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Mathematical reasoning in Differential and Integral Calculus classes: an analysis from exploratory tasks

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ABSTRACT

This paper synthesizes the results of studies that discuss possibilities for the development of mathematical reasoning in Differential and Integral Calculus classes, based on exploratory tasks. This is a qualitative, interpretative research, with data collected in the context of classes of this subject, composed of (a) protocols containing written records of the responsibilities of small groups of students and (b) audios of these tasks, and (c) video of the collective discussion mediated by the teacher from the students' resolutions. Among the results, we present which mathematical concepts were mobilized by students in the elaboration of arguments that seek to justify their solutions in one of the tasks; we list the mathematical reasoning processes that students mobilize in a second task, with emphasis on conjecture and justification; and we recognize the teacher's actions that contributed to the development of mathematical reasoning in the discussion of a third task.

KEYWORDS: Mathematics teaching. Differential and Integral Calculus teaching. Mathematical Reasoning. Exploratory tasks.

INTRODUCTION

The Differential and Integral Calculus (DIC) subject is an essential part of the basic core curriculum of Exact Sciences majors in Brazil, especially Engineering, and can contribute to the development of reasoning processes that are necessary for the formulation and solution of problems from different areas, the analysis and understanding of phenomena, their validation by experimentation and effective oral, written and graph communication (BRASIL, 2019). It is an important tool for developing essential criteria for the interpretation and resolution of daily professional problems (GUIMARÃES, 2019). However, in addition to gaps in the students' prior mathematical knowledge (GHEDAMSI; LECORRE, 2021), the didactic-pedagogical structure of Engineering courses, in which a traditional teaching methodology that prioritizes teacher-centered lectures still prevails (CABRAL, 2015), contribute to students failing the DIC discipline and dropping out (ZARPELON; RESENDE; REIS, 2017; THOMPSON; HAREL, 2021).

Research developed in the field of Mathematics teaching (with results that can contribute to reversing the situation mentioned above) indicate that promising teaching approaches are those in which students work collaboratively (GRANBERG; OLSSON, 2015; CARLSEN, 2018), getting involved in mathematical discussions (RODRIGUES; MENEZES; PONTE, 2018; RIBEIRO at al., 2021), solving tasks of an exploratory nature, or mathematically complex (PONTE, 2005; WHITE; MESA, 2014), thus contributing to the development of creative reasoning (LITHNER, 2008, 2017).

In recent years, as a research group[1], we have been investigating proposals that are feasible in actual Higher Education classrooms. Considering the importance of students assuming an active role in his learning process, in our classroom practice, we developed a work proposal that involved task resolution, which is different from a "usual" DIC class, regarding methodological aspects (COUTO; FONSECA; TREVISAN, 2017; TREVISAN; MENDES, 2018; TREVISAN; ALVES; NEGRINI, 2021) and curricular organization, with content in a spiral format, and "postponing" to the end of the course a rigorous training on the concept of limits and the formal definitions of derivative and integral (TREVISAN; MENDES, 2017). Despite the promising results of this form of work, little was known about the development of students' mathematical reasoning, the concepts mobilized and the role of the teacher's actions during episodes of task resolution.

Thus, in recent years, we have gone further into and refined the investigation about the development of mathematical reasoning in students of the DIC discipline who are studying Engineering, and the mobilization of different processes when working with exploratory tasks. This is in line with the current movement of renewal of Engineering lesson models, to adapt to new global realities, with more modern teaching methodologies and training through skills[2] that meet market needs.

The new National Curriculum Guidelines for undergraduate courses in Engineering (BRASIL, 2019) should be highlighted. They establish that undergraduate courses must provide, throughout their training, the development of skills related to the formulation and development of creative solutions, as well as the use of appropriate techniques. They also point to the importance of developing broader competencies, involving aspects related to communication, teamwork and an investigative attitude. Thus, they encourage the development of a professional who, in addition to "strong technical training," is also human, critical, thoughtful, creative, cooperative, and ethical (GAFFURI; BAZZO; CIVIERO, 2020). This discussion has a direct interface with guidelines from research on Mathematics Education about mathematical skills that need to be developed with students at different stages of their education (NISS, 2003), especially mathematical reasoning, both in terms of its processes (LANNIN; ELLIS; ELLIOT, 2011; JEANNOTTE; KIERAN, 2017) and regarding its creative character (LITHNER, 2008).

This article expands on a summarized version of the research, presented at the VIII – International Seminar on Research in Mathematics Education, which happened online in 2021. It compiles results of studies developed within the scope of the Professional Master's Program in Mathematics Teaching (PPGMAT) – UTFPR CP/LD on the topics of *mathematical reasoning and the DIC subject* (TREVISAN; ARAMAN, 2021a, b; NEGRINI, 2022; TREVISAN; VOLPATO, 2022).

Thus, our objective is to discuss possibilities for promoting mathematical reasoning in DIC 1 classes, based on the work with some exploratory tasks. More specifically, we intend to: (i) identify the mathematical concepts mobilized by the students when elaborating arguments to justify their resolutions; (ii) discuss the mathematical reasoning processes that students mobilize; and (iii) understand how the teacher's actions can contribute to the development of mathematical reasoning.

