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Changes in previous ideas about point, line and plane through interaction with the FARMA tool

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ABSTRACT

In this paper, we discuss the answers to a questionnaire involving basic concepts of geometry, which was applied to students of the ninth grade of Elementary School, before and after they interact with a tool called FARMA (Learning Tool for Remediation of Errors with Learning Mobility). The analysis of the answers came from a strategy of determination of substantive categories and theoretical categories. We observed that before the interaction with the tool the concepts presented by the students were closer to everyday ideas. After the interaction, there was a significant displacement of the answers in the direction of the correct concept, especially in the case of the point concept. This demonstrates the potential of the FARMA tool in teaching and learning Mathematics, especially Geometry.

KEYWORDS: Previous ideas in Geometry. Point, line, plane. FARMA. Quasi-Concept.

INTRODUCTION

We have known for a long time that when children arrive at school, they bring some conceptions related to different areas of knowledge. Such ideas, also called spontaneous conceptions, alternative conceptions, pre-conceptions or, more generally, previous ideas, have been systematically studied in some subjects, such as Physics, since the beginning of the 1970s.

Some authors consider that the expression “misconceptions” applies only to Physics, “errors” only to Mathematics and “misunderstandings” to both areas (NEIDORF et. al. 2020, p. 4).

In fact, in Mathematics there are several publications that address prior knowledge as errors. The report *Adding It Up: Helping Children Learn Mathematics,* published by the US National Research Council (NRC, 2001), opens Chapter 5 (The Mathematical Knowledge Children Bring to School) with the following sentence:

> Children start learning Mathematics well before they enter Elementary School. Beginning in childhood and continuing through the preschool period, they develop a foundation of skills, concepts, and misconceptions about numbers and mathematics (NRC 2001, p. 157).

Ojose, in his book *Common Misconceptions in Mathematics: Strategies to Correct Them*, is more emphatic in identifying prior ideas as errors. He defines misconceptions as “misunderstanding or misinterpretation based on incorrect meaning”, associated with “naive theories that impede rational reasoning of students” (OJOSE, 2015, p. xii). In this book, aimed especially at teachers in training or in service, the author addresses the misconceptions of students in arithmetic and algebra.

Mohyuddin and Khalil (2016) go in the same direction by stating that:

> Errors and misconceptions that students develop during past classes or bring with them to school from the community can create obstacles to the continuous learning of Mathematical concepts, consequently producing poor performance in Mathematics (MOHYUDDIN; KHALIL, 2016, p. 134).

However, prior ideas have been seen for some time not only as problems or mistakes, but also as essential conditions for learning to happen. Ausubel was one of the educators who most called attention to the importance of prior ideas in teaching (MOREIRA, 2011, p. 171).

In fact, even in the early months, children are already beginning to aquire some basic notions in several areas that will become essential for them to advance in scientific knowledge (NRC, 2007, p. 3-1). For instance, some researchers have shown that very young children already have expectations about the physical behavior of objects:

> Children aged 2.5 to 3.5 months are aware that objects continue to exist when they are hidden by other objects, that objects cannot remain stable without support, that objects move in continuous spatial trajectories, and that objects cannot move through the space occupied by other objects (BAILLARGEON, 1994, p. 133).

Regarding the concepts of size, weight and density, Smith, Carey and Wiser (1985) carried out tests with children aged 3 to 9 years old, in which they were presented with objects made of different materials (plastic, rubber, wood and metal), asking what they were made of and whether, when cut into smaller pieces, would they continue to be the same kind of material. The researchers found that:

> [...] for all ages, the children knew that the objects [...] were still made of the same type of material. Justifications changed with age. Half of the younger children (4 to 7 years old) mentioned only noticeable properties of the cut pieces to explain why it was still the same material (eg, it's still shiny or it's still thin). In contrast, the older children and the rest of the younger children [...] explicitly said that "cutting doesn't affect the material", "it's still paper because the cup was made of paper" or that "you just cut it, it it's still the same" (SMITH; CAREY; WISER, 1985, p. 42).

The perception that the social world is very different from the physical world also arises early in childhood. Children, although in their early years tend to attribute intentions to physical objects, they soon learn that entities in the social world have intentionality and that, for example, a ball cannot behave like a living being (NRC, 2007, p. 3- 9).

This knowledge of the natural and social world, although based on common sense, serves as a basis for understanding the subjects that children will later encounter in school.

This is the view expressed in the report *How Students Learn: mathematics in the classroom* (NRC, 2005), which takes as its first principle the title Engaging with Prior Knowledge, stating that "new knowledge is built on an existing foundation of knowledge and experience" (NRC, 2005, p. 4).

Specifically in relation to previous conceptions in Geometry, Piaget, a pioneer in research on the development of children's thinking (PIAGET, 1929), published, in 1960, the book *The Child's Conception of Geometry*, which focuses mainly on issues related to measurements in geometric shapes (PIAGET; INHELDER; SZEMINSKA, 1960). Some recent works, inspired by the theory of meaningful learning, address previous conceptions in plane geometry. For example: Silva and Schirlo (2013) report a case study carried out with students from the sixth year of Elementary School; Pivatto (2014), presents a didactic experience carried out with second-year High School Students; Puhl and Feltes (2017), present a didactic unit applied to a third-year High School class.

