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Operational invariants in schemes used by future engineers in a contextualized approach to ordinary differential equations

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ABSTRACT

This article is part of a completed doctoral qualitative research and addresses the learning of ordinary differential equations of separable variables. Drawing from the theory of conceptual fields, it aims to explain and analyze some operational invariants about knowledge linked to concepts of mathematics, physics and calculus used by 21 second-term Calculus students, majoring in civil engineering at a private institution, in the resolution of a contextualized event (problem integrating different areas of knowledge). Data collection was constituted by audio recordings of the discussions held during the work with the event, created according to the theoretical-methodological framework of the theory of Mathematics in the Context of Science. Results indicate that students made use of operational invariants related to the notions of function, graphical representation of a function, dependent and independent variable and logarithm (basic mathematics); rate of change, derivative, integration as an operation, variable in a differential equation, order, and degree of a differential equation (calculus); heat transfer, temperature, heat, and thermal conductivity (physics). In addition, data analysis suggests that group discussions and interactions among students should be more frequent in classes as they contribute to promoting learning and student cognitive development.

Keywords: Mathematics Education in Higher Education. Theory of Mathematics in the Context of Sciences. Theory of Conceptual Fields. Engineering. Operational Invariants.

INTRODUCTION

This study intends to explain and analyze some operational invariants related to basic mathematics, physics and calculus used by students majoring in civil engineering to solve a contextualized event, which, as will be discussed in further details, is a problem integrating different areas of knowledge. This event was based on a heat transfer problem and aimed to investigate how research participants learned ordinary differential equations (ODEs) of separable variables in accordance with the theory of mathematics in the context of sciences (TMCS) and the theory of conceptual fields (TCF).

The following sections present the notion of contextualized event and describe the theoretical and methodological framework of TMCS once they were deemed fundamental to create and put the aforementioned event into action. Theoretically, the cognitive phase of TMCS and Vergnaud's TCF are brought to the fore given their importance for data analysis.

During this investigation TCF was focused because it is believed to validate the analysis of the activity in a situation (VERGNAUD, 2009) and to allow the investigation of the operational form of knowledge in the resolution of learning situations as they are experienced by students. This theory is interested in the properties, relationships, and techniques explicitly or implicitly employed by students to solve a situation, what Vergnaud (1996a) calls operational invariants of schemes, materialized as concepts-in-action or theorems-in-action. This investigation chose to highlight operational invariants related to basic mathematics, calculus and physics, since they were applied by the research participants and were therefore meaningful to concept construction for learning ODE.

STARTING POINT: THE IDEA OF A CONTEXTUALIZED EVENT

Given the fundamental place the notion of contextualized event has to the doctoral dissertation from which this study is a part, it will be presented before the theoretical framework to which it belongs. Reflecting upon the teaching and learning of mathematics in undergraduate courses where it is present, but not their main object of study, Mexican researcher Patricia Camarena Gallardo introduced the notion of contextualized events as non-routine activities, i.e. as “problems, projects, or case studies that behave as integrating entities among diverse areas” (CAMARENA, 2021, p. 179). According to her, they have the potential to "make students face a cognitive conflict when they read a statement so that they get encouraged and intrigued enough to pursue the task of solving it” (CAMARENA, 2021, p. 179).

In the case of the doctoral research conducted by the first author of this article advised by the second author, the contextualized event was created to investigate how students of a civil engineering program learned ODEs of separable variables in a contextualized approach. The event as developed linking differential and integral calculus and heat transfer, as shown in Figure 1.

Figure 1 - The contextualized event

Thermal comfort in a building

Intending to improve the thermal comfort of non-air-conditioned environments, reduce the power consumption of air-conditioned environments, and rationalize energy consumption, the civil engineer seeks solutions that enhance the energy efficiency of a building project. To achieve the desired thermal comfort, the civil engineer needs to consider the heat transfer from the external environment to the interior of buildings. Thus, we present three walls built as follows:

WALL n. 1: Constructed with exposed solid bricks, laid in the dimension of 10 cm, with coating on all faces. The brick dimensions are 10.5 cm x 6 cm x 23 cm. The bricks were laid with 1 cm of laying mortar whose mix ratio is 1:6 (1 part cement and 6 parts sand), and the external coating of each face of the wall was done with 3.5 cm of the same mortar. The total thickness of the wall is 17 cm.

WALL n. 2: Constructed with exposed solid bricks, laid in the dimension of 10 cm, with coating on all faces. The brick dimensions are 10.5 cm x 6 cm x 23 cm. The bricks were laid with 1 cm of laying mortar whose mix ratio is 1:6 (1 part cement and 6 parts sand), and the external coating of each face of the wall was done with 3.5 cm of plaster. The total thickness of the wall is 17 cm.

WALL n. 3: Constructed with exposed solid bricks, laid in the dimension of 10 cm, with coating on all faces. The brick dimensions are 10.5 cm x 6 cm x 23 cm. The bricks were laid with 1 cm of laying mortar whose mix ratio is 1:6 (1 part cement and 6 parts sand), and the external coating of each face of the wall was done with 1.5 cm of the same mortar. On this wall, an expanded polystyrene (EPS) of 2 cm was glued to its external and internal faces. The total thickness of the wall is 17 cm.

According to the specifics of each wall, answer:

a) Which of the three walls presents greater thermal comfort in a building? Why?

b) What is the thermal behavior of the materials used in each wall?

c) What needs to be done to reduce thermal losses in a building?

Source: Lopes (2021, p. 118-119).