To this end, further into this work, we highlight some research results that characterize mathematical reasoning and its processes, and discuss what teacher's actions can support it. We present the context in which the study was carried out and the methodological procedures used in the research. Finally, we analyze and discuss data from tasks involving graphic representations, which have implications for teaching and research on DIC regarding the development of mathematical reasoning in students who take this subject.

MATHEMATICAL REASONING

A challenge for educators at all stages of learning is how to develop mathematical reasoning in their students. Despite the different theoretical perspectives on this topic, discussed in detail by Araman, Serrazina and Ponte (2019), there is a consensus among scholars that mathematical reasoning – a dynamic process of conjecturing, generalizing, investigating reasons, and developing and evaluating arguments – is an essential skill for learning mathematics.

Jeannotte and Kieran (2017) discuss, within the scope of procedural aspects of mathematical reasoning, those related to the search for similarities and differences between situations (conjecturing, identifying patterns, generalizing, comparing and classifying), and others related to validation (such as justifying, proving and proving formally). In addition to these, and supporting them, the process of giving examples. Despite the relevance of all these processes in the development of students' mathematical reasoning, this work highlights the processes of conjecturing and justifying, since they were more evident in the analyzed data.

According to Jeannotte and Kieran (2017), conjecture is a cyclical process of: (i) stating the conjecture; (ii) verifying that it covers all cases and examples; (iii) attempting to refute it; and (iv) finding a reason to prove that the conjecture is true, or trying to modify it. Students can create valid or invalid conjectures, based on valid or sometimes invalid reasoning; the latter, although not desired, can serve as a starting point for understanding mathematical ideas. As for how to present a conjecture, it can be written in many ways, or even exist only in the minds of students.

Justifying, in turn, has to do with explaining a conjecture, presenting reasons (arguments) to change the epistemic value first from "probable to most likely," and then to true or false. Araman, Serrazina and Ponte (2019) state that justifying is related to "identifying relationships that allow us to understand why a statement can be true or false" (p. 468). Thus, justifying is closely linked to "investigating why" and students' elaboration of arguments "to convince themselves and others that a specific statement is true" (LANNIN; ELLIS; ELLIOT, 2011, p. 35), using different mathematical concepts to do so.

Regarding the mobilization of these processes, whether in moments of discussion in small groups of students, or in plenary, with the whole class, Ponte, Mata-Pereira and Quaresma (2013) propose a model for analyzing the actions of teachers in four categories. The first is to *invite*, which is the students' first interaction with the topic to be addressed. The second, to *guide/support*, in which the teacher, through questions, guides students to continue participating in the resolution of a task that has been started. The third, to *inform/suggest*, is the moment when the teacher validates the students' answers, introducing new information and providing new arguments. The last is to *challenge*, in which the teacher "puts the student in the situation of advancing alone into new territory, whether in terms of representations, the interpretation of statements, the establishment of connections, or of reasoning, arguing or evaluating" (PONTE; MATA-PEREIRA; QUARESMA, 2013, p. 59). Another similar model, which also organizes the teacher's actions into categories, and for each of them details possible actions, is TMSSR (*Teacher Moves for Supporting Student Reasoning*) by Ellis, Özgür and Reiten (2018).

Based on these two models, Araman, Serrazina and Ponte (2019) organized an analysis framework that describes teaching actions that support mathematical reasoning, shown in Chart 1. It will serve as a support for analysis in the context of one of the tasks presented in this article.

Chart 1 – Analysis framework of the teacher's actions

Categories	Actions
Inviting	Requests answers to specific questions. Requests reports on how they did the tasks.
Guiding/ Supporting	Provides clues to students. Encourages explanations. Leads the student's thinking. Focuses the student's thinking on important facts. Encourages students to repeat their answers. Encourages students to rethink their answers.
Informing/ Suggesting	Validates correct answers provided by students. Corrects wrong answers provided by students. Re-elaborates answers provided by students. Provides information and explanations. Encourages and provides multiple resolution strategies.
Challenging	Requests students' reasoning (justifications). Proposes challenges. Encourages assessment. Encourages reflection. Presses for precision. Presses for generalizations.

Source: Araman, Serrazina and Ponte (2019, p. 476).

METHODOLOGICAL PROCEDURES

The study that originated this article was designed as a qualitative, interpretive research (BOGDAN; BIKLEN, 1994). The participants were undergraduate engineering students from a Federal University in the state of Paraná, who took the DIC 1 course (taught by the author) in the 2nd semester of 2017 (tasks 1 and 2), and in the 1st semester of 2019 (task 3). This course, with a 90-hour class load, includes the study of functions, limits, derivatives and integrals of real functions, of a real variable, and was organized in an "unusual" curriculum structure (TREVISAN; MENDES, 2017) with contents presented in spiral format (revisited at various times throughout the semester).

