

Students learning with Interactive Virtual Math: an exploratory study in the classroom

ABSTRACT

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Interactive Virtual Math (IVM) is a visualization tool to support secondary school students' learning of graphs by dynamic events. In the prototype version students construct a graph and try to improve it themselves and with the feedback of the tool. In a small-scale experiment, which involved four classes at secondary and tertiary education and their mathematics teachers we investigated how the students used the tool in the classroom. In this study we focus on the students learning experience and the results are expected to provide knowledge and directions for further development of the tool. The corpus data consists of self-reported questionnaires and lessons observations. One main finding is that students, at different school levels, find the tool useful to construct or improve graphical representations and it can help to get a better understanding of the subject. The tool features that helped students most were the self-construction of the graphs and to get feedback about their own graph at the end. Other findings are that the students can work independently with the tool and we know more about the tool features that are attractive or need to be improved.

KEYWORDS: Students learning. Interactive Virtual. Math. classroom.

INTRODUÇÃO

Students' difficulties with tasks involving dynamical events are well documented in the literature. And there is also a body of knowledge that shows that conventional curricula have not been effective in promoting the learning of this ability (CARLSON; LARSEN; LESH, 2003). New technologies can allow for studying dynamic events and therefore be valuable for students to analyse and interpret dynamic function situations.

The Interactive Virtual Math-project aims to develop a digital tool for learning context-graphs or graphs from dynamic events at secondary education and to explore the use of new technologies in classroom (also in higher education and teacher education). The project started in 2016 as a proof of concept in which a prototype tool was developed and tried out with 14-15 years old students (PALHA; KOOPMAN, 2016). In the present stage we explore how a prototype-version of the tool is used in classroom by teachers and students. The aim of the research is to assess and understand how students work with the IVM-tool in the classroom and to collect directions for its further development.

In this article we report on the results of part of the whole research. We investigated students' experience and use of the tool through questionnaires and lesson observations. The following research questions guided the study:

1. How did the students think to have learned with the tool?
2. How did the students experience and used the tool?

We start by briefly discuss the nature of students difficulties with graphs from dynamic events and how new technological tools can be used to help learning. We introduce the digital tool Interactive Virtual Math (IVM), its main features and how it works. In the method section the learning experiment is described and information about the participants and the instruments as well. The results concerns students' responses to the questionnaire and its discussion follows in the final section.

THEORETICAL BACKGROUND

LEARNING CONTEXT GRAPHS

An example of a dynamic event is the situation presented in Fig. 1. The task was adapted from an article of (CARLSON; OEHRMAN; ENGELKE, 2010), which address students difficulties with constructing graphs about dynamic events.

Fig. 1 - The jar-task

Imagine this jar filling with water at a constant rate.
Sketch a graph of the water's height in the jar as a function of the amount of water in the jar.
Explain the thinking you used to construct your graph.



To solve the jar-task the students will need to consider how the dependent variable (height) changes while imagining changes in the independent variable (volume). The coordination of such changes requires the ability to represent and interpret relevant features in the shape of the graph (CARLSON et al., 2010). In a previous study Carlson et al. (2002) used a similar version of the task to investigate students' ability to reason about covarying quantities in dynamic situations. The subjects of the study were 20 high-performing 2nd-semester calculus students who had successfully completed a course emphasizing rate and changing rate. Results from this study showed that the majority of the students were able to coordinate changes in the direction and amount of change of the dependent variable but were unable to consistently coordinate changes in the average rate of change with fixed changes in the independent variable or consistently coordinate the instantaneous rate of change with continuous changes in the independent variable. Moreover, they had difficulties explaining why a curve is smooth and what is conveyed by an inflection point on a graph.

In an exploratory study (PALHA, 2017) investigated 14-17 years old students' understanding of graphs by dynamical events by three distinct questioning forms: construction of graph, selection of graph, construction of object. One main finding of this study is that students' ability to solve the jar-task varies with the type of task and, it is influenced by the task questioning. The findings showed that the majority of the students failed to successfully solve tasks in which they were required to construct for themselves a graphical representation. As in the study of Carlson et al. (2002) the great majority of the students draw an increasing straight line or an increasing but not correct curve to the jar-task. An interesting finding was that students gave much more wrong answers in tasks in which they were required to construct a graph or to sketch an object by a graph than tasks in which options with graphs were given and the student had to select one of them. In addition, the study documented the main strategies used by the students to approach the task. Examples of these ones include: reflecting upon the constructed representation or explanation; visualization of the jar as whole or the three main parts or imagining the filling process; adding attributes and measurements to the representations.