The reason that explains why heat transfer was considered to contextualize ODE learning is shown in the following section. It also brings the framework of TMCS, which based the theoretical and methodological perspectives used to create and implement the contextualized event.

THE THEORY OF MATHEMATICS IN THE CONTEXT OF SCIENCES: THEORETICAL-METHODOLOGICAL FRAMEWORK FOR THE CREATION AND IMPLEMENTATION OF THE CONTEXTUALIZED EVENT

As highlighted in the introduction, TMCS was developed by Camarena to substantiate discussions about the teaching of mathematics in higher education courses that do not aim at the training of mathematicians. It proposes the learning environment as a system constituted by five interacting phases: curricular, didactic, epistemological, teaching, and cognitive, each phase containing specific theoretical elements and methodological processes (CAMARENA, 2013).

The theoretical and methodological assumptions of TMCS was used to construct and implement the contextualized event presented in Figure 1. To conceive this event, the first author of this article resorted to the procedures of *Dipcing* (CAMARENA, 2002), a methodology found in the curricular phase of the theory. Through the analysis of a document known as Course Completion (Integralização Curricular, in Portuguese), which lists the courses that make up the curriculum of civil engineering of the institution in which the research was developed, the researcher determined all the non-mathematical courses which use ODEs.

The reading of the document provided her with the opportunity to learn about the specifics of each curriculum nucleus according to which the program is organized. Then, a survey was conducted with faculty members to understand what knowledge related to concepts of mathematical courses is needed in their classes. The next step was to analyze the textbooks of the non-mathematical courses listed by these teachers given the need to understand in which situations of the specific and professionalizing areas of this engineering program ODEs are used. Based on their answers, the researcher chose to analyze the textbooks from the following courses: General Physics, Statistics and Probability, General Chemistry, General Mechanics, Fluid Mechanics, Strength of Materials, and Structural Analysis.

The reading of books related to heat transfer demonstrated the employment of a wide range of mathematical concepts. A historical-epistemological analysis carried out on the mathematical content with which the researcher chose to work allowed her to infer that the problems related to heat transfer were linked to the epistemological development of differential equations. Therefore, the decision to advance a contextualized event related to heat transfer, since it was a context that was, in fact, present in those problems in which, historically, differential equations were developed. In addition, one of the educators who contributed in the survey pointed out that an important demand in the area is the investigation of the various forms of sustainability and economy of existing energy resources, since the use of materials with greater thermal resistance can represent a great reduction in electricity consumption in buildings with air-conditioned environments. It also represents a greater comfort for residents who live in buildings with no air conditioning because these materials provide these spaces with greater thermal insulation, reducing heat exchanges with their external environment.

As a consequence, the event at hand was thought to be the heat transfer from the external to the interior environment of flat masonry walls. It aimed to seek mathematical solutions to problems related to the thermal comfort in buildings using the resolution of ODEs. Aiming to work with ODE in a contextualized way, this investigation followed the five stages considered by Camarena and González (2001) in the epistemological phase of TMCS to create the event. These were: (1) analyzing engineering textbooks to verify how ODEs are applied in civil engineering; (2) examining the course description and curriculum of this program as to understand how this content is supposed to be taught and in which classes or other types of curricular units it is addressed; (3) carrying out an epistemological study of differential equations; (4) studying mathematical texts for engineering to learn how one should approach ODEs according to the didactic materials for training future engineers, and (5) analyzing the cognitive aspects related to teaching and learning ODEs, considering background research conducted by other authors.

The research participants were 21 second-term Calculus students majoring in civil engineering from a private institution in the state of Minas Gerais, Brazil and did not yet have knowledge on the fundamental concepts of differential equations. To create the contextualized event, we used the constructivist assumptions of the Didactic Model of Mathematics in Context (MoDiMaCo), inherent to the didactic phase of TMCS (CAMARENA, 2017). The goal was to teach students mathematical concepts to help them develop competency to transfer mathematical knowledge[1] to specific areas, in this case, heat transfer.

The two guiding axes of MoDiMaCo, contextualization and decontextualization, were worked. In the contextualization axis, which is interdisciplinary, knowledge about calculus and heat transfer was explored. As for decontextualization, the first author taught a class explaining basic knowledge on differential equations and had students solve ODEs of separable variables in a disciplinary way through individual and group activities, with the formality and rigor required in an undergraduate course aimed at training a civil engineer.

For group activities, also in line with MoDiMaCo, groups of three students assuming distinct positions were formed: an **emotional** leader, an **intellectual** leader, and an **operational** leader. Such organization is justified by Camarena (2017) who contends that these positions have complementary characteristics for the good development of a collaborative work. The emotional leader is the student who motivates the group, the intellectual leader is reflective and analytical, with well-constructed prior knowledge, and the operational leader is the one who effectively performs the tasks and who reports the arguments of the group to the whole class, among others (CAMARENA, 2017). The choice of these leaders was made through a questionnaire, adapted from Camarena (2003) and shown in detail in Lopes (2021). The questionnaire was applied by the teacher and consists of 80 Yes/No questions about the way students see themselves and how they make choices in different areas of their lives. Their answers are directly related to their learning style, that is, to their preferences or personal tendencies to use some strategies to the detriment of others when they want or need to learn something (BARROS, 2008; HERNÁNDEZ; ALONSO, 2013).