In general, 25 hours of the course (about 10 meetings of 3 50-minute class hours) were focused on working with task-solving episodes. The 50 students in the class were divided into groups of three to four, and at first, they worked autonomously. The teacher monitored the work and made interventions when necessary. Finally, there was a collective discussion, mediated by the teacher based on the students' resolutions, with a systematization of concepts.

Three of the tasks selected for this article were proposed at the beginning of the course. They sought to mobilize the ability to analyze, in a coordinated way, the changes that occur in two interdependent variables, articulating different representations. A fourth task was proposed at the beginning of the second half of the course. Considering the contents proposed in the teacher's biannual planning of the course, at this moment, the students were learning the concept of limits of numerical sequences, and the development of the task intentionally preceded the extension of this concept to the case of real domain functions, in the case study of limits in infinity.

In task 1 (Figure 1), discussed in detail by Trevisan and Araman (2021a), when students were asked to describe the graph to a person who was not seeing it, the objective was to identify the concepts that would be mobilized when elaborating this description, and the arguments given to elaborate it. Although both functions present in the task are growing, $f(x)$ values grows at a decreasing rate (therefore its graphic representation is concave downwards), while h(x) values grows at an increasing rate (with a concave upwards graphic representation).

Figure 1 – Task 1

How would you describe to a person who is not seeing the graph of each of the functions described using the tables below?

x	0	1	3	6
$f(x)$	1	1.3	1.7	2.2

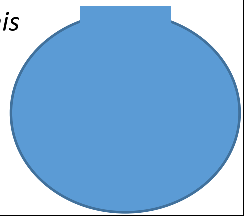
x	12	15	18	21
$h(x)$	21.40	21.53	21.75	22.02

Source: Trevisan and Araman (2021a, p. 602)

In task 2 (Figure 2), detailed in Trevisan and Araman (2021b), it was necessary to (i) recognize quantities involved in the situation (in this case, water height in the container and time); (ii) recognize the direction of growth (e.g., height and volume increase with time, but the radius increases and then decreases); (iii) identify the "way" in which the water height varies (initially growing at a decreasing rate, then increasing); (iv) sketch a graph with change in concavity.

Figure 2 – Task 2

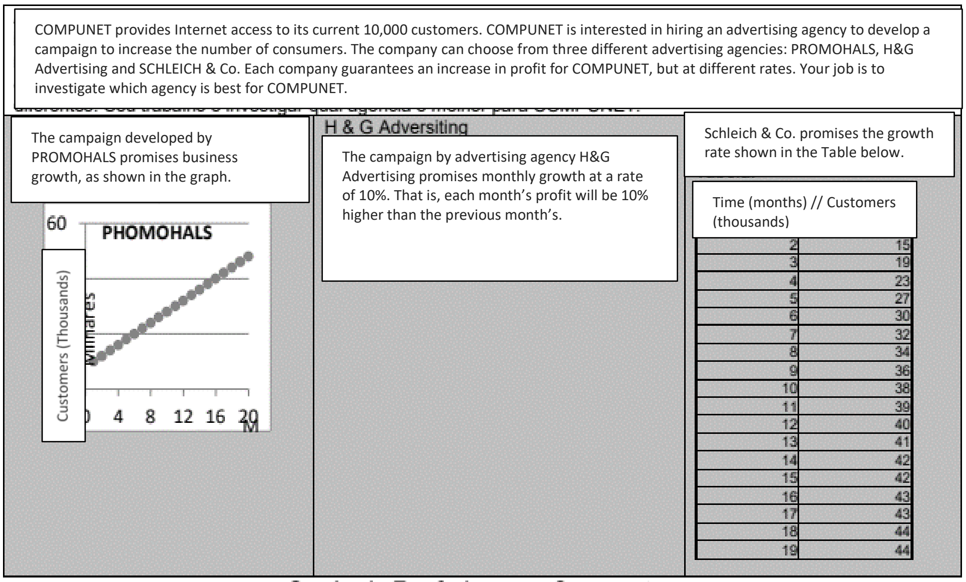
Water is poured into a bottle/vaseat a constant rate Use this information and the shape of the container(pictured to the side) to answer the questions below . Draw a graph that relates the water heightin the container over time Explain the reasoning that led to this graph.



Source: Trevisan and Araman (2021b, p. 166).

Task 3, addressed by Trevisan and Volpato (2022) (Figure 3), called The Compunet Case, explores different representations of numerical sequences. The task presents an opportunity to explore an arithmetic progression, in the case of the first company; a geometric progression, in the second company; and, in the third company, it provides evidence of "stabilization" in its behavior.

Figure 3 – Task 3



COMPUNET provides Internet access to its current 10,000 customers. COMPUNET is interested in hiring an advertising agency to develop a campaign to increase the number of consumers. The company can choose from three different advertising agencies: PROMOHALS, H&G Advertising and SCHLEICH & Co. Each company guarantees an increase in profit for COMPUNET, but at different rates. Your job is to investigate which agency is best for COMPUNET.

The campaign developed by PROMOHALS promises business growth, as shown in the graph.

H & G Adversiting

The campaign by advertising agency H&G Advertising promises monthly growth at a rate of 10%. That is, each month's profit will be 10% higher than the previous month's.