We have found only one study, published in Turkish, which addresses misconceptions about point, line and plane by Öksüz (2010). The results of this study illustrated that the research subjects, gifted Elementary School students (seventh grade) have some difficulties in understanding the concepts of point, straight line, straight segment, radius and plane, such as:

> [...] misconceptions about understanding real-world applications of geometric concepts, misunderstandings when using basic geometric concepts in complex problem-solving situations, misunderstandings about understanding different shapes (symbolic, visual, etc.) of the same geometric concepts, mistakes in the process of materializing undefined geometric concepts in their mental models, etc. (ÖKSÜZ, 2010, p. 509).

However, none of the works quoted in the previous paragraphs, including that of Öksüz, overlap with what we report here.

In general, the research question addressed in this article can be stated as follows: What conceptions about point, line, and plane can be found in answers from Elementary School students when interacting with learning objects through the FARMA tool and which can change in these conceptions be observed after this interaction?

Based on this question, this article aims to present and analyze the responses to a questionnaire involving basic concepts of geometry, which was applied to ninth-grade students, before and after they interacted with a tool called FARMA - Authoring Tool for Error Remediation with Mobility in Learning (MARCZAL, 2014). In addition to pointing out some previous ideas about the mathematical concepts of point, straight line, and plane, the analysis of the answers allowed us to measure the displacement of answers from everyday ideas to mathematically accepted concepts between the two moments.

Hereafter, we present the research methodology, discuss the data and make final considerations.

METHODOLOGY

This research is part of a professional Master's thesis that applied a tool called FARMA (Authoring Tool for Remediation of Errors with Mobility in Learning) to 33 ninth grade students of a public school in the state of Sao Paulo.

FARMA is a tool that enables the creation of Learning Objects (LO), extracting from it several mathematical concepts to be explored through interactions with the LO. According to Marczal and Direne (2012), one of the important features of the tool is its error feedback mechanism, allowing the student to restore the session to the moment the error occurred, exploring it in more detail.

For this investigation, several learning objects were elaborated for Geometry teaching. More details about the research context and the interactions that students had with the tool can be found in Pereira (2018).

The data survey was obtained through different collection procedures, including observations, notes, questionnaires, and activities. This article analyzes the students' answers to question number 2 of the questionnaire applied before and after the interaction with a Learning Object (LO) called Basic Elements of Geometry. Data were collected between May and July 2017. The question presented before and after the interaction with the Learning Object was as follows:

> What idea do you have when it comes to: a) point; b) line; c) plane; how do you define these concepts?

According to Maxwell and Chmiel (2014, p. 22), there are two main approaches to data analysis: those based on similarity relationships and those based on contiguity relationships. Similarity relationships involve the observation of similarities or differences and are based on the comparison; this procedure is used to define categories and groups and compare data by category. Contiguity-based relationships involve looking for connections between things, the influence of one thing on another, relationships between parts, and so on. Similarity and contiguity refer to two fundamentally different types of relationships between things, which cannot be reduced to one another. From these two approaches, two general data analysis strategies are derived: categorization strategies, which are based on similarity relationships, and connection strategies, which are based on contiguity relationships (MAXWELL and CHMIEL, 2014, p. 22).

In our case, we used the categorization strategy with two types of categories: substantive categories, which are primarily descriptive, close to the data and, in general, relate to the concepts and beliefs of the research subjects; and theoretical categories, which insert the coded data into an explicit theoretical framework, previously defined by the researcher (MAXWELL; CHMIEL, 2014, p. 25).

The substantive categories, which emerged from the units of analysis (answers to the questionnaire), had their names and descriptions partially based on the meanings and/or expressions of the word point, line, and plane in the Houaiss dictionary. Subsequently, the substantive categories were grouped into theoretical categories called: Concept, Quasi-concept, Daily Life, and Others, as described in Chart 1 (with an example for the concept of point). We emphasize that the substantive categories vary for each concept while the theoretical categories remain the same for the three concepts.

Chart 1 – Theoretical categories

Theoretical category	Description
1 – Concept	When the student's answer was compatible with the definition of point, as presented in the LO.
2 – Quasi-concept	When the answer was located between the mathematical concept and some previous idea of a point.
3 – Daily	When the answer was directly related to the meanings of the word point used in everyday life.
4 – Others	When the answer was undefined or "I don't know".

Source: Pereira (2018, p. 61).

It is important to emphasize that the theoretical categories in Chart 1 form a qualitative scale, ranging from the closest (Concept) to the smallest approximation (Others) with the mathematical concept involved.

The novelty introduced in the analysis was the idea of quasi-concept, inspired by Bernard's (1999) notion of quasi-concept. A quasi-concept would be a hybrid mental construction, which has two faces: a realistic face, based on data analysis of a situation; and an imprecision that makes it adaptable to various situations.

DATA PRESENTATION

The presentation of data, for each concept, is performed using a chart and a graph in which we can see the variation in the distribution of responses between theoretical categories, before and after the student's interaction with the LO (Object of Learning). After this presentation, we analyze the differences in this variation for each of the concepts (point, line, and plane).

Displacement of responses on the concept of POINT

Chart 2 summarizes the displacement of responses between the two moments for the concept of point. This chart was built from Charts 7 and 8, shown in the Appendix.