To facilitate the exploration of the guiding axes of MoDiMaCo, the work with the contextualized event was organized in two situations, each with three activities carried out in six face-to-face meetings. Each activity lasted four hours approximately. During the six meetings which took place in different environments (classroom, laboratory of construction materials and library), data were collected through audio recording of the discussions, participant observation, written records registering the development of the activities, and notes taken by the researcher during the activities.

Situation I, called **"Thermal comfort in a building"**, served to build theoretical foundations for the resolution of the contextualized event. For Activity I, the researcher took her students to the laboratory and presented the event to them, as shown in Figure 1. The three walls previously built for the experiment were also shown to the students. After making their initial conjectures about how to answer the contextualized event, in Activity II of Situation I, the students were taken to the library to conduct some research aiming to answer questions which required basic knowledge on heat transfer that would also serve as a theoretical foundation for the resolution of the event. Students were expected to learn that heat transfer by conduction is governed by an equation, called Fourier's law, which involves a rate of change. As one can read in Lopes (2021):

> [...] for a one-dimensional flat wall with a temperature distribution T, the rate of transfer equation is $\ddot{q_x} = -k\frac{dT}{dx}$, where the thermal flow $\ddot{q_x}(W/m^2)$ is the rate of heat transfer in the x direction per unit area perpendicular to the direction of transfer, proportional to the temperature gradient $\frac{dT}{dx}$, in this direction. K is the thermal conductivity of the material. Its values vary in a wide range depending on the chemical constitution, physical state, and temperature of the materials. The sign on the second member of the

> equation $\dot{q_x} = -k\frac{dT}{dx}$ is negative because heat is transferred in the direction of decreasing temperature (LOPES, 2021, p. 123).

During Activity II, the groups were asked to write at least three questions to be made to the teachers who would participate in Activity III. In the latter activity, the doubts and questions brought by the students were object of reflection of a class co-taught by three teachers: the researcher, a physics teacher, and a heat transfer teacher.

In Situation II, called **"Performing the experiment and solving ODEs"**, Activity I aimed to provide students with a real civil engineering experience: in the laboratory, they installed temperature sensors on the walls and collected data on the thermal behavior of each of them. To do this, the researcher previously built a thermal chamber with internal dimensions 60 cm x 40 cm x 40 cm, with one of the 40 cm x 40 cm faces being hollowed out. For its manufacture, plywood, 50 mm expanded polystyrene, laminated paper, two lamps, one dimmer (device also known as a brightness regulator, which serves to regulate the brightness of the light), and hardware were used. The plywood was screwed, forming the shell of the chamber, and its interior was coated with expanded polystyrene lined with laminated paper. To measure the temperature at each point of the wall, the wall under study, properly instrumented with the temperature sensors, was coupled. At the open end of the chamber, at this coupling, the center of the wall coincided with the center of the lamp. This experiment is deemed relevant because it uses real walls, constructed with the proper materials; in other words, it is a real, not virtual, experience, with non-simulated temperature behavior.

In Activity II, having reached the conclusion that Fourier's law is used to determine the heat rate of transfer per unit area, students were asked to solve an ENADE[2] question related to this subject matter. One of the reasons why this question was proposed was to explore Fourier's law and to construct the concept of ODE of separable variables. The question portrays the experiment conducted with the walls and requires important knowledge, such as: property of materials, heat transfer by conduction, permanent regime, and the general equation of conduction – Fourier's law. It also involves knowledge about thermal properties of materials, fluid mechanics, transformations of physical units, and it basically explores two types of materials: ceramics (concrete and its constituents) and polymers (wood and its constituents).

After solving the question, the researcher asked students if there was any function that could provide the temperature at any point along the width of the wall that was being considered. This questioning was meant to trigger the need to resolve an ODE.

Lastly, in Activity III, through a dialogued lecture, the researcher formalized the notion of ODE and some characteristics of this type of equation in a decontextualized way:

> [...] the differential equations were classified considering the number of independent variables of the function, the number of unknown functions, and the structure, order and degree of the equation. We also defined general solution and particular solution and checked if a function was a solution of the given equation. We then transformed a normal equation to a differential equation and vice versa, identified the various types of differential equations, and solved differential equations of separable variables. (LOPES, 2021, p. 209).

In a second moment of Activity III, the groups solved some ODEs and these resolutions were discussed collectively. Afterwards, they resumed the resolution of the contextualized event, analyzed the data collected in the laboratory and, based on the work developed, answered the central questions of the event.

The ideas proposed by Camarena (2000, 2012) were considered throughout the creation and implementation of the event, for just as the contexts of other sciences give meaning and significance to mathematics, it is believed that mathematics gives meaning to the themes and concepts of other sciences. The contextualized event was approached to consider the epistemological perspective of interdisciplinarity, in which some knowledge of calculus and heat transfer are intertwined in a network, maintaining relationships between them, giving meaning to each other.

The researcher, for the development of her doctoral dissertation, did not adhere to the teaching phase proposed by TMCS. The cognitive phase, however, was considered for data analysis articulated to Vergnaud's TCF, as detailed in the following section.

COGNITIVE PHASE OF TMCS AND TCF: ASSUMPTIONS FOR THE ANALYSIS OF DATA OBTAINED IN THE IMPLEMENTATION OF THE CONTEXTUALIZED EVENT

Both the assumptions of the cognitive phase of TMCS and Vergnaud's TCF served as the basis for the analysis of the data collected by the researcher in the context of implementation of the event.

