitoramento e avaliação permita aos decisores públicos uma melhor visão do desempenho das ações dos Estados brasileiros no enfrentamento ao COVID-19 ao longo do tempo.

PALAVRAS-CHAVE: COVID-19. Funções de crescimento. Monitoramento. Avaliação.

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INTRODUCTION

Pandemic and major epidemic outbreaks are not rare events, contrary to what common sense may suggest. They are real threats. History tells us of the effects on mankind of the Black Death in the 14th century and the Spanish flu in 1918. Recently, there have been outbreaks of i) SARS in Canada and Taiwan (2003); ii) dengue fever in Singapore (2005); iii) H1N1 in Canada (2009) (WANG; WU; YANG, 2012), to mention some examples.

However, a pandemic might be catastrophic if it is not taken seriously, due to the nonlinearity of its transmission in a world that is highly interconnected through long-range transportation (BAR-YAM, 2016; NORMAN; BAR-YAM; TALEB, 2020), which represents an ideal setting for the widespread transmission of COVID-19. On 16-November-2020, more than 54 million people worldwide were infected, and more than 1.3 million people had died, according to the Centre for Systems Science and Engineering at Johns Hopkins University (DONG; DU; GARDNER, 2020).

In an attempt to avoid any “naïve empiricism” (NORMAN; BAR-YAM; TALEB, 2020), several initiatives are carrying out careful research worldwide to better understand the behavior of COVID-19, such as regarding the reproductive ratio (ANAAN; HARGREAVES, 2020), the mortality rate (e.g. VASCONCELOS *et al.*, 2020), and the impact on the global economy in the short-run (e.g. McKIBBIN; FERNANDO, 2020).

Likewise, this article intends to contribute to that scientific effort showing that a set of growth functions can help policymakers to i) adequately monitor the evolution of cumulative cases in Brazilian states and ii) assess the performance of their actions in the face of the spread of COVID-19.

GROWTH FUNCTIONS

An empirical growth function is a nonlinear curve of some measure $f(t)$ against variable t , which has an S-shaped or sigmoidal pattern. This type of function is important in many fields of study (biology, chemistry, agriculture, medicine, social sciences, etc.), to understand its underlying structure (SEBER; WILD, 1989).

Concerning time-dependent biological events, such as the outbreak of COVID-19, the growth rate does not decrease constantly: first it increases to a certain maximum value before it starts to decrease to zero. In other words, growth functions have a very identifiable characteristic on the curve: the moment when the growth rate is highest, also known as the inflection point, within a biologically plausible framework (RITZ *et al.*, 2015).

In this study, we use this set of growth functions, as shown in Table 1. For small values of time t , the onset of outbreaks tends to be exponential, which is an attribute of many growth models. The beta (β) coefficient is the growth rate and the alpha (α) coefficient is the exponential growth multiplier.

However, this model leads to unlimited growth [$t \rightarrow \infty, f(t) \rightarrow \infty$], while biological growth almost invariably stabilizes over time [$t \rightarrow \infty, f(t) \rightarrow asymptote$]. Regarding the remaining functions, all of them have an S-shape (SEBER; WILD, 1989).

Table 1- Types of growth functions

FUNCTION	EQUATION
1.Exponential	$f(t) = \alpha \exp(\beta t)$
2. Logistic	$f(t) = Asym / \{1 + \exp[-k(t - Infl)]\}$
3. Gompertz	$f(t) = Asym \{ \exp\{-\exp[-k(t - Infl)]\} \}$
4. Weibull	$f(t) = Asym \{ \exp\{-\exp[-k(\ln(t) - \ln(Infl))]\} \}$
5. Richards	$f(t) = Asym / \{1 + M \exp[-k(t - Infl)]\}^{(1/M)}$

Source: Seber and Wild (1989), Tjørve and Tjørve (2010), Ritz *et al.* (2015).

The logistic function describes a symmetrical sigmoidal growth curve, whose inflection point (*Infl*) occurs halfway through the asymptote (*Asym*), which in turn is the maximum estimated value of infected people [*f(t)*]. The *k* coefficient works as a scale parameter on *t*, influencing the flattening (or slope) of the growth rate: the closer to zero, the more flattened the curve. Moreover, it is increasing when $t < Infl$ and decreasing when $t > Infl$ (SEBER; WILD, 1989).

