

Mathematical connections through problem solving: a study in higher education

ABSTRACT

Mathematics is characterized by different connections that can be formed between intra- and extra-mathematical fields; these connections are vital for understanding new concepts. In this article, we present the initial results of a study that focused on the construction of mathematical concepts through the establishment of connections. Students in a Mathematics Education Undergraduate Program enrolled in the Fundamentals of Analysis curriculum course participated in the study. During the study, we analyzed the connections made by students when solving problems that involved concepts used during the discipline, such as continuous functions and intermediate value theorem, among others. The study was conducted in the context of the COVID-19 pandemic. We used a problem solving methodology for classroom work and a qualitative research approach with data obtained from the work produced by students and their observations. As a result, we found that the study contributes to understanding the learning of different mathematical concepts and the types of connections used, as shown through the procedures developed by the students to solve the proposed problems.

KEYWORDS: Connections. Mathematics Education Undergraduate Program. Teaching. Fundamentals of Analysis. Problem Solving.

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INTRODUCTION

Improving the processes of teaching and learning mathematics has become a central focus of research in the field of Mathematics Education. Increasingly, attempts are being made to create environments and propose activities that enable students to build connections between the studied concepts. As highlighted by Allevato and Onuchic (2019), connections are one of the foundational perspectives of current teaching guidelines. However, the question remains as to what exactly we are referring to when we discuss the concept of connection in mathematics?

According to Canavarro (2017), two transversal ideas are found in the literature that conceptualize connections. The first of these considers the diversity of connections, including those between mathematics and a range of other areas of knowledge, such as medicine, arts, physics, among others. Moreover, connections can be made within mathematics itself, among content from different areas, such as arithmetic and geometry, for example, or between concepts and procedures. The author also points out the existence of connections related to the different stages of development of mathematical concepts that, in general, are given less emphasis.

The second idea refers to the purpose of connections, which is to expand on the understanding of thoughts and concepts involved, enabling students to perceive mathematics as a coherent and interrelated discipline, not just a collection of rules (CANAVARRO, 2017).

Similarly, De Gamboa (2015) points out that mathematics is characterized by strong links between all of its different fields, which are decisive elements in the development of new mathematical knowledge. He also points out that mathematics can be applied to various fields of knowledge, allowing one to analyze, interpret, and operate across different processes. As such, mathematics employs multiple concepts and procedures.

Regarding school mathematics, De Gamboa (2015) also reinforces the existence of different connections between the concepts used. The author emphasizes that students must establish relationships between the different concepts that are being developed.

Similar to the arguments made by these authors in terms of understanding the importance of working with connections, Allevato and Onuchic also raise the following questions:

How can teachers enable students to have experiences that allow them to perceive and establish these connections? How can we train or prepare teachers to carry out this work with their students in the classroom? (ALLEVATO; ONUCHIC, 2019. p. 2).

In light of these concerns, we see a promising path that involves working with the concept of connections during the initial training of mathematics teachers. This article builds on a presentation given at the International Seminar on Research in Mathematics Education - VIII SIPEM, held in 2021. During this event, we presented and discussed early findings of the study with researchers in the area, further enhancing the analysis.

Here, the objective is to present the initial results of a study conducted with Undergraduate Mathematics Education students enrolled in the Fundamentals of Analysis curriculum course. Throughout the course, connections made in the solution of two proposed problems were analyzed. The problems required the use of concepts such as continuous functions and intermediate value theorem, among others. The study was carried out during the COVID-19 pandemic, and the methodology used was problem solving.

In the sections that follow, the theoretical basis of the study are considered, and the connections that students created when solving the proposed problems are analyzed.

MATHEMATICAL CONNECTIONS

Certain concepts related to mathematical connections have already been highlighted in the introduction, particularly the research of Canavarro (2017). To expand the theoretical framework, it is important to consider other researchers who have dedicated themselves to the subject, including Businkas (2008) and Flores and García-García (2017). Several aspects of these authors' studies are highlighted below to explore the different types of connections that can be established.