According to Moreira (2002), although TCF has originated from the studies Vergnaud conducted with children focusing on the conceptual fields of additive and multiplicative structures, it is neither specific to these fields nor to mathematics. In his words, "In physics, for example, there are several conceptual fields—such as mechanics, electricity, and thermology—that cannot immediately be taught either as systems of concepts or as isolated concepts." (MOREIRA, 2002, p. 8). This also holds true for biology, history, geography, physical education, among others. Thus, despite its contextual origin, TCF is currently used for cognitive analysis with subjects of different educational levels. This is confirmed by Cunha and Ferreira (2020). Through a systematic literature review which searched articles published in national and international journals of science education between 2008 and 2018 and in the proceedings of the National Conference of Research in Science Education (Encontro Nacional de Pesquisas em Educação em Ciências) from 2007 to 2017, the authors aimed to analyze if and how TCF was used in analyses related to the learning of natural sciences (chemistry, physics and biology). As for the educational level, they observed the prevalence of TCF in higher education (28 of the 66 articles analyzed).

TCF is a theoretical framework for the study of concept learning and allows one to grasp how individuals construct knowledge and how it is organized within a cognitive structure. In accordance with Vergnaud (1983), knowledge is organized into conceptual fields, which are sets "of problems and situations whose treatment requires concepts, procedures, and representations of different, but closely related types" (VERGNAUD, 1983, p. 127). Teachers must provide students with problematizing situations, understood as tasks or sets of tasks that are meaningful to students and that, in a specific conceptual field, may vary in complexity. The goal is to develop potentialities for the emergence and acquisition of a concept and its structure.

Teaching activities should be diversified to motivate individuals to apply knowledge related to a given concept in various situations and test their explanatory models in various contexts. According to Vergnaud (1993, 1997), the construction of concepts involves three sets symbolically indicated by S I R, where: (S) is a set of situations that give meaning to the concept; (I) it is a set of operational invariants associated with the concept, which represents what is preserved in it and allows it to be recognized and operationalized in different situations; and (R) is a set of symbolic representations that can be used to indicate and represent the invariants.

Although it is clear that the construction of the conceptual field related to ODE requires students to work with a wide variety of situations and that the researcher has only worked with one contextualized event, she resorted to TCF, which can be found in the cognitive phase of TMCS, to analyze her data because she understood that the implementation of this event could trigger the beginning of the construction of fundamental knowledge related to concepts of this field and the conceptual field of heat transfer.

Corroborating Camarena and Muro (2012), this investigation considered the contextualized events of TMCS as equivalent to the situations worked in TCF. In more specific terms, given the case at hand, the situations are similar to that category of situations in which individuals do not have all the necessary competencies for solving them. This incompetency leads them to a process of reflection and exploration in which different schemes, that is, invariant organizations for a certain class of situations, will be accommodated, separated or combined, resulting in the construction of new schemes for new situations.

According to Vergnaud *et al.* (1990) and Vergnaud (1983, 1993, 1997, 2009), the operational invariants are responsible for directing the recognition, by individuals, of the elements pertinent to the situation and, therefore, guiding the construction of mental models. The operational invariants constitute the implicit conceptual basis that allows to obtain appropriate information and to infer the most appropriate rules of action from such information and the objectives to be achieved. Two types of operational invariants are identified: **concepts-in-action** and **theorems-in-action**. The concepts-in-action are an object, properties and relations or a category of thought considered relevant among those that make up the repertoire of the subjects that will be selected for a certain action. Theorems-in-action, on the other hand, allow individuals to articulate propositions that can be true or false. Both concepts-in-action and theorems-in-action mostly remain implicit in the actions performed by those individuals, however they can also become explicit. A concept-in-action is not a concept and a theorem-in-action is not a theorem, because, in science, concepts and theorems are explicit and their accuracy can be discussed and disputed. Concepts-in-action and theorems-in-action can nonetheless progressively become true scientific concepts and theorems.

According to TCF, one way to analyze the knowledge-in-action (concepts-in-action and theorems-in-action) subjects carry is monitoring the various moments in which they are asked to give answers to problems, the strategies adopted when solving a problem, the schemes used, and the mental models built to tackle new situations. Looking into the work of students with a contextualized event, Camarena and Trejo (2011, p. 138) contend that a cognitive action focuses “on the thought operations students make of the operational invariants in the schemes that are constructed, which directly or indirectly affect knowledge about the structure of the sciences that are linked to the contextualized event”.

Therefore, interested in this cognitive analysis, the first author of this article, in her doctoral dissertation, chose to use TCF as a foundation for the investigation of the operational invariants adopted by students in situations of resolution and discussion of the contextualized event and during the choice of schemes used to develop it. The invariants, that is, the theorems-in-action and concepts-in-action related to calculus and heat transfer, were analyzed by monitoring the moments in which students were asked to give answers to problems, the strategies used when solving a problem, the schemes used, and the mental models built to tackle new situations arising in the development of the CE. This analysis allows us to create a framework that fosters the understanding of the affiliations and ruptures between knowledge produced in physics and in calculus, as well as shows the temporal evolution of the explanatory models produced by the students in relation to their learning of ODEs, inferred from the concepts-in-action and theorems-in-action used in the activities.

Lopes and Lima (2021) describe some concepts-in-action and theorems-in-action related to calculus that were utilized by the participants of the present research (LOPES, 2021). The concepts brought to the fore are seen to significantly impact how students cope with situations related to ODEs. This article, beyond synthesizing the analyses presented in Lopes and Lima (2021), highlights other operational invariants related to both mathematics and physics as identified by Lopes (2021) in her research.