The Gompertz function is often used when growth is not symmetrical about the inflection point, which occurs at 37% of the asymptote (SEBER; WILD, 1989). The Weibull function is an alternative way to describe the Gompertz function, using the logarithms of *t* and *Infl* instead (RITZ *et al.*, 2015), and thus allowing for higher values.

Finally, the Richards function, which has a particular link with the Susceptible-Infected-Removed model (WANG; WU; YANG, 2012), constitutes a common generalization for several other well-known models, including the logistic ($M = 1$) and Gompertz ($M \approx 0$) functions. Its main feature is the *M* parameter, which properly adjusts the inflection point to cover all values between 0% and 100% of the asymptote (TJØRVE; TJØRVE, 2010; OSWALD *et al.*, 2012). The other parameters have the same meaning as that of the logistic function given above.

A PROPOSAL FOR AN EVALUATION FRAMEWORK

Daily data were collected from the *Corona Virus Brasil* website¹ between 25-February to 14-November-2020. Among several variables provided by that database, we chose the daily reported cumulative cases.

This choice, instead of the daily number of accumulated deaths, is based on the following reasoning. Decreasing the number of deaths does not decrease the number of likely new ones infected, but the opposite is true: decreasing the number of new people infected will decrease the number of likely new deaths, even taking into account the uncertainties regarding the number of underreported cases. Besides, this choice is motivated by previous studies related to outbreaks and epidemics (CHOWELL; HYMAN, 2016; RODRIGUEZ *et al.*, 2020).

Then, the equations' coefficients were estimated² and presented in Tables 2-5 with the percentage of deaths and the number of days since the first confirmed case in each State. The equation that best explains the evolution of COVID-19 for each Brazilian State was chosen according to the lowest Root Relative Squared Error (RRSE): the closer to zero, the better.

Table 2 - Results for the Richards function on 14-November-2020

Quadrant	State	Asym	k	Infl	M	% of death	Nº days
I	RO	86,713.2	0.02	199.3	-0.50	2.00	240
	AM	209,299.7	0.03	223.6	-0.15	2.77	245
	SE	86,220.0	0.08	215.1	1.21	2.63	245
	SC	296,800.0	0.03	157.8	0.31	1.13	247
	GO	304,589.9	0.03	171.2	0.48	2.26	247
II	AP	55,165.0	0.08	155.8	1.81	1.42	240
	MS	94,761.2	0.03	157.6	0.34	1.93	244
III	PE	181,534.9	0.08	133.8	0.79	5.17	248
	BA	392,507.7	0.02	144.3	0.12	2.13	254
	SP	1,239,000.6	0.02	155.7	0.38	3.47	263
	PR	271,963.1	0.02	152.7	-0.10	2.39	248
	DF	230,860.3	0.02	137.4	0.08	1.74	253
IV	RS	353,594.9	0,02	183.3	0.27	2.24	250

Table 3 - Results for the Weibull function on 14-November-2020

Quadrant	State	Asym	k	Infl	% of death	Nº days
I	RR	73,143.1	-2.27	122.3	1.17	238
	PI	179,308.6	-1.91	151.2	2.10	240
II	AC	44,374.9	-1.53	117.0	2.15	241
	PA	358,813.5	-1.63	120.0	2.61	241
	MA	253,095.5	-1.66	109.6	2.20	239
	PB	172,493.1	-2.05	117.5	2.31	241
III	RN	97,914.8	-2.31	120.4	3.17	247
IV	CE	417,642.3	-1.46	127.3	3.32	243
	ES	258,159.3	-1.66	161.3	2.37	254
	RJ	721,750.5	-1.12	208.9	6.52	255

Table 4 - Results for the Gompertz function on 14-November-2020

Quadrant	State	Asym	k	Infl	% of death	Nº days
I	MG	462,236.2	-0.02	161.3	2.49	252
II	AL	94,840.1	-0.03	114.6	2.48	252
	MT	167,751.0	-0.02	141.8	2.63	240

Table 5 - Results for the Logistic function on 14-November-2020

Quadrant	State	Asym	k	Infl	% of death	Nº days
I	TO	79,693.7	-0.04	153.1	1.45	241

Figures 1-3 show the results for the Richards, Weibull, and Gompertz equations. No figure was drawn up for the exponential because it does not explain any State, and for the logistic functions because it explains only one State. The x-axis is the cumulative number of days since the first confirmed COVID-19 case, and

the y-axis is the inflection point (Infl) converted into the number of days that have passed since the first confirmed COVID-19 case.