Businkas (2008) points out that mathematical connections are referred to in a variety of ways in the literature, namely: a relationship between mathematical ideas, making clear that connections exist independently of the learner; a relationship built by those who learn mathematics, or a process that is part of the activity of doing mathematics. In addition, there are also discussions about the role of connections in both teaching and learning, as well as how these connections are built in order to better understand the theme.

Flores and García-García (2017) propose an approach that is consistent with Businkas (2008). On one hand, these scholars argue that connections are the relations that structure mathematics and are independent of the student. On the other, it is through the relations that connect thought processes that mathematics is constructed. In this sense, the cited authors classify mathematical connections into two types: intra-mathematical connections and extra-mathematical connections.

Intra-mathematical connections are those established between concepts, procedures, theorems, arguments, and within mathematical representations. Extra-mathematical connections, meanwhile, establish relationships between a mathematical concept or model and a problem from a (non-mathematical) context or vice versa.

When it comes to intra-mathematical connections, Businkas (2008) presents a set of categories, defining connection as the true relationship between two mathematical ideas, A and B. Each of the categories is described below with some examples to provide an illustration.

Different representations: this category includes alternative representations and equivalent representations. It is said that A is an alternative representation of B if both are expressed in two different ways (verbal-algebraic, algebraic-geometric, among others). For example, the graph as a line is an alternative

representation of $f(x) = ax + b$ (BUSINSKAS, 2008). On the other hand, A is an equivalent representation of B, when both are expressed in two different ways, yet using the same representation (e.g., $f(x) = (x - 2)^2$ and $f(x) = x^2 - 4x + 4$) (FLORES; GARCÍA-GARCÍA, 2017).

Part-whole relationship: A is included in (is a component of) B, or else B includes (contains) A or is a generalization of A. In this case, there is a hierarchical relationship between two concepts. For example, a vertex is a component of a parabola, or the parabola contains a vertex, or even $f(x) = ax + b$ is a generalization of $f(x) = 2x - 4$.

Implication: this category includes logical relations, such as A implies B, among others. This connection indicates dependence between the concepts in a logical way. For example, the degree of an equation determines the maximum number of possible roots.

Procedure: A is a procedure used when working with object B. One example is a tree diagram, which is a procedure used to describe a sample space (probability).

Connections between mathematical concepts: these are identified when a student relates concept A to concept B, either to argue about his/her answer to a given problem or to explain concept C. This category appears in the work of Flores and García-García (2017), which, in addition to the categories defined above, is also included in intra-mathematical connections.

Finally, extra-mathematical connections establish relationships between mathematical content with other areas of knowledge or with everyday situations. In this case, Flores and García-García (2017) highlight modeling as a type of extra-mathematical connection. This happens when the student builds a mathematical model to solve a problem applied to a specific context; based on the model, the student incorporates a range of knowledge and various procedures to arrive at his or her answer.

These categories served as the basis for the analysis of the student problem solving work used in this study.

PROBLEM SOLVING

In Mathematics Education, both in Brazil and in other countries, problem solving has been the focus of several studies. The emphasis initially was on the use of models and strategies in problem solving. However, as research in the field has developed, "Problem Solving began to be thought of as a teaching methodology, a starting point and a way to teach mathematics" (ZUFFI; ONUCHIC, 2007, p. 81).

Building on this understanding, Onuchic and Allevato (2011) conceive of working with mathematics through problem solving using the concept of teaching-learning-evaluation. Thus, they understand it as a methodology that considers three elements that occur simultaneously: the teacher teaches, the student learns, and assessment is carried out by both. Furthermore, the authors point out that:

In the Teaching-Learning-Assessment Methodology of Mathematics through Problem Solving, the problem is the starting point and, in the classroom, through problem solving, students must make connections between different

areas of Mathematics, generating new concepts and new content. (ONUCHIC, ALLEVATO, 2011, p. 81).

There is no strict way of working with problem solving in the classroom. However, some steps are important to follow for those who want to use this methodology. To this end, Onuchic and Allevato (2011) present a script consisting of the following steps: preparation of the problem; individual reading; reading in a group; problem solving; observation and encouragement; recording of the solutions on the blackboard; discussion and consensus building.