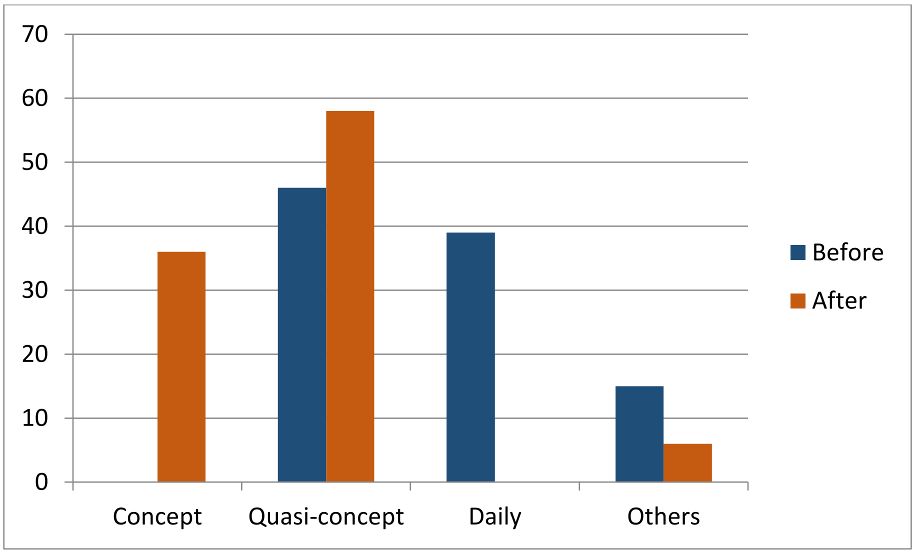
Chart 2 – Distribution of responses for the concept of POINT

Theoretical category	Substantive category (point)	Before (Chart 7)	After (Chart 8)
Concept	Form, Position and Dimension	0%	36%
Quasi-concept	Continuity, Geometry, Intersection, Sharp object, Mark, Single object, Place, Point object, and Symbology	46%	58%
Daily	School, Grammar, Local, Medicine and Regionalism	39%	0%
Others	Don't know, Undefined	15%	6%

Source: Pereira (2018, p. 66).

Figure 1 shows how the general categories for the concept of point varied before and after using FARMA:

Figure 1 – Variation of categories for the concept of POINT



Source: Pereira (2018, p. 66).

It is possible to clearly see that after using FARMA, the students' responses shifted to the left side of Figure 1, that is, they came closer to the mathematical concept of point. There was significant growth in the Concept column and the Quasi-Concept column also increased. The conclusion is that this shift involved shifts from everyday point ideas to ideas closer to the mathematical concept.

Displacement of responses on the concept of LINE

Chart 3 summarizes the displacement of responses between the two moments for the concept of line. This chart was built from Charts 9 and 10, shown in the Appendix.

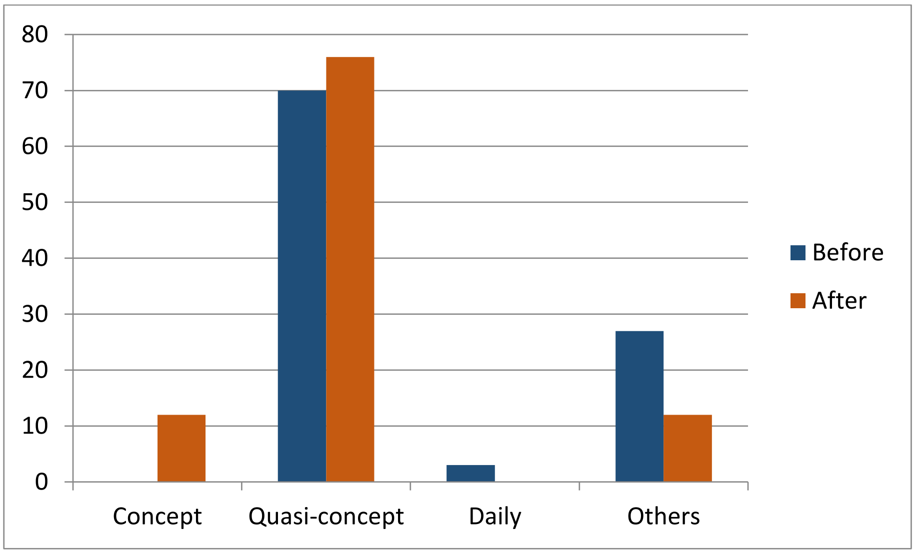
Chart 3 – Distribution of responses for the concept of LINE

Theoretical category	Substantive category (line)	Before (Chart 9)	After (Chart 10)
Concept	Dimension, infinite	0%	12%
Quasi-Concept	Points, Line, Plane object, No curve object and Symbology.	70%	76%
Daily	Honesty	3%	0%
Others	Don't know, Undefined	27%	12%

Source: Pereira (2018, p. 70).

Figure 2 shows how the general categories for the concept of line varied before and after using FARMA:

Figure 2 – Variation of categories for the concept of LINE



Source: Pereira (2018, p. 71).