Figure 1 – The Richards function framework

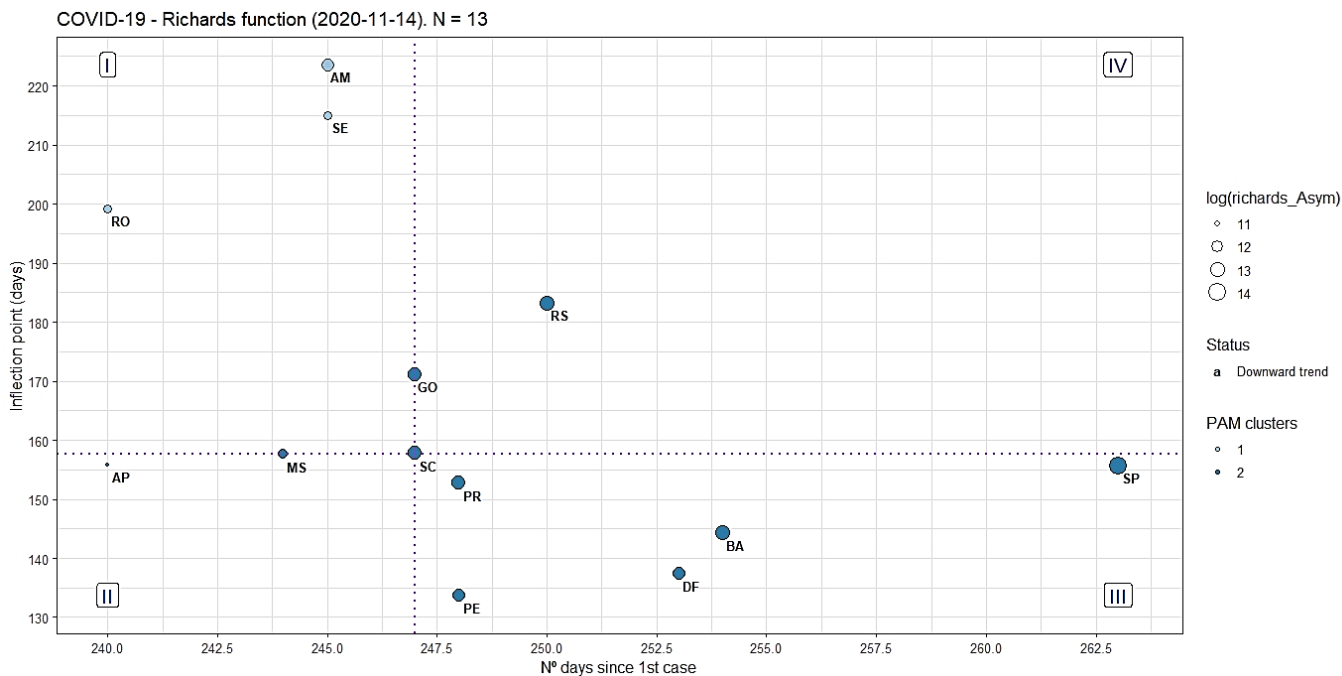
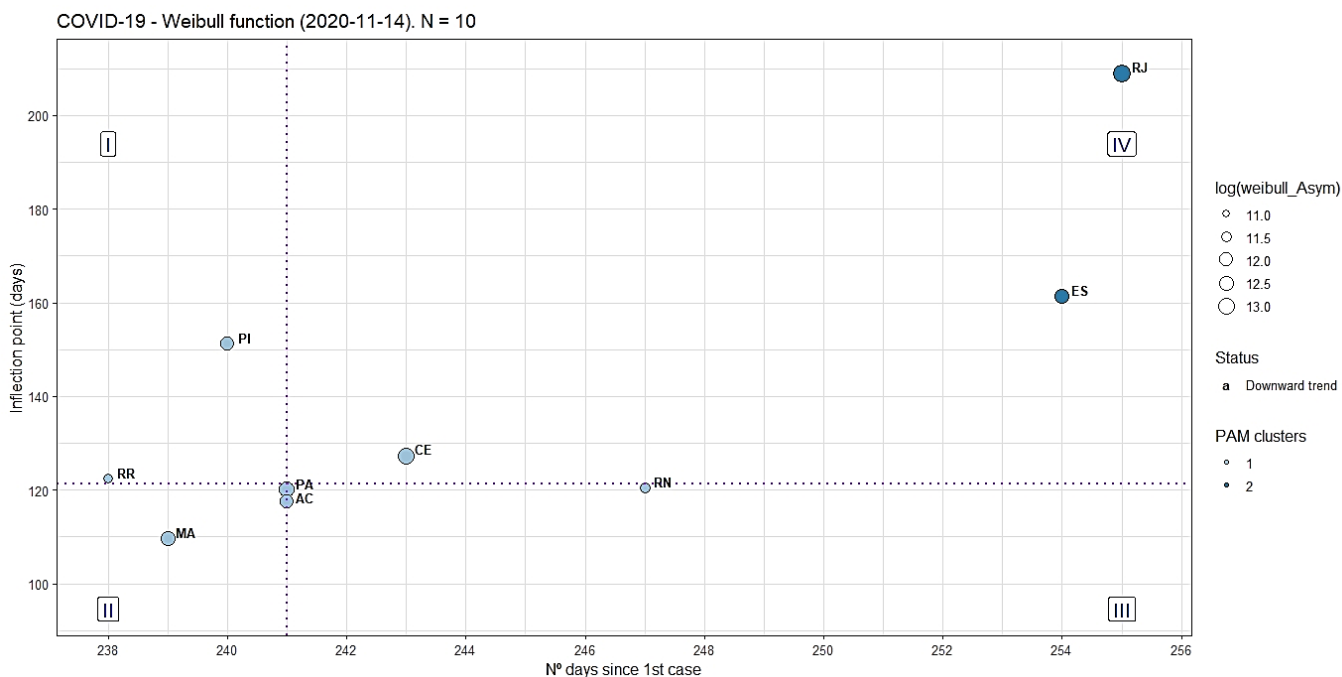