When analyzing how problem solving is currently conceived and the steps used to conduct it, we can see how it relates to the formation of mathematical concepts, the stimulation of thought, and the construction of mental structures to enable students to solve the proposed problem. Therefore, the use of this methodology is conducive to the establishment of mathematical connections, justifying its use in this study.

METHODOLOGICAL PROCEDURES

A qualitative approach was used in this research as it asks particular questions, involving meanings, values, and attitudes, and provides a space in which we can profoundly explore processes and phenomena (MINAYO, 2001). During classroom work, the problem solving steps were followed as detailed in the previous section.

Participants of the study included eight students from an Undergraduate Program in Mathematics Education at a Federal Institute of Education, Science, and Technology. Students were grouped into four pairs, identified as D1, D2, D3, and D4. Data collection was based on the written work produced by the participants, obtained during the solving of two chosen problems.

In the following section, the respective solutions presented by the students for the proposed problems are presented. These, in turn, are accompanied by discussions and reflections, seeking to identify the mathematical connections made.

DATA ANALYSIS

The first stage consisted of choosing the problems, detailed below, which were presented to the students individually. This step was to ensure that each student had contact with, and read, the problem. Students were then organized in pairs for discussion and problem solving.

As the research occurred during the COVID-19 pandemic, the students used online platforms such as Google Meet to hold meetings, interact, and discuss the solution paths and the concepts involved. At this stage, we provided support to each pair of students, encouraging discussions and the development of steps to solve the problem. This was also done via the online platform or through messaging applications.

In what follows, we describe and analyze the steps taken by the pairs in solving the problems. As mentioned, the first problem presented below was discussed in the paper given at the VIII SIPEM event in 2021. However, the second problem was

not included in that paper, and what follows is an expansion on the previously presented results.

Table 1 shows the first problem. After presenting the problem, the connections made by the students are detailed considering the categories defined above and based on their attempts to solve the problem.

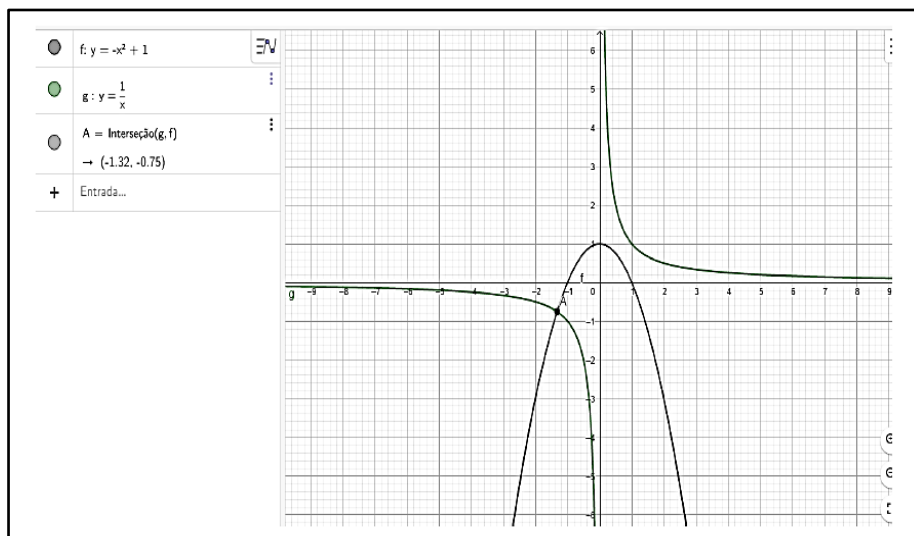
Table 1 - Statement of the first proposed problem

Calculate the coordinates of the intersection points of the parabola equation $y = 1 - x^2$ and the hyperbola equation $y = \frac{1}{x}$

Source: Adapted from Santos (2018).

Different Representations: this type of connection was identified in the work produced by pairs D1 and D3. These two pairs moved from the algebraic representation that the problem proposes to a graphic representation. For this, they used the software GeoGebra, as seen in Figure 1. The intersection point can be seen as point A in the figure.