It is possible to notice that, after using FARMA, the answers related to the line (although in lesser intensity) also moved to the left side of Figure 2, that is, they came closer to the mathematical concept of line. There was an increase in the Concept column and the Quasi-Concept column also had a small increase. In analogy to the previous case (point), this shift involved changes from everyday ideas from a straight line to ideas closer to the mathematical concept.

Displacement of answers on PLANE

Chart 4 summarizes the shift in responses between the two moments for the concept of plan. This chart was built from Charts 11 and 12, shown in the Appendix.

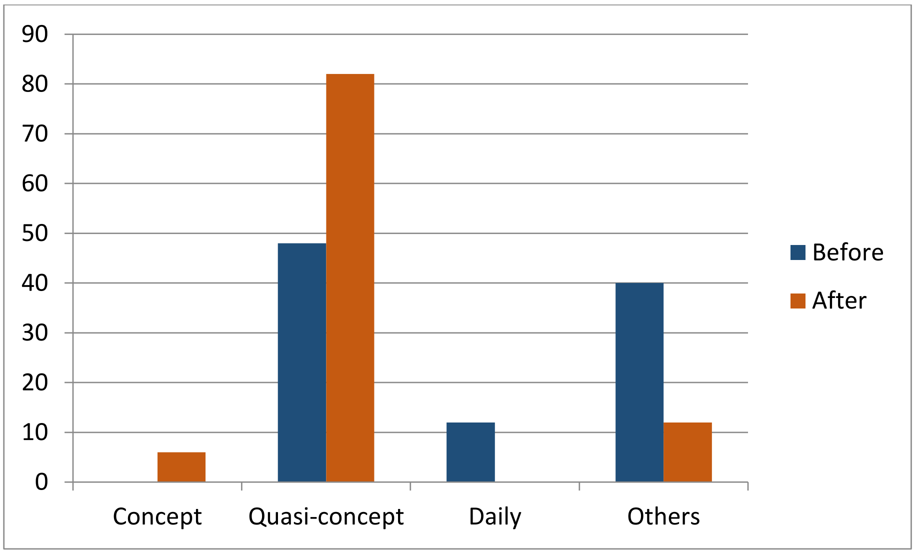
Chart 4 – Distribution of responses for the concept of PLANE

Theoretical category	Substantive category (plane)	Before (Chart 11)	After (Chart 12)
Concept	Infinite	0%	6%
Quasi-Concept	Plane object, Quadrilateral, Line, Plane surface, Region, Symbology	48%	82%
Daily	Planning	12%	0%
Others	Don't know, Indefinite, didn't answer	40%	12%

Source: Pereira (2018, p. 75).

Figure 3 shows how the general categories for the plane concept varied before and after using FARMA:

Figure 3 – Variation of categories for the concept of PLANE



Source: Pereira (2018, p. 76).

It is possible to notice that, after using FARMA, the answers related to plane (although to a lesser extent) also moved to the left side of Figure 3, that is, they came closer to the mathematical concept of plan. There was a small increase in the Concept column and the Quasi-concept had a significant increase. Similar to previous cases, this shift involved shifts from everyday ideas from plan to ideas closer to the mathematical concept.

ANALYSIS

From the observation of Figures 1, 2 and 3, for the three concepts, the enormous incidence of responses in the Quasi-Concept theoretical category, both before and after FARMA, is immediately noteworthy, which can be seen in Chart 5, where the numbers are the quantities of units of analysis (AU):

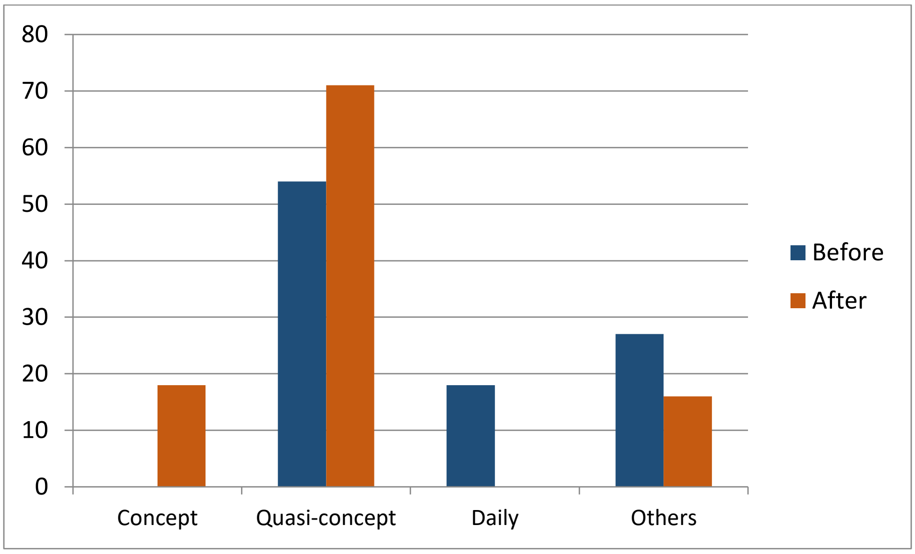
Chart 5 – Total distribution of AUs before and after FARMA

Theoretical category	**Substantive category (before)**				**Substantive category (after)**			
	Point	Line	Plane	*TOTAL*	Point	Line	Plane	*TOTAL*
Concept	0	0	0	*0*	12	4	2	*18*
Quasi-Concept	15	23	16	*54*	19	25	27	*71*
Daily	13	1	4	*18*	0	0	0	*0*
Others	5	9	13	*27*	2	12	4	*16*
TOTAL	33	33	33	*99*	33	33	33	*99*

Source: Pereira (2018, p. 76).