Figure 2 – The Weibull function framework

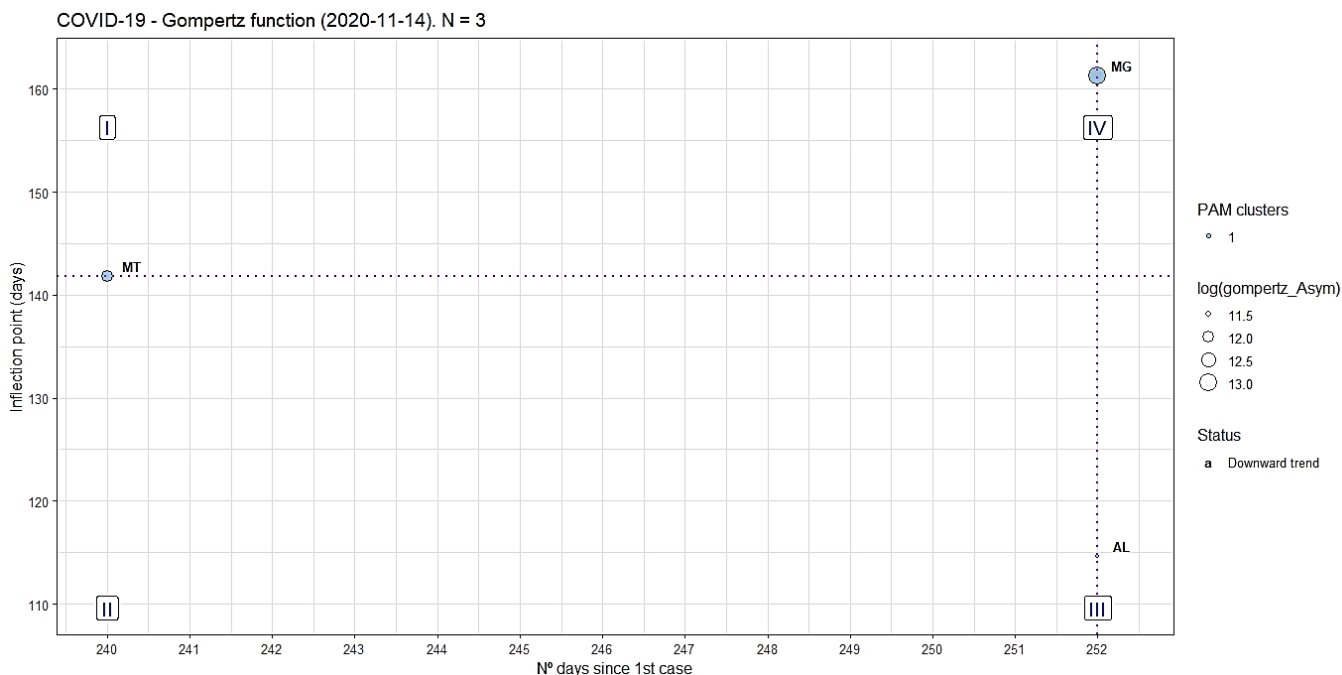


To mediate them, two more variables are considered. If the inflection point is greater than the current day (e.g. 14-November-2020), the growth rate of new cases is on an upward trend, and the State abbreviation is in red; otherwise, it is on a downward trend and the abbreviation is in black. Finally, the point size is

defined by the logarithm of the maximum number of estimated accumulated cases (Asym), to facilitate a relative comparison between them.

The quadrants were defined from the medians of the axes, dividing the Cartesian plane into four regions (I-IV), interpreted in a counterclockwise direction.

Figure 3 – The Gompertz function framework