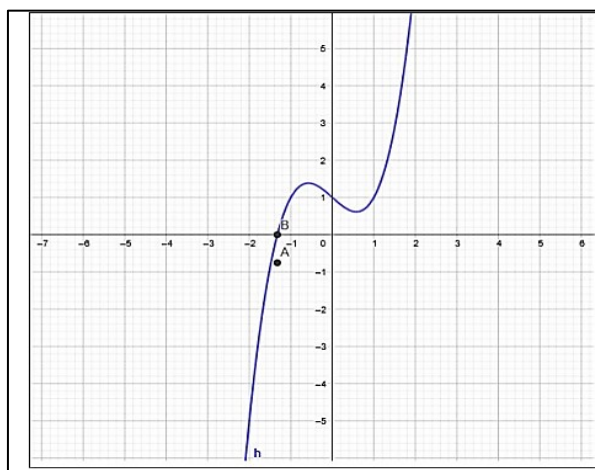
Figure 1 – Graphic representation of problem produced by D3



Source: Research data (2020).

In addition, pair D1 also produced a graphic representation, in which the students clarify that, to find the common point, it is necessary to equate the equations. According to their work, from this, a single expression can be constructed, given by the cubic equation $x^3 - x + 1 = 0$. In this case, again, they resorted to graphic representation to interpret the situation, as shown in Figure 2.

Figure 2 – Graphic representation of the cubic equation of problem 1 by D1



Source: Flôres and Bisognin (2021, p. 766).

It was not possible to identify different representation connections in the solutions provided by pairs D2 and D4.

Procedure: this connection was identified in the work of all pairs. After finding the cubic equation representing the problem, pair D1 concluded that to find the solution, they could use already known methods, as highlighted in the following sentence transcribed from their solution protocol.

- Thinking about this idea, if we can determine the root indicated by point B, we can have an approximation of the x coordinate we are looking for. To determine this root, a very effective way is to use the bisection method.

Continuing along this rationale, they develop a table, in which they try to identify which interval contains the root. They then proceed with the bisection.

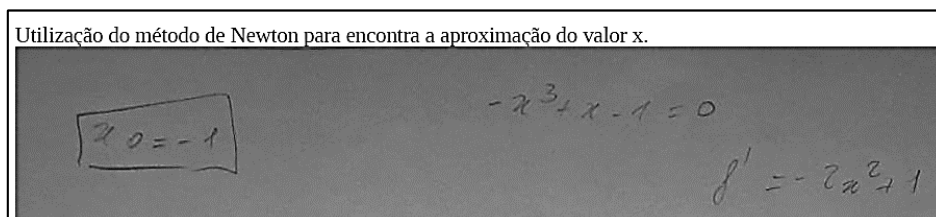
Figure 3 – Sign change table for problem 1 by D1

x	-3	-2	-1	0	1	2	3
y	-23	-5	1	1	1	7	25

Source: Flôres and Bisognin (2021, p. 766).

Newton's Method was also used to solve the problem, as mentioned by pair D3 and shown in Figure 4.

Figure 4 – First steps of Newton's Method used to solve problem 1 by D3



Source: Research data (2020).

Similarly, D2 and D4 used the bisection method, although D4 did not specify it as such in the presented solution.

Part-whole: connections of this type can be seen when the pairs D2 and D3 presented the reference to the intermediate value theorem as justification for the solution and as support for the bisection method used by the students.

Implication: pairs D2, D3, and D4 established implication connections when they related the degree of the equation with the number of roots. These pairs justified the fact that only one of the roots was the sought-after solution, as we can see in the excerpt below, extracted from D2 pair's protocol.

- As the situation will lead us to a third-degree polynomial, we will soon have three solutions. However, the root that satisfies this solution must belong to the set of real numbers. For it is the point of the intersection of the conics.

In this case, we can infer that the students presented implication connections, as they related the dependence of the concepts in a logical way.

Connection between concepts: in the resolution of the problem by the pair D1, we identified connections between concepts when the students presented in their written work different observed relations. For example, they observed the relations between the graph of the functions, point of intersection, and root of the equation. For the D3 pair, it is important to highlight the connections between concepts used: graph of functions, intersection point, coordinates, and root of the equation. The students mentioned in their written work the fact that the approximate values for x and y should be in the third quadrant, since the parabola was defined with $a < 0$, and the hyperbola was likewise defined in the third quadrant.