Based on Chart 5, Figure 4 can be build:

Figure 4 – Variation of categories for the three concepts



Source: Pereira (2018, p. 77).

According to the figure, the highest incidence of responses was located in the Quasi-Concept category, both before and after FARMA. The migration of responses from the lower categories (Others and Daily life) to the higher categories (Quasi-Concept and Concept) was expressive.

It is also interesting, due to its importance in the responses, to compare the substantive categories found in the Quasi-Concept category, as shown in Chart 6.

Chart 6 – Substantive categories of the Quasi-concept category

	Quasi-Concept	Description
POINT	1.Continuity	Relates point to the idea of continuity, sometimes referring to a concept of Geometry
	2.Geometry	Relates point to some concept or content of Geometry
	3.Intersection	Relates point to the idea of intersection
	4.Sharp object	Relates point with sharp objects, which have a point or corner, sometimes mentioning Geometry objects
	5.Mark	Relates point to a sign, design, stain, or mark, sometimes made by an object
	6.Single object	Refers to the point as something singular
	7.Place	Relates point to a delimited place in the plane or in space. The place is closer to the mathematical concept than the place
	8.Point object	Associates the point with a point object
	9.Simbology	Refers to the fact that points are usually indicated by capital letters
LINE	10.Points	Relates line to two connected points or to several points together
	11.Line	Relates line to the drawing (scratch) made with a ruler or the abstract idea of a line
	12.Plane object	Associates line with objects that are straight
	13.No curve object	Relates line to the absence of curves
	14.Simbology	Refers to the fact that lines are generally indicated by lowercase letters
PLANE	15.Plane object	Associate plane with plane objects
	16.Quadrilateral	Associates plane to a figure with four sides or formed by four lines
	17.Line	Associate plane with line
	18.Plane	Associates the plane with a flat surface, without ripples
	19.Region	Associates plane to a place, space or region occupied by bodies, points or lines
	20.Simbology	Refers to the fact that plans are generally indicated by Greek letters

Source: Pereira (2018, p. 77).

We can observe some similarities, such as, for example, some substantive categories refer to observable or ideal objects, such as the number 4, 6, 8, 12, 13, 15; in addition, for all concepts, a category called Symbology emerged. Further investigations can likely shed more light on such connections.

FINAL CONSIDERATIONS

In this article, we analyze changes in the conceptions of point, line, and plane of students in the ninth grade of Elementary School caused by the interaction with a Learning Object about basic concepts of Geometry, via the FARMA tool. We observed that before interacting with the tool, the concepts presented by the students were closer to everyday ideas. After the interaction, there was a significant shift in responses towards the correct concept, especially in the case of the point concept. This demonstrates the potential of the FARMA tool in teaching and learning Mathematics, especially Geometry.

The grouping of responses also revealed the importance of the proposition of the category called Quasi-Concept. In other words, between the concept accepted by the scientific community and the previous or everyday idea, there is an intermediate zone that could work for some students, such as Vygotsky's Zone of Proximal Development (REGO, 1998, p. 73) for the concepts of point, line, and plane. The Quasi-Concept idea could be explored in other areas and other teaching and learning situations.

Finally, we would like to comment that the analysis of the mathematical concepts that we have done here reminded us of the idea of a conceptual profile.

The conceptual profile is a research instrument created by Mortimer (1995), based on Bachelard's (1984) idea of epistemological profile. Both the epistemological and the conceptual profile are usually represented by a set of zones, which resembles a bar graph. One zone of the conceptual profile may differ from the other both epistemically and ontologically. The conceptual profile was mainly applied to the teaching of Science (chemical reactions, heat, energy, mass, radiation, etc.), but it can also be found in some areas of Mathematics as a function (CARRIÃO, 1998; RIBEIRO, 2013) and fractions (GUABIRABA, 2008).

In a way, we could say that Figures 1, 2, and 3 show changes in the conceptual profiles of point, line, and plane, respectively, before and after the interaction with FARMA. We emphasize, however, that for this case the profile has four fixed zones, which follow a scale that varies according to the greater or lesser approximation with the mathematical concept. Another difference is that while the conceptual profile, in general, applies to only one subject, in our case it is the profile of a class of 33 students.

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MUDANÇAS NAS IDEIAS PRÉVIAS SOBRE PONTO, RETA E PLANO POR MEIO DA INTERAÇÃO COM A FERRAMENTA FARMA

RESUMO

Nesse artigo discutimos as respostas a um questionário envolvendo conceitos básicos de Geometria, o qual foi aplicado a estudantes do nono ano do Ensino Fundamental, antes e depois dos mesmos interagirem com uma ferramenta denominada FARMA (Ferramenta de Autoria para Remediação de Erros com Mobilidade na Aprendizagem). A análise das respostas se deu a partir de uma estratégia de categorização e determinação de categorias substantivas e categorias teóricas. Observamos que antes da interação com a ferramenta, os conceitos apresentados pelos estudantes estavam mais próximos de ideias do cotidiano. Após a interação, houve significativo deslocamento das respostas na direção do conceito correto, principalmente no caso do conceito de ponto. Isso demonstra o potencial da ferramenta FARMA no ensino e aprendizagem de Matemática, em especial da Geometria.