Schleich & Co. promises the growth rate shown in the Table below.

Time (months) // Customers (thousands)	
2	15
3	19
4	23
5	27
6	30
7	32
8	34
9	36
10	38
11	39
12	40
13	41
14	42
15	42
16	43
17	43
18	44
19	44

Source: Trevisan and Volpato (2021, p. 7)

Finally, task 4 considers a tank that initially contains 5000 liters of pure water and receives a mixture containing 750g of salt diluted in 25 liters of water, every minute. The task is to investigate the behavior of the concentration of the mixture over time. With this, the following can be analyzed: (i) the amount of salt (in grams) as a function of time (in minutes), that is, $Q_1(t) = 750t$; (ii) the amount of water (in liters), also as a function of time (in minutes), that is, $Q_2(t) = 5000 + 25t$; and (iii) the concentration of the mixture (in grams per liter), as a function of time (in minutes), that is $C(t) = \frac{750t}{5000+25t} = \frac{30t}{200+t}$ $(t \geq 0)$. From the data collected in this task, analyses were carried out with two focuses: the processes of mathematical reasoning mobilized by the groups of students (NEGRINI, 2022), and the teacher's actions in a moment of collective discussion with the class (plenary).

The collected data are composed of (a) protocols containing written records of discussions in small groups of students, (b) audios of these discussions, and (c) video of the collective discussion mediated by the teacher based on the students' resolutions. The audio and video recordings were transcribed in full (as part of a larger project), in articulation with the protocols, thus enabling the organization and analysis of data.

In order to recognize a greater variety of reasoning processes mobilized, as well as to evidence arguments presented by the students, in the case of tasks 1, 2 and 4, data of types (a) and (b) were considered. The criterion for choosing the groups was those in which there was greater involvement of students in the "presentation, justification, argumentation and negotiation of meanings" (RODRIGUES; MENEZES; PONTE, 2018, p. 399). In turn, in order to understand how the teacher's actions can contribute to the development of mathematical reasoning, we highlight data in (c), considering two excerpts from the plenary session in which we believe there was significant participation of students and a greater variety of these actions (one in task 3, and another in task 4). The results are organized, sequentially and separately, for each of the proposed tasks, considering the objectives stated in the introduction of the article. The letters S1, S2, ... are used to refer to the students.

DATA ANALYSIS

Task 1

For the analysis of Task 1, we present the transcript of a dialogue between team members and the researcher, in which they attempted to synthesize the way they had reasoned.

S1: We did it by looking at the values, drew the graphs and observed the graphs as we could explain them... we said that it was a growth curve and to represent it to a blind person, we would take their hand and take it to the starting point, then the end point. We would explain that at the start point there was so-and-so value. And then the x value grew following one pattern and the y value grew by another pattern, only these patterns are proportional and form this curve. The growth of [values of] x is faster than the growth of [values of] y. We explained how the extension is, if one grows faster and the other grows slower, more or less like this.

Researcher: Here [referring to the hand gesture made by S1, while speaking] the two were kind of concave upwards, is that it?

S1: Yes.

S2: It really is about guiding their hand and saying: here, there was a growth like this, not very accentuated.

S1: Guide their hand like: here is the beginning of the graph, it started with these values and from there it grew with the face of this curve, here you go. Here it bends, you guide their hand over the curve. Here the curve will grow faster... I think they would feel that this growth is faster than this one, so you could explain it. You would explain why this growth is faster than the other. The rate of change of x in relation to y. And of the other x in relation to the other y.

The group recognizes that the graphs are growing (*using the expression growth curve*), and built their argument based on the marking of points made on paper (*we did it by looking at the values, we drew the graphs*). They resort to identifying a pattern to independently analyze the absolute variations of the independent and dependent variables for each of the functions (*the x value grew following one pattern and the y value grew by another pattern*). They also mobilize actions to compare the growth rates (absolute) of the variables (*the growth of x is faster than the growth of y*).

However, the hand gestures made by student S1 were similar for both functions, suggesting the formation of concave upwards graphs. Although the teacher called attention to this fact (*the two were kind of concave upwards*), the group was not able to recognize this, correcting the description already presented. Although student S1 uses the expression *rate of change of x in relation to y*, he does not show that he understands its meaning, as well as its relationship with the concavity of the graphs, which means his justification lacks evidence.

Task 2

The group prepared a first sketch (Figure 4A) and, in the following discussion excerpt, tried to validate it:

S1: So, but then the graph will look like this, won't it?

S2: It is, like, because it grows fast at first, then it decreases and then it grows faster again, because of this [pointing to the central part of the bottle].