This approach allows for a relative comparison between them regarding the performance of its actions to confront COVID-19.

If a State is in region I, it is inefficient in the short-term; in region II, efficient in the short-term; in region III, efficient in the long-term; and in region IV, inefficient in the long-term.

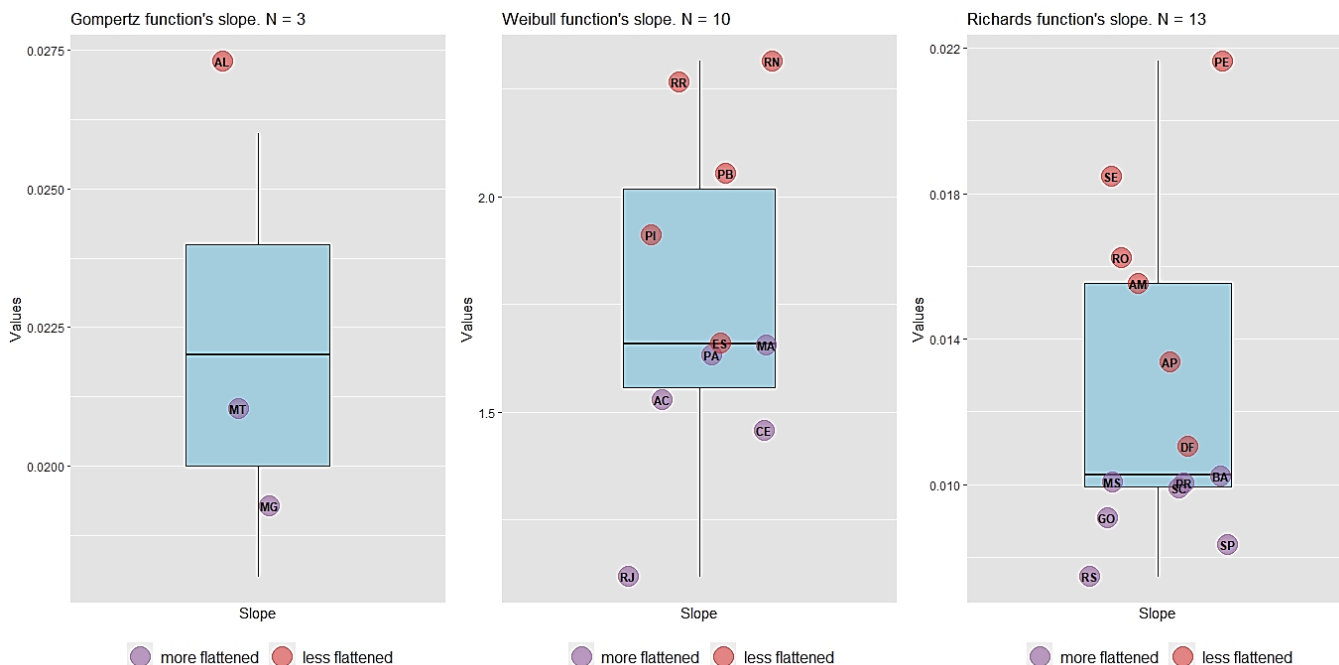
Therefore, the key concept of efficiency is to prevent the inflection point from increasing over time, which in turn also prevents the increase in the maximum number of estimated cases. Moreover, using the **Partitioning Around Medoids** technique (PAM), two clusters were found in figure 1 and 3, showing heterogeneity on their performance, while one cluster was found in figure 3.