PALAVRAS-CHAVE: Ideias prévias em Geometria. Ponto, reta e plano. FARMA. Quase--conceito.

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APPENDIX

The idea of POINT, LINE and PLANE before and after the interaction with the Learning Object (FARMA).

Chart 7 – POINT (before FARMA) by category

Theoretical category	Substantive category (point)	Description	Unity of analysis	Student
Quasi-Concept 15 UA (46%)	Continuity	Relates point to the idea of continuity, sometimes referring to some concept of Geometry	a) POINT: Somenthing that continues	A2
			a) POINT: It's something that can be continued in a line	A3
	Geometry	Relates point to some concept or content of Geometry	a) POINT: Graphs	A28
			a) POINT: Sides, angles	A32
			a) POINT: where the line pass	A33
	Intersection	Relates point to the idea of intersection	a) POINT: Connection between two lines	A18
			POINT: Connection between two lines	A9
	Sharp object	Relates point with sharp objects, which have a point or corner, sometimes mentioning Geometry objects	a) POINT: the corner of the blackboard	A7
			a) POINT: It's when something is pointed, such as a triangle, it has a point	A4
	Mark	Relates point to a sign, design, stain, or mark, sometimes made by an object	a) POINT: They are points similar to the final point marked by the chalk.	A20

			a) POINT: Something that marks	A21
			a) POINT: A little ball	A29
			a) POINT: A little dot	A30
			a) POINT: A pen point	A6
			a) POINT: a point	A26
Daily 13 UA (39%)	School	Relates point to evaluative activities at school	a) POINT: The point of note when you take off.	A5
			a) POINT: Teacher's grade	A22
	Grammar	Relates point to some grammar punctuation mark	a) POINT: It's the period I use when there's a sentence.	A1
			a) POINT: Period	A15
			a) POINT: End, exclamation.	A19
			a) POINT: A period that ends sentence	A27
	Place	Relates point to a place, indicating a place in everyday life.	a) POINT: A public place.	A10
			a) POINT: It's a particular place.	A16
			a) POINT: From point to point, from place to place	A24
			a) POINT: Starting point.	A31
	Medicine	Relates point to surgical stitch or suture in Medicine (the word used is the same in Portuguese)	a) POINT: When you cut and then you have to stitch.	A8
			a) POINT: You get hurt and have to go to the doctor to stitch.	A23
	Region	Relates point to some everyday or regional expression	a) POINT: The point when there's something ready, like, there's something in point.	A13
Others 5 UA (15%)	Doesn't know	Write explicitly that you doesn't know	a) POINT: I don't know.	A12
			a) POINT: I don't know.	A14
			a) POINT: I don't know.	A17
	Undefined	Refers to the point in an indefinite or incomprehensible way	a) POINT: In Mathematics there is a point in Portuguese.	A11
			a) POINT: Link to each other	A25

Source: Pereira (2018, p. 62-63).

Chart 8 – POINT (after FARMA) by cathegory

Theoretical category	Substantive category (point)	Description	Unity of analysis	Student
Concept 12 UA (36%)	Figure	Relates point to some geometric figure, in particular to the crossing of two straight lines	a) POINT: it's the geometric figure formed from two lines	A23
	Position	Relates point to the determination of a position (in space)	a) POINT: it's an element that indicates a position	A10
	Dimension	Relates the point to an object without dimension, size, or unmeasurable	a) POINT: point is a point with no dimension	A1
			a) POINT: It has no dimension in relation to the other points	A2
			a) POINT: It is an object in space that cannot be measured	A3
			a) POINT: point without measure	A4
			a) POINT: It may be something you cannot measure	A7
			a) POINT: it's an object that has no definition, dimension and shape.	A12
			a) POINT: Point with no dimension	A22
			a) POINT:The point has no size	TA24
			a) POINT: The point has no dimension	A25
			a) POINT: The point is everywhere, because it has no size	A28
Quasi-Concept 19 UA (58%)	Singular object	Refers to the point as something singular	a) POINT: POINT is a unique thing, it is not formed by anything	A8
	Place	Relates point to a delimited place in the plane or in space	a) POINT: It's a delimited place on the plane	A9
			a) POINT: It is used to mark place in space.	A11
	Mark	Relates point to a sign, design, stain or mark	a) POINT: It is a hole.	A13
			a) POINT: Map marks to mark cities	A14
			a) POINT: I saw that it is when there is a little spot	A5

			a) POINT: A needle hole in a paper gives the idea of a spoint	A18
			a) POINT: The touch of a pencil on paper is a point	A26
			a) POINT: Graphite tip mark on a paper	A30
	Point object	Associates the point with a point object	a) POINT: a star in the sky is a point	A27
			a) POINT: Stars in the sky	A31
			a) POINT: a star in the sky is an example of point	A21
			a) POINT: The head of a nail.	A32
			a) POINT: A fan (refers to the center of the fan)	A33
	Simbology	Refers to the fact that points are usually indicated by capital letters	a) POINT: When using capital letters.	A6
			a) POINT: Quando usa letras maiúsculas	A15
			a) POINT: Quando usa letras maiúsculas	A17
			a) POINT: we use capital letters from our alphabet: A. B, C...	A20
			a) POINT: We always use capital letters to put a point.	A29
Others 2 UA (6%)	Undefined	Refers to the point in an indefinite way or with meaning that is difficult to understand	a) POINT: The study of a point.	A16
			a) POINT: At one point of one	A19

Source: Pereira (2018, p. 63-65).