Based on these findings Palha (2017) author suggests the need to use learning activities in the classroom that involve construction from the start and not just by filling in parts, selecting options or following what the teacher does on the board. Moreover, construction tasks offer much more insight in what students think (for themselves and for the teacher) than selecting or completing tasks.

The previous findings suggest the need for students to have opportunities to experience the covariational nature of functions and real-life dynamic events can be a rich context to this. Carlson et al. (2002) recommend that students should be involved in reflections of their own understandings of patterns of change (involving changing rates-of-change) and the use of technologies and specially designed physical devices for studying real-time dynamic events.

DIGITAL TOOLS

In the learning of functional reasoning Thompson (2011) states that it is critical that students first engage in mental activity to visualize a situation and construct relevant quantitative relationships prior to determining formulas or graphs. Also, learners should be helped to focus on quantities and generalizations about relationships, connections between situations, and dynamic phenomena (ELLIS, 2007). Digital tools can be valuable to achieve these aims. These experiences, should, however, be connected to proper curriculum materials and teacher support, in order to become rich opportunities for students to learn (CARLSON et al, 2003).

Following these ideas, the IVM-tool was designed and developed to help learners to focus on the relevant quantitative relationships and engage them in the mental activity of visualizing these relationships. The tool is designed according to a socio-constructivists perspective, in which people learn through individual constructing knowledge and in interactivity. The tool support this process through providing specific didactical hints, by elicit reflection and to allow students to work at their own pace. The learners can go through the application as many times they need it and they get feedback on their own productions. In the classroom they can compare their drawings with the ones got by other students and improve them if needed.

INTERACTIVE VIRTUAL MATH

The tool and an instructional video about how it works can be respectively found at <https://virtualmath.hva.nl> (select EN for English) and <https://youtu.be/lc7mNUcZ8CQ>.

When entering the tool the students get the jar-task, which request them to try to imagine two variables changing simultaneously. They must construct the graphical representation and the verbal explanation for this relation, which stimulate them to try to represent their concept image graphically and verbally (VINNER, 1983). Through hints and feedback the student is challenged to improve his own construction. The tool also includes the use of Virtual Reality (VR), which is still very limited. The use of VR (sound, movement, interaction) is expected to improve the experience of the graphic situation

SELF-CONSTRUCTION

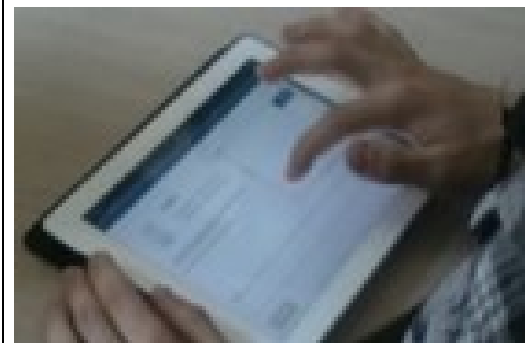
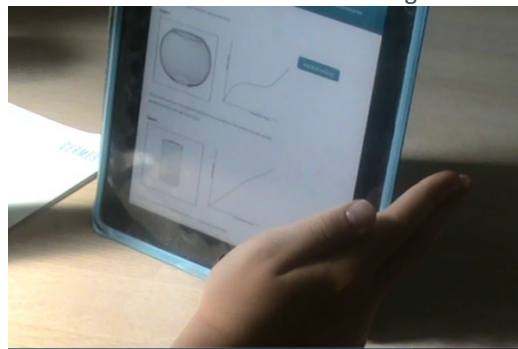
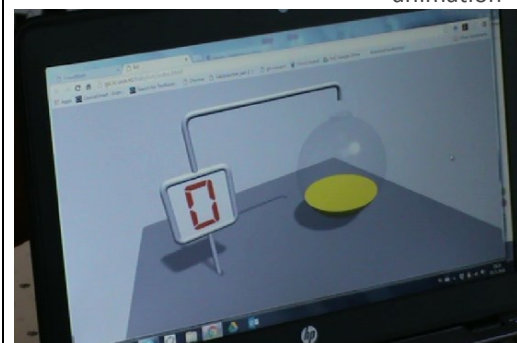
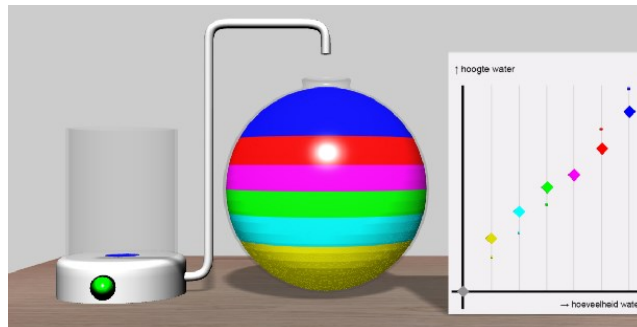