OPERATIONAL INVARIANTS RELATED TO BASIC MATHEMATICS AS EXPLAINED BY THE STUDENTS WHEN WORKING WITH THE CONTEXTUALIZED EVENT

The development of the contextualized event brought some contributions to the learning of ODE of separable variables to civil engineering undergraduate students. Amongst them, the construction of knowledge related to concepts of basic mathematics stands out. As one can find in Lopes (2021), there was an objective, a final goal, and a problem to be solved; all this served as a driving spring for the search for knowledge. In this process, all kinds of questions related to basic education mathematics, physics, calculus and heat transfer could be discussed according to the demands and interests of the students throughout the resolution of the event. This section then sheds light to some operational invariants related to basic mathematics mainly in relation to the notion of function, its graphical representation, dependent and independent variables and logarithm.

It is widely known that one can represent a function verbally (by describing it with words), numerically (by using a table of values), visually (by designing a graph), and algebraically (adopting an explicit formula). The four ways of representing a function can be observed in a dialogue portrayed in Lopes (2021), when one of the research participants asks: "teacher, when you say that it is a function of type $f(x) = ax + b$, are you talkin´ about the one where we draw a little table and add the points on the graph to draw a line?”. A short while later, another student confirms the information by saying: “that's right, if x is positive, the concavity goes upward and if it is negative, it goes downward”. Upon this statement, the student disputes: "'Wait up, man', the one you are referring to is the second-degree parabola, whereas the one under study is a first-degree line”.

When the first student mentions the "little table”, he uses the concept-in-action of the table of values in which, for each value of x defined, one consequently obtains a value for $f(x)$. He does not refer to the concept of function, but asks the researcher about the procedure needed to construct the graphical representation of a first-degree polynomial function. When talking about mathematics, the students use imprecise language that lacks clarity regarding basic knowledge, for instance, second-degree parabola and first-degree straight line. The degree refers to the polynomial associated with the algebraic representation of the function and not to the curve that graphically represents the function. The classmate who agreed with him also employs the concept-in-action that associates parabola with quadratic function, as well as line with affine function. At this point, he utilizes the theorem-in-action related to the quadratic function $f(x) = ax^2 + bx + c$, whose graphic representation is a concave upward parabola if $a > 0$ and a concave downward parabola if $a < 0$. The student does not mention the sign of a, but the sign of x, making it clear that he cannot understand the meaning of the literal terms of the algebraic expression of a function. It seems that he does not associate this idea only with the second-degree polynomial function, hence the term theorem-in-action. Probably, for this student, whenever $x > 0$ (and again it would not be x, but a), the parable of the equation $f(x) = ax^2 + bx + c$ is the graphical representation of the function whose concavity faces upward and if $x < 0$ (correctly $a < 0$), the concavity faces downward. In other words, he thinks this idea can be applied to any function.

In the same dialogue, a student asks: "Is this $\mathrm{ax} + \mathrm{b}$ the one related to the problem featuring a taxi that we saw in the placement test for Calculus, the one which has a fixed price for a ride and a varying part referring to the km driven?”. This student refers to a problem-situation that uses the affine function as a mathematical model for a taxi ride in which x is the number of kilometers driven and $f(x)$ is the price charged by the taxi driver. She uses the concept-in-action where b is the amount charged by the taxi driver (fixed amount) and a is the amount charged per km driven. As one can see, she does not refer to x as an independent variable and $f(x)$ as a dependent variable in $f(x) = ax + b$.

According to Vergnaud *et al.* (1990), the transformation of operational invariants in words and texts or in any other semiotic system (graphs, diagrams, algebraic notation, among others) is neither direct nor simple; there are important gaps between what is represented in the mind of an individual and the usual meaning of signs, since students can associate certain linguistic or graphic expressions with different meanings, resulting from their knowledge-in-action. This can be observed when, in $f(x) = ax + b$, the student associates the angular coefficient with the independent variable and the linear coefficient with the dependent variable. It also happens when, in $f(x) = ax^2 + bx + c$, the student does not consider the parameter a when he says that “if x is positive, the concavity goes upward and if it is negative, it goes downward".

In a different moment, while the students were solving the ODE $\frac{\mathrm{dx}}{\mathrm{dt}} = 3\mathrm{xt}^2$, one of them confessed that he had never understood the Neperian logarithm, to which his classmate explained: “This is logarithm to the base *e*. It's the same as an exponential. Look, take *e* and raise *t* to third power plus *c*", referring to the equation $\ln|x| = t^3 + C$. After reflecting upon the explanation offered by his classmate, the student answers: "if it is the same thing, then why do logarithms exist?” For the student who explained, the theorem-in-action is: logarithm to the base ***e*** and exponential are equivalent ideas. It is possible that he has learned that the logarithm is a mathematical operation directly related to the exponential function, but he does not seem able to distinguish both operations.

According to Vergnaud (1996a), to be meaningful for students, learning conditions must emerge from problem situations. In addition, teachers are responsible for identifying which knowledge their students explicitly have and which knowledge they can use correctly, but not yet to the point of being made explicit. In this sense, the question "why do logarithms exist?" was very important as it offered an opportunity to help the subject-in-action construct explicit and scientifically accepted concepts and theorems.

It was observed that many questions emerge when students are faced with a situation in which they need to put their knowledge into action, especially if they are in situations different from the usual school-type situations. Therefore, to progress in a conceptual field, it is essential that students can explore several situations that allow their operational invariants to be made explicit and their meanings to be negotiated.