In the case of D4, the students presented connections between mathematical concepts when they highlighted the Rational Root Theorem to search for the root of the cubic equation, as we can see in Figure 5.

Figure 5 – Resolution of problem 1 by D4

$x^3 - 1 = 0$
 pois temos as raízes possíveis, então
 $a_0 = -1$ e $a_n = -1 \Rightarrow p/a_0$ e q/a_n
 $p = \{1, -1\}$ e $q = \{1, -1\}$ então
 $p = \{1, -1\} \Rightarrow p/x = 1; 1 - (1)^3 - 1 = 0$
 $1 - 1 - 1 = 0$
 $-1 \neq 0$
 $p/x = -1; -1 - (-1)^3 - 1 = 0$
 $-1 + 1 - 1 = 0$
 $-1 \neq 0$
 Então, podemos observar que o termo não
 funciona, pois os termos com as variáveis se
 anulam com valor 1.

Source: Flôres and Bisognin (2021, p. 769).

Next, we analyze the solutions given for the second proposed problem, presented in Table 2. It is important to point out that the pair D4 did not present a solution for this problem, so the analysis will focus on the other three pairs.

Table 2 – Statement of the second proposed problem

An item, with a cash price of R\$500.00, is sold in 6 equal installments of R\$120.00. What is the monthly interest rate being charged, knowing that the plan includes compound interest?

Source: adapted from Santos (2018).

Procedure: after deducing the expression that relates to the sought-after rate with the other data of the problem, pair D1 pointed out that it is a polynomial. Also, according to the pair, the solution would be to find the root of this polynomial. For this, the bisection method was used, as shown in Figure 6.

Figure 6 – Bisection method used in problem 2 by D1

n	a	b	xn	f(a)	f(b)	f(xn)	Erro
1	0,001	0,08	0,0405	-0,0987	2,453754	-1,65314	0,0395
2	0,0405	0,08	0,06025	-1,65314	2,453754	-0,41492	0,01975
3	0,06025	0,08	0,070125	-0,41492	2,453754	0,801161	0,009875
4	0,06025	0,070125	0,0651875	-0,41492	0,801161	0,140408	0,0049375
5	0,06025	0,065188	0,06271875	-0,41492	0,140408	-0,15021	0,00246875
6	0,062719	0,065188	0,063953125	-0,15021	0,140408	-0,00817	0,001234375
7	0,063953	0,065188	0,064570313	-0,00817	0,140408	0,065301	0,000617187
8	0,063953	0,06457	0,064261719	-0,00817	0,065301	0,028363	0,000308594
9	0,063953	0,064262	0,064107422	-0,00817	0,028363	0,010048	0,000154297
10	0,063953	0,064107	0,064030273	-0,00817	0,010048	0,000928	7,71484E-05

Source: Research data (2020).

D2 used concepts from financial mathematics to begin solving the problem, more specifically the formula that relates present value, installment, interest rate, and time (Figure 7). Subsequently, the pair used algebraic manipulation, finding a seventh-degree polynomial, which represents the situation presented.

Figure 7 – Resolution of the second problem by D2

Handwritten mathematical work on lined paper showing the derivation of a seventh-degree polynomial from a present value formula.

$$P = Pv \frac{(1+i)^n \cdot i}{(1+i)^n - 1}$$

$$\frac{120}{500} = \frac{(1+i)^6 \cdot i}{(1+i)^6 - 1}$$

$$0,24 = \frac{(1+i)(1+i)(1+i)(1+i)(1+i)(1+i) \cdot i}{(1+i)(1+i)(1+i)(1+i)(1+i)(1+i) - 1}$$

$$0,24 = \frac{i + 6i^2 + 15i^3 + 20i^4 + 15i^5 + 6i^6 + i^7}{(1+i)^6 - 1}$$

$$\times \Delta \quad 6i + 15i^2 + 20i^3 + 15i^4 + 6i^5 + i^6$$

$$0 = -0,44i + 2,4i^2 + 10,2i^3 + 16,4i^4 + 13,56i^5 + 5,76i^6 + i^7$$

Source: Research data (2020).