The students get two jar-assignments. The first one is the same one as in Fig. 1. The students must try to construct the graph and write the explanation for themselves. They can use a mouse, the finger or a digital pen to construct the graph and they can erase it and try again as many times they want (Fig. 2A) Only when they complete the drawing and write the explanation the tool allows them to proceed to the next assignment.

COMPARISON

The second assignment is similar to the first one but the jar has a cylinder-shape. In previous studies with this task on paper and pencil (CARLSON, 1998;

PALHA, 2017) the majority of students produced a straight line by the round-jar. We thought that students who draw a straight line by a round jar would be puzzled when having to draw a graph by a cylinder because they would have to draw a straight line again. Indeed, in Palha and Koopman (2017) we observed four students working with the tool and the ones who saw a straight line in the first assignment go backed and tried to improve the graph.

When the students complete both assignments they get an overview of both assignments (Fig. 2B) and they can choose to improve their answers or to submit their work.

	<p>Fig. 2A - Self-construction</p>  <p>Fig. 2B - Comparison</p>
	<p>Fig. 2C - Help 3D-animation</p>  <p>Fig. 2D - Help interactive animation</p>
<p>Fig. 2E - Reward</p>	 <p>Fig. 2F - Virtual Reality</p>



We notice that the drawing so far was also possible to do also within a pen and paper assignment (although the contrast aspect would demand more preparation work from the teacher). But, from this point on the tool offers specific feedback and support that would be difficult (if not impossible) to make available in a paper and pencil setting. Results from Ellis (2007) indicate that instruction encouraging a focus on quantities can support generalizations about relationships, connections between situations, and dynamic phenomena. We applied this result in the tool-design to help students to visualize the relation between the height of the water and the volume. Two kinds of help with the tool were developed: Help 3D animation and Help interactive animation.

HELP 3D ANIMATION

When the students select 'help 3D animation' they get the image of the jar in 3D (Fig. 2C). The water starts to fall in the jar when the green button is pushed. Each time the button is pushed falls the same amount of water; because the jar is round the increment of the height will vary. The different increments are easily visualized because they have a different color. We expect that this animation helps students to imagine the height-values co-varying with the volume-values. And, to realize that the increment between the height-values is not constant.

HELP INTERACTIVE ANIMATION

When the students select 'help interactive animation' they get to see the image of the jar and a cartesian graph side by side (Fig. 2D). In the horizontal axis there are six dots with different colors; each dot will correspond to a value of the height for a certain volume of water. The dots are placed by the student at the height they think the water will attain in the jar. When the first dot is placed the water starts falling in the jar; the amount of water is always the same for each time. The students observe if the guess for the height was good or not; the dot can be placed at the right height. In the second and following guesses students must assume the height of the water in the jar and represent it in the graph with dots. It is expected that while guessing where to put the dot for the height, they

will notice that the difference in height between consecutive dots (values of the height) decreases in certain situations and increases in others.

FLOW

The way a student can go through the tool is the 'flow'; it is the adequacy of the steps of the application for the user. It is connected with the order of the tasks and the possibility of choice. The students can choose to consult one, both or none type of help. They can, after consulting the help try to improve their graphs and explanations. The options help appears only after the student has first tried to construct a graph and explanation by the assignments (and not before). The reason for this is to force the student to represent their concept image from the beginning.

REWARD

When the students are satisfied with their drawing and explanation they can submit it and get formative feedback from the tool. The students get to see the corresponding jar-figure to the graph they draw (Fig. 2E). The idea is to provide the students with the possibility to evaluate by themselves their production.

VIRTUAL REALITY

The tool also includes the use of Virtual Reality (VR), which is still limited to Help 3D animation. The use of VR (sound, movement, interaction) is expected to improve the experience of the graphic situation (Fig. 2F).