Chart 9 – LINE (before FARMA) by category

Theoretical category	Substantive category (point)	Description	Unity of analysis	Student
Quasi-Concept 23 UA (70%)	Points	Relates line to two connected points or to several points together	b) LINE: It is connected by two points	A29
			b) LINE: points together, lots of points close and glued	A32
			b) LINE: When you have two points and go through the middle.	A33

	Line	Relates line to the drawing (scratch) made with a ruler or the abstract idea of a line	b) LINE: A line or when something is in the same position, that is, when it is linear.	A4
			b) LINE: A line.	A6
			b) LINE: Can be to lines.	A7
			b) LINE: A line using the ruler.	A8
			b) LINE: Two lines that can cross or be parallel.	A9
			b) LINE: A line or ruler to measure lines.	A11
			b) LINE: A line, object, etc., wtih no curves.	A16
			b) LINE: A stretched line.	A21
			b) LINE: A scratch with the ruler.	A23
			b) LINE: It's like a ruler that traces and stands the line.	A27
			b) LINE: A straight line.	A30
	Plane object	Associates line with objects that are straight	b) LINE: It's a road, for example.	A1
			b) LINE: A ruler.	A5
			b) LINE: A fishing rod.	A13
			b) LINE: A linear road.	A19
			b) LINE: Lines are the same as guitar strings or meeting something.	A20
			b) LINE:Almost everything is a line blackboard, floor, eraser.	A24
			b) LINE: A linear thing.	A25
	No curve object	Relates line to the absence of curves	b) LINE: Something with no curves.	A2
			b) LINE: It's something literally linear, with no curves or anything like that.	A3
Daily 1 UA (3%)	Honesty	It associates line with integrity, probity and rectitude	b) LINE: When people are linear.	A10
Others 9 UA	Undefined	Refers to the line in a redundant, indefinite	b) LINE: What is a line is linear.	A15

(27%)		or incomprehensible way	b) LINE: Straight lines are those with corners.	A18
			b) LINE: A mark.	A22
			b) LINE: A line.	A26
			b) LINE: Graphs.	A28
	Doesn't know	Write explicitly that doesn't know	b) LINE: I don't know.	A12
			b) LINE: I don't know.	A14
			b) LINE: I don't know.	A17
			b) LINE: I don't know.	A31

Source: Pereira (2018, p. 67-68).

Chart 10 – LINE (after FARMA) by category

Theoretical category	Substantive category (point)	Description	Unity of analysis	Student
Concept 4 UA (12%)	Dimension	Relates line to a geometric object of one dimension	b) LINE: it is a geometric object infinite in one dimension.	A10
	Infinite	Relates line to infinite points	b) LINE: it is formed by infinite points.	A8
			b) LINE: On a line there are infinite points.	A19
			b) LINE: There are infinite points.	A27
Quasi-Concept 25 UA (76%)	Line	Relates line to the drawing (scratch) made with a ruler or the abstract idea of a line	b) LINE: Straight line that has two points.	A2
			b) LINE: It is a line.	A4
			b) LINE: A well-stretched line, guitar string.	A13
			b) LINE: A line with no beginning and no end.	A22
			b) LINE: It is the geometric figure formed from a line.	A23
			b) LINE: The line of a notebook.	A33
	Plane object	Associates line with objects that are straight	b) LINE: A tightly stretched rope gives the idea of a straight line.	A18
			b) LINE: A stretched wire.	A21

			b) LINE: A well-stretched strip is a straight one.	A26
			b) LINE: A well-stretched strip.	A30
			b) LINE: Awell-stretched string.	A31
			b) LINE: a pen.	A32
	Points	Relates line to two connected points or to several points together	b) LINE: A line can be the meeting of two points.	A1
			b) LINE: The meeting of two points through a line.	A3
			b) LINE: It has two little dots and a line	A5
			b) LINE: There are many points in a line.	A7
			b) LINE: Row sets of points.	A11
			b) LINE: Set of many points in a straight line.	A12
			b) LINE: The study of a line that has two points.	A16
			b) LINE: Linked points, I saw it can have two points.	A28
	Simbology	Refers to the fact that lines are generally indicated by lowercase letters	b) LINE: When we use lowercase letters.	A15
			b) LINE: When we use lowercase letters.	A17
			b) LINE: When we use lowercase letters of the alphabet: a, b, c...	A20
			b) LINE: We always use lowercase letters to set a straight line.	A29
			b) LINE: When we use lowercase letters.	A6
Others (12%)	Undefined	Refers to the line in a redundant, indefinite or incomprehensible way	b) LINE: Colinear lines.	A14
			b) LINE: The line has only one size.	A24
			b) LINE: The line has a left and right direction.	A25
			b) LINE: It is an infinite “trace”.	A9

Source: Pereira (2018, p. 69-70).