OPERATIONAL INVARIANTS RELATED TO CALCULUS: A SYNTHESIS OF THE RESULTS PUBLISHED IN LOPES AND LIMA (2021)

Considering differential and integral calculus as well as fundamental elements for the construction of knowledge related to ODEs, one can notice different operational invariants being brought to the fore by students during their discussions with classmates and the researcher aiming at the resolution of the contextualized event.

To this analysis, two concepts-in-action concerning the relationship between the notions of rate of change and derivative are highlighted: **the derivative is a rate of change and a rate of change is derived**. Articulated to these ideas, one can find the following theorem-in-action: **if an equation involves a rate of change, then it is an ordinary differential equation**. Regarding these invariants, although recognizing the link between the notions of derivative and rate of variation is essential for an engineering student, we underline that it is of utmost importance that he/she can realize, through working with different situations, that a derivative is an instantaneous rate of change, thus not every rate of change is a derivative.

Considering the idea of instantaneous change, the following theorem-in-action stands out: **the instantaneous rate of change of x regarding t is always equal to the instantaneous rate of change of t regarding x.** As analyzed in detail in Lopes and Lima (2021), the theorem-in-action "writing $\frac{dx}{dt}$ is the same as writing $\frac{dt}{dx}$" and "finding x or finding t is the same thing; you have to find a letter, so it is indeed the same thing” brought to discussion by the students is conjectured to have originated from schemes used in certain situations but are not suitable for every case.

Implicit in these ideas, one can find schemes that were efficient in some situations involving polynomial equations but are no longer when applied to the work with ODEs. As Lopes and Lima (2021) point out, the following theorem-in-action is implicitly present in the statements: **since it does not matter which letter is used to denote a variable in an equation, if there are two letters in an equation, it does not matter to determine the value of one letter or another to solve the equation.** In addition to it, there exists the concept-in-action: **determining the roots of an equation means finding the values assumed by a specific letter, $\mathbf{x}$, so that equality is satisfied**. From these observations, Lopes and Lima (2021) conclude that situations in which the unknowns of an equation have been denoted by letters other than x or situations in which the letter x was present in the equation but not denoting an unknown were not sufficiently explored in the formative path of the student. He seems to have internalized that **being denoted by letter x** is an operational invariant of the concept of variable.

Still related to the notion of derivative, through a dialogue among the participants, one of them seems to have realized an operational invariant related to the determination of the derivative of a composite function when he said: “Teacher, the chain rule is applied when the function is composite, isn't it? I had not realized that before.” In other words, this assertion indicates a theorem-in-action that, at that moment, seems to have been internalized by him: *if* $h(x) = f(g(x))$, *then* $h'(x) = f'(g(x)) \cdot g'(x)$.

In her research, Lopes (2021) also identified that one of the research participants does not associate the calculation of an indefinite integral with an operation, but with a change in the formula, which develops and modifies itself from a symbolic point of view. When debating with his peer, the student mentions: "it is the same formula, it keeps evolving, and then it changes its symbol. It works like the integral: when we solve the calculation, it loses the symbol of both s and dx. Then the constant C enters the scene". The knowledge-in-action related to such idea is that: **at the end of the calculation of the indefinite integral of a function of x, neither the integration symbol nor the dx will be present in the result obtained, but a constant C will**. It becomes clear, therefore, how much the non-attribution of meanings to a mathematical object can impair students' understanding of it.

The non-attribution of meanings is also evident when one student argues with the researcher about the method of solving an ODE of separable variables: "you told us that we should integrate with x on one side[3] and with y on the other. That is what we did, so this dx and this dy change nothing." This assertion points to the following theorem-in-action: **Given that in notation $\int \mathbf{fdx}$, the element $\mathbf{dx}$ does not indicate anything about the mathematical operation to be performed, then $\int \mathbf{fdx} = \int \mathbf{fdy} = \int \mathbf{fdz} = \int \mathbf{fdt}$... etc**. Once the student does not understand the meaning of the elements dx and dy, he resorts to the aforementioned theorem-in-action and to the concept-in-action **solving an ODE of separable variables means integrating a function of x into a member of the equation and integrating a function of y into another member of the equation** to reach the conclusion that as $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{y}-1}{\mathrm{x}+3} \Leftrightarrow (\mathrm{x}+3)\mathrm{dy} = (\mathrm{y}-1)\mathrm{dx} \Leftrightarrow \int(\mathrm{x}+3)\mathrm{dy} = \int(\mathrm{y}-1)\mathrm{dx}$, then if one integrates the left member of the last equality in relation to x and integrates the right member in relation to y , the last equality will remain valid because he will be adopting the scheme that, to his knowledge, gives operability to the method of solving ODEs of separable variables: **integrating a member of the equation in relation to $\mathbf{x}$ and the other in relation to $\mathbf{y}$.**

Associated with the idea of what it means to solve an ODE, we observed yet another concept-in-action that emerged from a student questioning "in the equation of Fourier's law, which value do we determine: x or T?", which is: **determining the roots of an equation means finding numerical values that make the equality true**. This is, in fact, an operational invariant that helped subjects solve the equations they had encountered until then. However, in situations involving ODEs, new schemes need to be elaborated. Such fact appeared to be already taking place at the time of the implementation of the contextualized event, when another participant stated: "Neither x nor T: we need to find a function". This statement shows the concept-in-action: **the resolution to a differential equation lies in a function**.