METHOD

EXPERIMENT

We conducted a small scale experiment at secondary and tertiary education involving four classes and their students and teachers that used IVM during one lesson (45-50 minutes). Because we wanted to explore how students use the tool in the regular classroom practice the teachers were encouraged to setup the lesson from themselves. The corpus data consists of students' responses to questionnaires about their experience with the tool. We also use data from the lesson observations and teachers' questionnaires to describe the learning setting.

PARTICIPANTS

We collected data from seventy nine students and four teachers in different traditional schools in The Netherlands (Table 1). The study involved different class levels: one class with 9 students from the first year of the bachelor mathematics teacher, one eleventh grade with 28 students, pre-university stream with mathematics B; one tenth grade with 21 students, pre-university stream

with mathematics B and one tenth grade with 21 students, vocational stream with mathematics A. The four classes vary in their mathematical knowledge and ability. It is expected that the 10th grade vocational is the class with less pre-knowledge. No student had, as far as we know, worked before with the tool before the experiment.

The four teachers were invited to take part of the study; they knew about the tool but they were not used to work with it. The teachers were connected with the university (they have studied or were teaching at the University of Applied Sciences of Amsterdam) and they were teaching in the area of Amsterdam with one exception, who was teaching in a school at 100 km east from Amsterdam. The teachers are two men and two women with ages varying between 28 and 40 years and with teaching experience varying between 5 to 15 years. The teachers were selected by their teaching experience (we wanted to have a different range of experience since this is a factor that influences classroom performance). And, because they had previously showed interest in using the tool with their students. Not all teachers dare to experiment new approaches especially technological tools that are still in development. We should therefore be careful with the generalization of the results of the experiences of these teachers as they are not representative for the Dutch teachers.

Several devices were used by the students taking part of this experiment but, the same device was used in the same class. In two classes (DS and RJ) it was used the school computer and students draw with the mouse; in one class (JV) the students used their own laptop or tablet and the graph was draw with the touchpad or the finger; and in one class (FS) the students used their own smartphone.

Table 1 Participants of the study

Class	School level	Teacher	Students	Device
DS	11th grade pre-university education ¹⁾	DS, 40 years, 15 years teacher experience	28 students, 17 boys and 11 girls; average 17 and range 16-19	Computer
FS	10th grade pre-university education ²⁾	FS, 28 years old, female, 5 years teaching experience	21 students, 15 boys and 6 girls, Average 15,4 and range 15-17	Smartphone
RJ	10th grade vocational education with applied mathematics A ³⁾	RJ, 31 years old, men, 7 years teacher experience		Computer
JV	First year students Bachelor teacher education AUAS	JV, 40 years old, female, 15 years teaching experience	9 students, 5 boys and 4 girls, Average 20 and range 18-28	Laptop, tablet, smartphone

¹⁾ in the Dutch education system is 5 vwo wiskunde B/

²⁾ 4 vwo wiskunde B/

³⁾ 4 havo wiskunde A

INSTRUMENTS

The main instrument of the study was a questionnaire aimed at assess and understand students' use of the tool and collect suggestions for its improvement. It is a self-made digital questionnaire, with open questions, multiple choice (MC) and multiple answers possible (MAP). By the MC and MAP an option 'other' is added, which allows students to write other possibilities than the ones mentioned in the questioning. It takes about 10 minutes to answer the questionnaire and it has 17 questions organized in four sections: Q1-6 background information, Q7-10 how they learned with the tool, Q11-13 devices used by the students, Q14-17 usability of the tool and suggestions. Students responses to Q7-10 and Q14-17 provide respectively answer to the first and second research question and they are presented in the results' section.

In order to get a more objective view of the way the students and the teachers' use of the IVM during the lessons we used a semi-structured observation. The entire class and the teacher are observed. We look at aspects like how the tool is introduced by the teacher, students' working with the tool, asking questions, interacting with other students and teacher and classroom discussions. Part of the lessons were recorded in audio and video and these records were used to complete the observation report.

ANALYSIS

The results of the questionnaires were organized in an excel document. Each student and teacher got a code number; in the excel document all variables are given and the analyses are summarized. The results of the questionnaire were first analyzed per classroom (results are in the section quantitative results). Analysis of the observations reports were descriptive and its result provide a description about teachers and students practices with the tool in a particular classrooms.