Chart 11 – PLANE (before FARMA) by category

Theoretical category	Substantive category (point)	Description	Unity of analysis	Student
Quasi-Concept 16 UA (48%)	Plane object	Associate plane with plane objects	c) PLANE: Computer table.	A21
			c) PLANE: Computer screen.	A22
			c) PLANE: a plane thing.	A25
	Quadriateral	Associates plane to a figure with four sides or formed by four lines	c) PLANE: It's a rectangular form.	A1
	Line	Associate plane with the line	c) PLANE: Something linear.	A2
			c) PLANE: When your base is linear.	A4
			c) PLANE: The surface to be flat must be linear.	A20
			c) PLANE: There's no curve, it's linear.	A33
	Plane surface	Associates the plane with a flat surface, without ripples	c) PLANE: It's when you have a flat view of something, where its base is a straight line.	A3
			c) PLANE: It's a plane view.	A7
			c) PLANE: It's a plane floor.	A8
			c) PLANE: It's a plane object; it is one that has nothing on the surface.	A14
			c) PLANE: It is a surface without ripples.	A16
			c) PLANE: The floor.	A24
			c) PLANE: A ripple-free table surface.	A27
			c) PLANE: A linear floor.	A28
Daily 4 UA (12%)	Planning	Associate the word plan to planning something	c) PLANE: A plan is for you to have a plan to do.	A5
			c) PLANE: When I'm going to make a deal and I have to have a plan.	A13
			c) PLANE: When I plan something.	A15

			c) PLANE: Planning.	A23
Others 13 UA (40%)	Undefined	Refers to the plan in a redundant, indefinite or incomprehensible way	c) PLANE: It's a horizontal thing.	A10
			c) PLANE: The ground.	A19
			c) PLANE: A plane.	A26
			c) PLANE: Where we make geometric drawings.	A29
			c) PLANE: Square, with no curves.	A30
			c) PLANE: Lines togheter.	A32
	Didn't answer	Didn't answer	c) PLANE: Didn't answer.	A6
	Doesn't know	Write explicitly that doesn't know	c) PLANE: I don't know.	A9
			c) PLANE: I don't know.	A11
			c) PLANE: I don't know.	A12
			c) PLANE: I don't know.	A17
			c) PLANE: I don't know.	A18
			c) PLANE: I don't know.	A31

Source: Pereira (2018, p. 72-73).

Chart 12 – PLANE (after FARMA) by category

Theoretical category	Substantive category (point)	Description	Unity of analysis	Student
Concept 2UA (6%)	Infinite	Associates plane to a set or to infinite lines	c) PLANE: It is formed by infinite LINES.	A8
			c) PLANE: Rows of straight lines.	A11
Quasi-Concept 27 UA (82%)	Plane object	Associate plane with plane objects	c) PLANE: Something spaceful, like the blackboard in the room.	A13
			c) PLANE: The blackboard in the classroom gives an idea of plane.	A18
			c) PLANE: Similar to a sheet.	A22
			c) PLANE: A notebook sheet.	A28
			c) PLANE: It's the blackboard from school.	A29

			c) PLANE: The floor of a basketball court.	A31
			c) PLANE: A sheet of cardboard.	A32
			c) PLANE: The bottom of a swimming pool.	A33
	Quadrilateral	Associates plane to a figure with four sides or formed by four lines or lines	c) PLANE: The union of four straight lines that form a portion.	A3
			c) PLANE: It's closed with four points.	A5
			c) PLANE: The union of fours lines.	A23
			c) PLANE: It has four linear sizes.	A24
			c) PLANE: It contains four straight lines and many points.	A27
	Region	Associates plane to a place, space or region occupied by bodies, points or lines	c) PLANE: A portion occupied by bodies.	A2
			c) PLANE: A portion where many points fit.	A7
			c) PLANE: a portion full of points and many straight lines.	A14
			c) PLANE: It has many points and many lines inside.	A16
			c) PLANE: In a plane there are infinite points.	A19
	Simbology	Refers to the fact that plans are generally indicated by Greek letters	c) PLANE: When we use Greek letters.	A6
			c) PLANE: It is represented by lowercase Greek letters; e.g.: α, β	A10
			c) PLANE: When we use Greek letters.	A15
			c) PLANE: When we use Greek letters.	A17
			c) PLANE: When we use Greek letters.	A20
	Plane surface	Associates the plane with a plane surface, without ripples	c) PLANE: It's a kind of room floor.	A1
			c) PLANE: It's easier with examples, in a room there's a floor plan etc.	A9
			c) PLANE: A gymnasium sports court is an example of the notion of a plan.	A21

			c) PLANE: The surfasse aof a table.	A30
Others 4 UA (12%)	Undefined	Refers to the plan in a redundant, indefinite or incomprehensible way	c) PLANE: Plane is área.	A4
			c) PLANE: Formed by even more points like straight lines, but they are together.	A12
			c) PLANE:It has two directions	A25
			c) PLANE: The sky is plane because it has stars that are points.	A26

Source: Pereira (2018, p. 73-75).

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