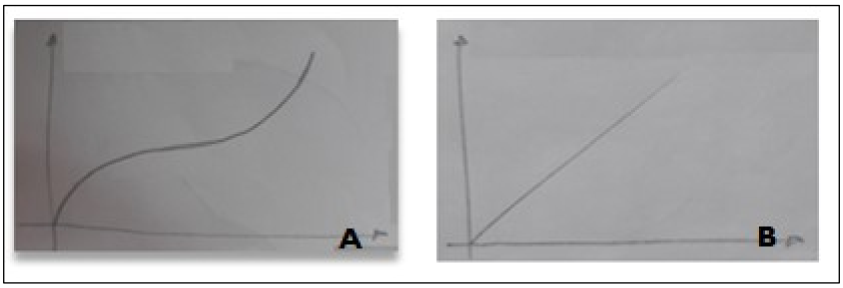
S3: So, but then if you measure the height of the vase by the diameter, it's a straight line.

S2: No, because, listen here! The radius is smaller than here [comparing the bottom with the central part of the bottle] so the amount of water you pour here will immediately appear here. By the time it starts to fill from that point on, it will take a while to grow and here it will grow faster [pointing out the diameters of the water surface in the bottle as it was being filled].

S3: Yes, but I'm talking about diameter.

S2: I think it would look something like this [Figure 4B. Would it be a straight line?

Figure 4 – Sketches proposed by the group of students for Task 2



Source: Trevisan and Araman (2021b, p. 168)

In order to validate the first sketch, S2 elaborated justifications based on the shape of the container, imagining what the diameters of the water surface would be as it was filled. He argued that height *grows fast at first, then it decreases and then it grows faster again*. S3, in turn, presented a new conjecture: that the graph relating the height of water to the diameter of the water surface would be a straight line. S2 again elaborated on his justification for the shape of the graph, now mentioning the radius in his argument. S3 seemed confused and failed to recognize that, implicitly, S2's arguments considered time as an independent variable, not the radius of the bottle. A new representation (a line) emerged, made by S1 based on S3's conjecture. The discussion followed:

S3: No. It depends, what are we relating? Volume or height.

S2: The height of the water in the bottle, see [pointing to the task statement]?

S3: Oh, yes.

S1: So, I think it would be a graph like this [Figure 4A], because when you start pouring water it will grow fast, then it will slow down, look at the size of this [pointing to the center of the container], then when it starts to taper here it will grow faster. This graph here [Figure 4B] would be the water height. The volume would be constant, it will always be filling at the same rate [Figure 4B].

In this passage, the group returns to the task statement and sees that the requested graph should relate the height of the water in the container over time. Student S1 then reformulated the arguments to validate the initial sketch, based on the "way" in which the water height varied. He also pointed to the other graph, recognizing that it represented the volume of water in the bottle; however, in his justification he incorrectly used the word constant, as if the rate of change in volume that was assumed in the task statement were constant.

Although the group validated the initial conjecture, the elaboration of arguments to justify the change in concavity, or the fact that the graph initially presents concavity facing downwards, and then upwards, was not part of the discussion. These aspects were "accepted" almost naturally by S2 and S3 from the sketch prepared by S1, although it was an incomplete justification, which did not address the concept of concavity.

Task 3

In order to expand a discussion about graph representations that had been brought up by some groups during the resolution of the task, the teacher invites the class for discussion.

Researcher: I want to hear a little from you, what you thought about the situation. Do any of the groups volunteer to start? (*Inviting*).

S1: We can start.

Researcher: OK, then. Tell us what you thought about. (*Guiding/Supporting*).

S1: First, we analyzed the *Promohouse* using the graph. The first five months, 5,000 clients, then, in the second month, 50,000 clients, then, in total, they got 40,000 new clients, then we went to H&G. Can we say what we concluded or should we do it like this [indicating if they should continue]?

Researcher: Could you tell us what you did then? (*Guiding/supporting*).

S1: We used the compound interest formula, but we analyzed it month by month to be able to compare the two companies. Then, with the third company, we also looked at that 5-month interval table, and then we concluded that *Promohouse* did not prove to be advantageous at any time, and the third company proved to be advantageous for a period of up to 16 months, then the best was H&G.

Researcher: Why would I want to make that number of customers increase so fast? (*Guiding/supporting*). Thinking about the situation, why would I want the number of clients to increase so fast? (*Challenging*).

S2: Maybe so your billing would be sufficient in that period.

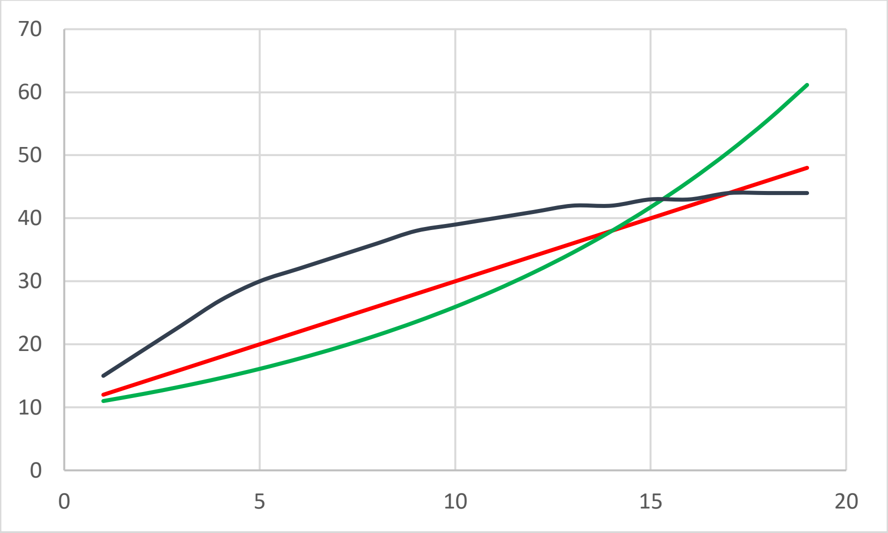
Researcher: Could I be willing to wait longer to reach that increase in customers? (*Guiding/supporting*)