After finding the polynomial, this pair employed Newton's Method to determine the root. In addition, they highlighted the approximation error and mentioned that other methods could be used, such as the secant and bisection methods.

The procedure used by D3 to begin the process of solving the problem was to use an online calculator to calculate the rate, as shown in Figure 8.

Figure 8 – First steps in solving the second problem by D2

Financiamento com prestações fixas

Simule o financiamento com prestações fixas

Nº. de meses

Taxa de juros mensal %

Valor da prestação
(Considera-se que a 1ª prestação não seja no ato)

Valor financiado
(O valor financiado não inclui o valor da entrada)

Metodologia

O total desse financiamento de 6,00 parcelas de 120,00 reais é 720,00 reais, sendo 220,00 de juros.

Source: Research data (2020).

However, the students themselves highlight the question transcribed below, and from this, they go in search of a justification for the answer.

- But how to really calculate this and know where this calculation comes from?

When exploring the calculation, the students found an explanation about the method used to obtain the answer. Substituting the values given by the problem, they arrived at the equation highlighted in Figure 9.

Figure 9 – Resolution of the second problem by D2

Metodologia do Financiamento com Prestações Fixas

Cálculo com juros compostos e capitalização mensal.

$$q_0 = \frac{1 - (1 + j)^{-n}}{j} p$$

Onde:
 n = Nº de Meses
 j = Taxa de Juros Mensal
 p = Valor da Prestação
 q_0 = Valor Financiado

$$-1,84j + 9,96j^2 + 42,4j^3 + 68,2j^4 + 56,4j^5 + 23,96j^6 + 4,16j^7 = 0$$

Source: Research data (2020).

To find the value of the requested rate, the students used the bisection method, similar to pair D1.

Connections between concepts: in the case of pair D1, we identified the association of the presented problem with geometric progression. In this case, the students used the idea of capital equivalence associated with the sum of the terms of a finite Geometric Progression to start solving the problem.

Figure 10 – Initial resolution of the second problem by D1

$$120 \cdot \left(1 + \frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \frac{1}{(1+i)^4} + \frac{1}{(1+i)^5} \right)$$

Soma de termos de uma PG: $S_n = \frac{a_1 \cdot (1-q^n)}{1-q}$

Substituindo $a_1 = 120$ e $q = \frac{1}{(1+i)}$:

$$S_n = \frac{a_1 \cdot (1-q^n)}{1-q} \Rightarrow S_n = \frac{120 \cdot (1 - (\frac{1}{1+i})^n)}{1 - \frac{1}{1+i}} \Rightarrow S_n = \frac{120 \cdot (1 - (1+i)^{-n})}{i}$$

Source: Research data (2020).

In the answer given by D2, connections between concepts of financial mathematics, polynomials, and roots of polynomials were identified. Also, in the solution presented by the pair D3, connections between the concepts of financial mathematics and those related to the idea of zeros of a function were found, as seen in the excerpt below.

- After performing the calculation using the formula found in the Citizen Calculator, we reached a point where it is no longer possible to use the formula to develop the rest of the calculation. For this reason, the result found is transformed into a function, as shown in Figure 11.

Figure 11 – Function found by D3

$$f(x) = -1,84j + 9,96j^2 + 42,4j^3 + 68,2j^4 + 56,4j^5 + 23,96j^6 + 4,16j^7 = 0$$

Source: Research data (2020).

Part-whole relationship: D2 highlighted that it is the mean value theorem that guarantees that the solution to the problem exists. D3, on the other hand, presented an explanation based on the bisection method, highlighting the intermediate value theorem as a justification for the operation.

Implication: pair D2 established implication connections when the students related the degree of the polynomial with the number of roots and justified the fact that only one of the roots was the sought-after solution. This is perceived in the transcription below, taken from the students' solution protocol.

- In this case, the polynomial will have seven solutions (roots), real and complex. Of these, three will be real, but two will be negative, therefore as we are finding a value for interest, negative values cannot be accepted. Thus, the interval to find the solution should start from 0.1.

In Table 3, the range of connections that the students used when solving the problems are listed.