The last operational invariant mentioned in Lopes and Lima (2021) regards the degree and order of an ODE. One of the participants tells a classmate*:* "It is the same thing as the equations we already know, the first-degree equation has one root, the second-degree equation has two roots, and so on." Such assertion reveals a theorem-in-action that stems from knowledge that the student could correctly mobilize in situations involving polynomial equations, but that needs to be reviewed if one works with ODEs: **if an equation has degree n, then it has n roots**. This theorem-in-action is linked to the concept-in-action: **the number of roots of an equation is given by the degree of the equation.** Lopes (2021) discusses that, during the implementation of the contextualized event, while all the valid knowledge-in-action was deemed as such by some students including in situations where a differential equation needs to be solved, another student was aware of the need for more information to have them assessed and incorporated into situations involving ODEs.

OPERATIONAL INVARIANTS RELATED TO PHYSICS EMPHASIZED BY THE STUDENTS WHEN WORKING WITH THE CONTEXTUALIZED EVENT

In addition to contributions related to basic education mathematics and calculus, as shown in the previous sections, the development of the contextualized event also contributed to the construction of knowledge related to physics concepts. This section then presents the analysis of some data related to the knowledge-in-action on heat transfer, temperature, heat and thermal conductivity, brought to the fore by the students when discussing and investigating the thermal behavior of three configurations of masonry walls during a laboratory practice in the work with the event.

When the contextualized event was in its very beginning, the students realized they needed to study some concepts fundamental for its resolution. This was observed when a student admitted: "I read about the three heat transfers, gradient, thermal flow, conductivity; there is even a table of this conductivity. I do not know how to explain, but I know that this subject matter is important in our course". When the student mentioned the "three heat transfers", he was referring to the modes of heat transfer (conduction, convection and radiation), concepts learned in physics during basic education. This dialogue showed that the students were resuming knowledge they already mastered to build new knowledge in higher education, especially knowledge related to gradient, thermal flow, conductivity and the conductivity table, studied in basic education and deepened in the course of Heat Transfer in tertiary education.

Data made evident that many students confuse heat with temperature. When asked about the properties of styrofoam as a thermal insulator, for instance, one student replied that “styrofoam demands a very high temperature to start exchanging heat with the environment; therefore, as it has a low specific heat, you have to subject it to either high or too low a temperature to start exchanging heat”. In the same dialogue, a group of students concluded that the first wall (coated with sand and cement) absorbs much heat, because its materials hold a high capacity to retain temperature; the second wall (coated with plaster) can also retain temperature, but the styrofoam-coated wall is the one that transmits lower temperature to the environment, thus ensuring greater thermal comfort. Another student reiterates that "styrofoam needs a very high temperature to start exchanging heat with the environment.” The students were then referring to heat transfer, which occurs when there is temperature difference in a medium or between media, that is, energy in transit due to a temperature difference. One of the students, as illustrated, talks about "retaining temperature “rather than retaining heat, and claims that styrofoam "has a low specific heat". These statements are incorrect when asserting that a body has heat, since it has thermal energy, as explained by the researcher during that dialogue. The student was seen to use the theorem-in-action regarding the coefficient of thermal conductivity, because he understands that styrofoam can be used as a thermal insulator (common sense perception). In fact, the student meant that styrofoam has a “low” coefficient of thermal conductivity, since the lower the coefficient, the greater the insulating capability.

Still on the notion of temperature and heat, during a dialogue among participants, one student demonstrated he had understood an operational invariant related to an important concept-in-action of physics regarding heat transmission. According to this invariant, whenever a body is at a higher temperature than another or yet when different temperatures exist in the same body, energy is transferred from the region of higher temperature to the region of lower temperature. The discussion about the possibility of heat transfer happening even across the same body, with distinct temperatures, took place when one of the research participants reported that "[...] heat transfer can only happen between two bodies with different temperatures". When analyzing the statements made by the students, one sees them as **subjects-in-situation** who, when faced with a variety of situations, make sense of concepts from physics, extending and redirecting these concepts to heat transfer.

Additionally, some important knowledge-in-action is advanced by students in a problem situation in which they need to transform temperature measurements from Celsius to Kelvin. Even though the authors did not expect students to present difficulties in this subject matter, they noticed the participants did not understand the need for these transformations and had doubts about the meanings and notations of the temperature scales. To illustrate, one student revealed that he had never understood the conversions of the temperature scales, but remembered that the Celsius scale had to do with the temperature of cold and hot water. His classmate explained that the Kelvin scale does not have negative numbers. During this dialogue they remembered the conversion tables. This was when one of them reiterated: “I do know the tables and how to do the conversions. My point is that I do not understand why it is the way it is. I remember I was never explained why the Celsius and Fahrenheit scales have degrees (°C and °F), but the Kelvin scale does not”.

This excerpt highlights important knowledge-in-action considered by students. When he remembers that the Celsius scale has to do with the temperature of cold and hot water, he is referring to meaningful content of physics regarding the Celsius scale, which has as reference points the freezing temperature of water under normal pressure (0 °C) and the boiling temperature of water under normal pressure (100 °C). The student who stated that there is no negative number on the Kelvin scale highlighted an important concept-in-action on this scale, because it is used to measure the absolute temperature of an object and is calibrated in terms of energy. As energy is a positive magnitude, there are no negative temperatures. Therefore, zero is the lowest possible temperature, called absolute zero or zero Kelvin.