S2: It really depends on the availability of your company. Because depending on whether you're in the red, you need a very quick increase in customers, but if you're OK and you have the opportunity to wait a little longer for more customers, why not?

Here, we can see that the teacher's actions focus on three categories: inviting, *guiding/supporting* and *challenging*. The *inviting* category occurred when the teacher called on the class to share how they thought about and solved the task at hand. In certain passages, the teacher asks questions in order to encourage students to explain how they solved the task, which is an action of the *guiding/supporting* category. The purpose of these actions is to allow students to explain and detail the way they reasoned, with no intention, at that moment, to judge whether the strategy was correct or not. Actions in the *challenging* category show that the teacher encouraged students to think, asking them to elaborate on some conjectures regarding what was requested in the task.

The discussion moved forward, with the intention of getting the class to associate information about growth rate of the functions that represent the number of customers of the three companies, in relation to time, with their graphic representations (which are presented in the same Cartesian plane, with three different colors, according to the solution of one of the groups, which was designed for the whole class – Figure 5).

Figure 5 – Simultaneous representation of the three companies



Source: Trevisan and Volpato (2022, p. 9)

Researcher: Let's explore these graphs[3] here a little more. How would you recognize which graph refers to which company by just looking at them? How do I know which graph represents company 1 [red]? And company 2 [green]? What about company 3 [blue]? Why do I know the blue graph is for company 3? (*Challenging*).

S6: Decreasing growth.

Researcher: Is decreasing growth related to what on that graph? (*Guiding/supporting*)

S7: In the beginning it shows more customer growth.

Researcher: How is the blue graph different from the green graph, for example? (*Guiding/supporting*).

S6: It has to form a horizontal line I think.

Researcher: And this horizontal line is related to what? (*Guiding/supporting*).

S7: Growth stagnation.

Researcher: I see less and less growth, and when it comes to the last few months, this growth is not expressive. (*Informing/suggesting*). And that green graph doesn't behave the same way? (*Guiding/supporting*).

S6: Because it's the opposite, the longer it goes, the line tends to be vertical.

Researcher: Try to explain this idea of why it stays vertical (*Guiding/supporting*).

S7: Maximum growth.

Researcher: Maximum growth. Would you say that these representations that I have here are situations of growth or degrowth? (*Guiding/supporting*).

S7: Growth.

Researcher: And does this growth occur in the same way in the three companies? (*Guiding/supporting*).

S6: No.

Researcher: So it means that it is not enough to say that it is growing, it means that you have to look at the way it is growing (*Informing/suggesting*).

The discussion, at the beginning of this excerpt, focused on getting the class to conjecture on which graph represents each of the companies and give their justifications, with actions of the *challenging* category. Throughout this part of the discussion, at various times the teacher re-elaborated conjectures and justifications presented by the students, in order to complement them with more precise information, or seeking to use more appropriate terms – actions linked to the *informing/suggesting* category. There were also moments when the teacher intended to guide/support, leading the student's reasoning, and focusing the discussion on important facts. At one of these moments, S2 hypothesizes that the graph representing the number of customers of the company whose growth is exponential, "will become vertical."

The teacher took advantage of the context of the discussion and explored the idea of concavity of the graph of the function, trying to associate it with the type of growth in the number of customers of each company (increasing, decreasing or constant), and anticipating how this discussion could be linked to the future study of derivatives.

Task 4

We selected one of the excerpts from the discussion that was brought up in four of the groups analyzed by Negrini (2022). Before, the group had used examples (concentration values for specific times) and assumed as valid the conjecture that the concentration of the mixture, in the long term, would approach the value of 30 grams per liter. They then elaborated a justification for specific cases, using different time values and reaching a generalization.

S1: Ok, let's do this process here again... 5000 liters.

S2: Ok, it's minute 0 here, right?

S3: Will it use 30 grams per liter of concentration?

S1: This value here will vary, the 5000 is constant.

S2: Yeah, the 5000 has to be in the function.

S3: What if it is $\frac{25}{750}.0.3x$?

S1: Think, there's always an increase of 25 from one to the other.

S3: Right.

S1: So, it'll be $5000 + 25$, and the 25, I'm almost positive it exists, times the time. No! Wait up.

S3: I think it's going to be like this $\left(5000 + \frac{25}{750}\right).0.3$, because it's the concentration?

S2: No! The concentration won't change.