Overall, the States clustered in figure 3 have the worst performance because they have the highest median at the inflection point, the highest asymptotes, and most of them present the growth rate in an upward trend.

On the other hand, it seems that the States grouped in figure 2 have a better performance than those in figure 1 because, in general, they tend to have a lower inflection point and asymptote.

Finally, figure 4 provides a comparison between the States related to the k coefficient, according to their respective growth function: those below their median are considered more flattened (purple) than those above (red). In other words, flatter asymmetric curves tend to take longer to reach their stabilization point.

Figure 4 – Boxplot for slopes of the functions



In this sense, on 14-November-2020, the results presented above for the Brazilian States can be summarized as follows:

- All states show a growth rate on a downward trend, highlighting the results of Acre (AC) for presenting the smallest Asymptote, i.e., the maximum expected number of infected people;
- Most of the states considered efficient are located in the North, Northeast and South regions;
- São Paulo (SP), Minas Gerais (MG) and Ceará (CE) have the highest asymptotes among all the States, thus presenting the worst results in the long run;
- The Richards, Weibull, Gompertz and logistic functions are the functions that better explain the evolution of COVID-19 in Brazilian states, in that order;

FINAL REMARKS

The purpose of this article was to propose a framework to monitor and evaluate, properly and daily, the performance of Brazilian States against COVID-19, for a direct and adequate understanding of policymakers on the evolution of the disease in each State, whose framework would be an important tool to be implemented. As a consequence, an improvement of the results in Brazilian public policies for mitigating or suppressing the effects of COVID-19 on society could be achieved while we do not yet have a vaccine.

Therefore, until 14-November-2020, the Richards, Weibull, Gompertz and logistic functions are the functions that better explain the evolution of COVID-19 in Brazilian states, in that order.

The adjustment of the functions to the data proved to be adequate, which allows policymakers to observe the evolution of the inflection point, the flattening of the curve, and the maximum expected accumulated number of cases daily, as well as to make predictions more accurate.

With these outcomes, it will be possible to check whether the public policies of the states against COVID-19 are having any effect.

Monitoring COVID-19 in Brazil: an application of growth functions to assess the performance of States

ABSTRACT

A proposal for monitoring and evaluating the performance of Brazilian states against COVID-19 is presented, employing a set of non-linear growth functions. This is necessary given its unprecedented nature in the face of public policies known to Brazilian society in the 21st century. The daily data on the accumulated cases of the 27 Brazilian states were collected on the Ministry of Health of Brazil's website. As of 14-November-2020, the Richards, Weibull, Gompertz and logistic functions, respectively in order, are those that best explain the evolution of COVID-19 in Brazilian states. Moreover, it was identified, through a quadrants system, the States that are being more/less efficient in the short/long term. Thus, it is expected that this monitoring and evaluation framework will allow policy-makers to have a better view of the performance of the actions of Brazilian states in confronting COVID-19 over time.

KEYWORDS: COVID-19. Growth functions. Monitoring. Assessment.

NOTES

¹ <https://covid.saude.gov.br/>

² The coefficients were estimated via **drc** and **FlexParamCurve** packages from R software.

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