All these questions, in addition to the doubt about the Kelvin scale in terms of degrees, were considered very important for they generated a scenario “[...] that allowed to understand the affiliations and ruptures among diverse knowledge” (VERGNAUD, 1996b, p. 8). Data analysis suggested that when students are faced with a situation that requires them to mobilize their knowledge, many questions end up emerging, especially if they are in situations different from the ones they are used to experiencing. The questions regarding the scales used to measure temperature revealed that the students learned the proper content without understanding why it came to be as it did. In this sense, Vergnaud (1996a) states that teachers must provide students with problematizing situations that are meaningful to them, which shows that, in each conceptual field, there is a wide variety of situations and the knowledge students hold is shaped by the situations that they progressively master. Ergo, the situations give meaning to concepts, becoming the entry point for a given conceptual field. As previously stated in this text, however, a single concept needs a variety of situations to become meaningful in the same way that a single situation needs several concepts to be analyzed.

FINAL REMARKS

According to the data collected and analyzed, the contextualized event constructed and developed in the research at hand is believed to have potential to favor the learning process of ODEs. Besides providing students with content that extrapolated ODE mathematical formulation and algebraic resolutions, it served to integrate mathematical and non-mathematical courses, namely, Calculus and Heat Transfer.

For the cognitive analysis of ODE learning by students, TCF was used and formed the basis for investigating the operational invariants applied by students in situations of resolution and discussion and during the choice of schemes considered in the development of the contextualized event. These invariants, i.e., the theorems-in-action and concepts-in-action related to basic mathematics, calculus and heat transfer, were analyzed through the study of the reflections shared by the students when facing the emergent situations occurred during the development of the event.

As explained in detail in Lopes and Lima (2021), data analysis allowed us, on the one hand, to identify concepts-in-action and theorems-in-action that were valid in situations involving the resolution of polynomial equations and that helped students successfully answer the proposed questions. On the other hand, when tackling situations regarding differential equations, they realized the invariants needed to be adapted, reassessed, recombined, and even discarded. It also shed light to other operational invariants related to physics, heat transfer or aspects of basic mathematics that, although not having as much impact on the study of ODEs as those related to calculus, can provide teachers with indications about the cognitive development of their students when they experience an approach to mathematics that considers extra-mathematical contexts.

This study also underlines the importance for teachers to acknowledge the schemes and operational invariants related to other areas besides mathematics which are applied by their students to solve a contextualized event. Aware of that, teachers can properly plan an approach to contribute to their students learning, in this case, ODEs. In these lines, Vergnaud (1994) points to the role teachers are called to play, which entails identifying what explicit knowledge their students have and what knowledge they use correctly, but have not yet mastered to make it explicit. He also recommends that learning conditions should emerge in problem situations as to become meaningful for students.

The results described suggest that the contextualized event fostered a favorable learning environment, with many questions and inquiries about concepts of basic mathematics, physics, and calculus, all essential for the learning of ODEs. To end this article, we invite new studies which articulate the theoretical framework of TMCS and TCF, as well as point to the need for further investigations aimed at constructing the conceptual field of ODE in engineering degrees through a variety of problem situations (which may be contextualized events), given that Lopes (2021) has focused on the study of ODEs of separable variables. Finally, we suggest investigations that consider the conceptual field of partial differential equations (PDE) in engineering degrees.

INVARIANTES OPERATÓRIOS EM ESQUEMAS UTILIZADOS POR FUTUROS ENGENHEIROS EM UMA ABORDAGEM CONTEXTUALIZADA DE EQUAÇÕES DIFERENCIAIS ORDINÁRIAS

RESUMO

Este artigo, recorte de uma pesquisa concluída de doutorado, de natureza qualitativa, que aborda a aprendizagem de Equações Diferenciais Ordinárias de variáveis separáveis, tem como objetivo explicitar e analisar, a partir da Teoria dos Campos Conceituais, alguns invariantes operatórios acerca de conhecimentos vinculados a conceitos da Matemática, Física e Cálculo mobilizados por um grupo de 21 estudantes do segundo período de Cálculo do curso de Engenharia Civil de uma instituição particular durante a resolução de um Evento Contextualizado (problema integrando diferentes áreas de conhecimento). A coleta de dados foi feita por meio de gravações em áudio das discussões realizadas durante o trabalho com o evento, implementado segundo os preceitos teórico-metodológicos da Teoria A Matemática no Contexto das Ciências. Dentre outros, evidenciam-se a mobilização, de invariantes operatórios relacionados às noções de: função, representação gráfica de uma função, variável dependente e independente e logaritmo (Matemática básica); taxa de variação, derivada, integração como uma operação, variável em uma equação diferencial, ordem e grau de uma equação diferencial (Cálculo); transferência de calor, temperatura, calor e condutividade térmica (Física). Além disso, a análise dos dados evidencia que, na sala de aula, momentos de discussões conjuntas e de interações entre os estudantes deveriam ser mais presentes por colaborarem na promoção de suas aprendizagens e de seus desenvolvimentos cognitivos.

PALAVRAS-CHAVE: Educação Matemática no Ensino Superior. Teoria A Matemática no Contexto das Ciências. Teoria dos Campos Conceituais. Engenharia. Invariantes Operatórios.

NOTES

1. According to Camarena (2021), knowledge transfer is the ability of an individual to use a certain constructed knowledge to solve problems in a diversity of contexts in which such knowledge is required.

2. The Brazilian National Student Performance Examination (Enade) assesses the learning outcomes of higher education students in relation to the curricular components found in the curricular guidelines of their undergraduate programs, the development of competencies and abilities necessary for strengthening general and professional training, and the level of educational expertise brought by students regarding national and global realities.

3. When referring to *members* of an equation, the student uses the term *sides of the equation*, revealing that he has not yet mastered a rigorous mathematical language.

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