S1: 25 times the time. So, $t = 1$, we get 5025. Fine. We got that, now we have to find a formula to apply to the concentration. Should I divide it by something?

S3: Plus $750t$?

S1: Now, how did we get that concentration. This concentration of 30 grams per liter?

S3: What if we add $750t$? It's not right?

S1: No, that would be a very large number.

S3: What if we divide everything by $750t$?

S1: That's not right either.

S3: It doesn't work either.

S1: See, $\frac{5000+25}{750}$, no! It's not. Think of 30 grams per liter, this is the amount of water... Is it this much?

S3: I think at some point we'll have to divide within the function. But by what?

S2: Because we're not dividing here, right? So I think we'll have to divide it in the function.

S3: How did we get to the limit?

S1: We used 5025, and what did we do? In minute 1 we have 5025 liters.

S2: We are dividing grams per liter, so we have to change it here [pointing to the formula]. $750x$ here, just change it here, see. I think if we just invert it it'll work.

S1: That's exactly right, man.

S2: I just don't know if it's right to express it like this.

S1: It's correct, $\frac{750x}{25x+5000}$. Just do the sanity check, when you do the sanity checks you'll find the values.

The group Looks for an algebraic expression for the function. If, on the one hand, one of the students presents some apparently random conjectures, trying to find a formula "that works," another student suggests redoing the calculations, for particular values of time, trying to recognize some kind of pattern.

Throughout part of the discussion, the group assumed as valid the conjecture that the concentration, in the long run, would approach the value of 30 g/L, and tried to incorporate this value in the algebraic expression. They could independently represent, as functions of time, both the amount of salt (750 times the time) and the amount of water (25 times the time, plus 5000).

They explain the conjecture by saying that in order to algebraically express the generalization found from numerical values, it would be necessary to do a division. New adjustments were made to the formula until the students ended up justifying it, using a procedure they call a "sanity check" (in this case, comparing concentration values, calculated using the formula, with those previously recorded in a table for particular time values).

At the beginning of the collective discussion, the teacher invited the students to participate, and student S1 then shared the way they had reasoned. The following excerpt is the start of this discussion, and the analysis performed by Trevisan *et al.* (2022) seeks to understand how the teacher's actions can contribute to the development of students' mathematical reasoning.

Researcher: So, let's go, let's explore this situation a bit... does any team want to present what they thought about? (*Inviting*).

S1: We do. First, we assessed how the amount of salt and the amount of liters of water varied. Then we made a table and then a formula that synthesized the table.

Researcher: And what kind of calculation did you do with this table? (*Guiding/supporting*).

S1: Concentration.

Researcher: What are we taking as concentration, by the way? (*Guiding/supporting*).

S1: Level of salt in the water.

Researcher: Right, so we are thinking of concentration here as a relation that is given by division (*Informing/suggesting*). Which one? (*Guiding/supporting*).

S1: The formula we did is concentration equal to 750 times the t [referring to time], because at $t = 1$ there will be 750g of salt. At $t = 2$, there will be 1500. Divided by the volume, which will be the fixed 5000 plus 25 times the t.

Researcher: So, let me write down the formula [writes on the board] (*Informing/suggesting)*: $C = \frac{750t}{5000+25t}$.

S1: So, we thought, when the time is very, very, very long, the 5000 will become practically nothing, so it means that the maximum concentration will be 750 divided by 25, which will be the limit, which is 30.

Researcher: So, what was your conclusion? (*Guiding/supporting*).

S1: That the concentration will not exceed 30.

Researcher: The concentration will not exceed 30. When we think that the concentration will not exceed 30, somehow it is approaching 30. (*Informing/suggesting*).

In this excerpt, we can see that the teacher's actions can fall within three categories: *inviting, guiding/supporting* and *informing/suggesting*. The action in the inviting category occurs when the teacher calls a group to report to their classmates how they tried to solve the task, and asks them to explain what they did, sharing their reasoning with the other students in the class. At times, the teacher asked questions in order to obtain specific answers. These actions aim to draw attention to some aspects of the resolution of the task, leading the student's thinking in order to clarify their reasoning, falling into the *guiding/supporting* category. This action is also present when the teacher asked students to explain the relationship they used to express concentration, and guiding them to draw a conclusion from their report, and the students stated that "the concentration would not exceed 30."

Some of the teacher's actions in this excerpt can be classified as *informing/suggesting*. For example, when the teacher gives a clue to the students, emphasizing that the concentration formula involves a division, and then guides them so that they can express the concentration formula they have found. Also, the teacher writes the formula on the board $C = \frac{750t}{5000+25t}$.

So, he confirms/validates the students' resolution and sharing it with the whole class. Finally, the teacher rewrites the student's answer ("the concentration will not exceed 30"), validating the statement that the concentration is approaching 30.

DISCUSSION AND FINAL REMARKS

This article discusses possibilities for promoting mathematical reasoning in DIC lessons, based on the work with some exploratory tasks, more specifically (i) identifying the mathematical concepts mobilized by students when elaborating arguments to justify their resolutions; (ii) discussing the mathematical reasoning processes that students mobilize; (iii) understanding what teacher's actions can contribute to the development of mathematical reasoning.