Table 3 – Connections observed in problem solving

Problem	Connection Type	D1	D2	D3	D4
P1	Different representations	X		X	
	Part-whole		X	X	
	Implication		X	X	X
	Procedure	X	X	X	X
	Mathematical concepts	X	X	X	X
P2	Different representations				
	Part-whole		X		
	Implication		X	X	
	Procedure	X	X	X	
	Mathematical concepts	X	X	X	

Source: Prepared by the authors.

Based on Table 3, we can see that pair D1 did not establish part-whole and implication type connections to solve any of the problems presented. Likewise, pair D2 did not present different representations to solve the problems, as did D4. D3 was the only pair that demonstrated the greatest variety of connections in their solutions. In the first problem solved by this pair, we identified all five types of connections.

When it came to the second problem, none of the pairs used different representations to solve it, something that had occurred in the first problem. It is possible that the nature of the problem influenced the students to use other connections in the search for a solution. In addition to the nature of the problem influencing the establishment of connections, it is likely that the students' prior knowledge also had an impact, as highlighted by Flores and García-García (2017).

In the study by Flores and García-García (2017), the use of procedural connections and connections between mathematical concepts were more common among participants. The authors point out that this fact may be a general reflection of the way in which students work in the classroom, favoring algebraic aspects of solutions. The present study corroborates these observations.

It is worth highlighting the part-whole connections made by two pairs, when they explained the intermediate value theorem, studied in the Fundamentals of Analysis course, to justify the existence of the problem solution. Furthermore, this pair used the numerical method to find them.

It is also important to point out the relationship that students established with the degree of the equation and the number of roots. According to Businkas (2008), this shows implication connections since the pair related the dependence of the concepts in a logical way.

Finally, not all students were able to establish connections of all kinds in the resolutions presented. This suggests that the students had little contact with this experience in their school and academic careers, which may have influenced their performance.

As these students are future teachers, experiences like this become even more enriching, as they can provide the foundation to develop similar activities when working in the classroom.

FINAL CONSIDERATIONS

The objective of this study was to investigate the connections made by students in an Undergraduate Program in Mathematics Education when solving proposed problems. For this purpose, the Problem Solving Methodology was used (ONUChIC; ALLEVATO, 2011). Considering the literature discussed herein, our study supports the concept that working with connections is a favorable method for the development of mathematical learning. Connections are regarded as fundamental for the student to understand the interrelationships between mathematical ideas, overcoming the tendency to compartmentalize concepts (CARREIRA, 2010).

The use of the chosen methodology proved to be favorable to establishing connections, as highlighted by Allevato and Onuchic (2019), who emphasize connections as essential components for successful problem solving. Furthermore, problem solving is an essential activity for the development of knowledge through connections.

Proposals such as the one presented herein also serve to encourage future teachers who undergo this experience to do the same in their classrooms. As such, it is possible to contribute to the increasingly significant development of pedagogical practices in mathematics. This, in turn, is important considering that "future teachers [must] experience practices that place them deeply within this context of connections, while learning Mathematics in Higher Education and preparing for their professional future" (ALLEVATO; ONUChIC, 2019, p. 13).

We hope this study will serve as inspiration for other researchers. Similarly, the objective was to present information that contributes to other analyses related to the theme.

MATHEMATICAL CONNECTIONS THROUGH PROBLEM SOLVING: A STUDY IN HIGHER EDUCATION

ABSTRACT

Mathematics is characterized by different connections that can be formed between intra- and extra-mathematical areas, which are vital for understanding new concepts. In this article, we present the initial results of a study that focuses on the construction of mathematical concepts through the establishment of connections. Students in a Mathematics Education Undergraduate Program enrolled in the Fundamentals of Analysis curriculum course participated in the study. During the study, we analyzed the connections made when solving problems that involved concepts used during the discipline, such as continuous functions and intermediate value theorem, among others. The study was conducted in the context of the Covid-19 pandemic. We used a problem solving methodology for classroom work and a qualitative research approach with data obtained from the work produced by students and their observations. As a result, we found that the study contributes to understanding the learning of different mathematical concepts and the types of connections used, as shown through the procedures to solve the proposed problems.

KEYWORDS: Connections. Mathematics Education Undergraduate Program. Teaching. Fundamentals of Analysis. Problem Solving.

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