Regarding the DIC concepts used by students to solve task 1, when elaborating arguments and justifications, we recognized that the group under analysis easily identified the growth direction of the $f(x)$ and $h(x)$ graphs functions. The hand gestures used to represent the graphs suggest that they are concave upwards. however, the group did not make evident the recognition of the existence of a horizontal asymptote in the curve that graphically represents $f(x)$. The group independently analyzed the variations in the values of variable x and variable y, seeking to recognize some type of pattern in these variations (without, however, calculating these rates), and not able to establish a relationship with the concavity of the graph of functions. Although the arguments presented did not cover all the aspects the teacher desired, the task mobilized in the students the need to elaborate arguments to explain the variations observed in the tables and, for that, they mobilized the reasoning processes of identifying and comparing patterns. Subsequently, they made validation attempts, presenting justifications (based on their mathematical knowledge) to support their explanations, as argued by Lannin, Ellis and Elliot (2011).

Regarding reasoning processes, the group that solved task 2 elaborated an initial conjecture, presenting, at that time, mathematical knowledge related to the rate and direction of growth. This conjecture was illustrated based on the sketch of a graph, which, following the discussion, the students tried to validate, identifying relationships that allow understanding why a statement was true or false (ARAMAN; SERRAZINA; PONTE, 2019). Based on new relationships between mathematical knowledge, which refers to the shape of the bottle, its height, and its diameter, they try to validate this conjecture.

Regarding task 4, the group initially used specific examples that served to investigate mathematical structures or think about justifications, even if they were not enough (LANNIN; ELLIS; ELLIOT, 2011). The group reformulated conjectures looking for an algebraic expression that generalized what they had understood. The generalization process aimed to reach valid conclusions by establishing a relationship, applying it to different mathematical objects and transferring this relationship to a larger set (JEANNOTTE; KIERAN, 2017).

At the end of the discussion, the group successfully validated an algebraic expression for the function that relates concentration and time. The discussions allowed the students to extend, to more general situations, regularities observed in particular cases (generalization) – such as, for example, the variation in the amount of water and the amount of salt, as functions of time and, more generally, the concentration.

Regarding the teacher's actions that can contribute to the development of mathematical reasoning, from the analysis of an excerpt of collective discussion from task 3 and another from task 4, we can see that it begins with the action of *inviting*, in which the Students are asked to report to others how they solved the task. After that, we can observe the actions of *guiding/supporting*, at times when the teacher sought to highlight some aspects of the students' speech, elaborating on the resolutions they had developed. According to Ellis, Özgür and Reiten (2018), these actions can highly impact the progress of students' reasoning, since the purpose of the questions, as well as their development, contributes to the generalization and justification of mathematical ideas.

In the *informing/suggesting* category, actions are developed to validate the correct answers and present information, nomenclatures and concepts that are new. Finally, actions of the *challenging* category become more present in the moments when the teacher sought to go deeper into justifications of facts brought up by the students (ARAMAN; SERRAZINA; PONTE, 2019).

One of the limitations of this study is that, at times, the teacher hurried to inform the students, instead of inviting them to re-elaborate and/or explain what they did in solving the task, an aspect that compromises the development of mathematical reasoning. This may also explain the low number of students who participated in the discussion.

RACIOCÍNIO MATEMÁTICO EM AULAS DE CÁLCULO DIFERENCIAL E INTEGRAL: UMA ANÁLISE A PARTIR DE TAREFAS EXPLORATÓRIAS

RESUMO

Neste artigo, estão sintetizados resultados de estudos que discutem possibilidades para o desenvolvimento do raciocínio matemático em aulas de Cálculo Diferencial e Integral a partir de tarefas exploratórias. Trata-se de uma pesquisa qualitativa de cunho interpretativo com dados recolhidos no contexto de aulas desta disciplina, compostos por (a) protocolos contendo registros escritos das discussões dos pequenos grupos de estudantes, (b) áudios dessas discussões e (c) vídeo da discussão coletiva mediada pelo professor a partir das resoluções dos estudantes. Dentre os resultados, apresentamos quais conceitos matemáticos foram mobilizados pelos estudantes na elaboração de argumentos que buscam justificar suas resoluções em uma das tarefas, elencamos processos de raciocínio matemático mobilizados pelos estudantes em uma segunda tarefa com destaque para conjecturar e justificar e reconhecemos ações do professor que contribuíram para o desenvolvimento do raciocínio matemático na discussão de uma terceira tarefa.

PALAVRAS-CHAVE: Ensino de Matemática. Ensino de Cálculo Diferencial e Integral. Raciocínio Matemático. Tarefas exploratórias.

NOTE

1. GEPEAM - Study and Research Group on Teaching and Learning Mathematics (UTFPR – câmpus Londrina).

2. “Competencies are formed by knowledge of the content, plus the skills to use this knowledge, and actions when doing so.” (ABENGE, 2020, p. 21).

3. The word “graph”, in the transcribed passages, refers to the curves that graphically represent the number of customers of the companies, in relation to time.

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