TEACHER PREPARATION

The teachers took part of a preparation meeting before the experiment. The goals of the meeting were to the teachers learn to work with the tool and to inform about the research. The meeting took about 1,5 hour and all teachers attended the meeting. Based on the agreements with the teachers the researchers setup a protocol with directions for the lesson preparation. Teachers could deviate from these directions and, indeed, all teachers have brought some new elements or made modifications to the plan, but none of these were great changes. Before the lesson the teachers discussed the lesson plan with the researchers and adapted it, when needed.

LEARNING SETTING

Teachers' goals for the use of the tool in the lesson were diverse but its general set up was similar. All teachers tried to combine individual and working in duo's. The tool was introduced by three teachers in a similar way: the teacher

told about the goal of the tool without explaining how it worked and then gave the handout to students. One teacher did not use the handout; he first logged in with the students and then the students went themselves through the tool. Both ways have worked well. By three teachers the tool was presented to students as a stand-alone assignment and without connection with the topic they were dealing at the moment of the experiment. The only exception was JV, which integrated the tool as part of the topic 'technology in the classroom'. At the end of the lesson the four teachers conducted a classroom discussion about the assignments of the tool.

RESULTS

DID THE TOOL HELP TO CREATE OR IMPROVE THE GRAPH?

Students' responses to question 7 (Fig. 3) provide information about their ability to construct the graph by the jar-assignment and the extend at which the tool have helped them.

Fig. 3 - Q7 You have created a graph with a spherical jar and delivered it with the tool. Has the tool helped you to create or improve the graph? Choose the option that suits you best.

Fig. 3A Class DS, N=28, 11th grade

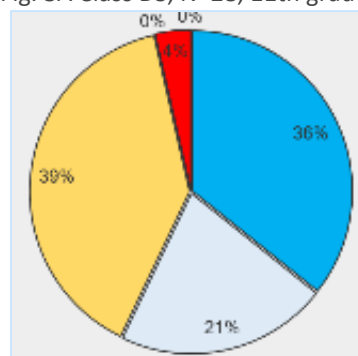


Fig. 3B Class FS (N=21, 10th grade pre-univ.)

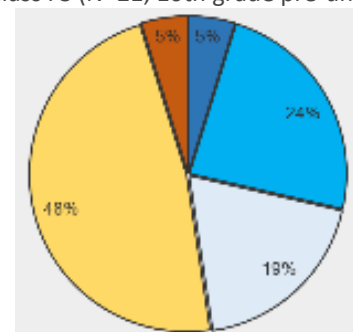


Fig. 3C Class RJ (N=21, 10th grade vocat.)

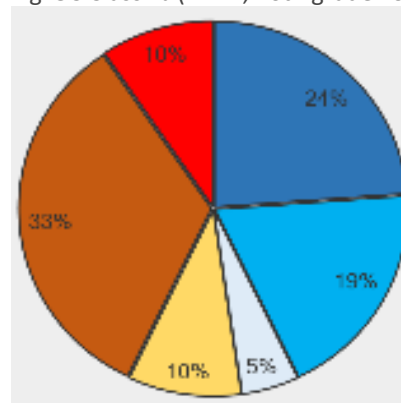
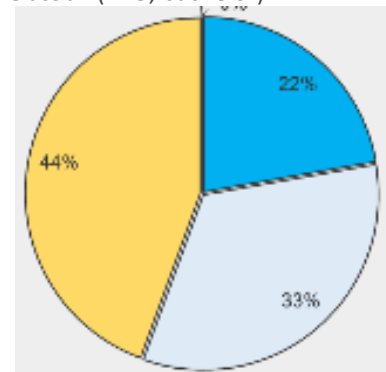


Fig. 3D Class JV (N=9, bachelor)



- a) tool helped to construct the graph (I have no idea)
- b) tool helped to improve the graph (I have a vague idea)
- c) tool helped to correct graph (I know how but I had a mistake)
- d) It didn't help (I knew it already)
- e) It didn't help (I still don't understand)
- f) other

Fig. 3A shows that more than the half of DS' students (11th grade) selected options 7b) or 7c), which means that they felt helped by the tool to accomplish the assignments. Ten of these students (36%) had an idea about the graph form but it was not completely correct and the tool helped to improve it (option b) and the other six students (21%) stated that they knew how to draw the graph but they made a mistake and the tool helped them to correct it (option c). One student who felt helped by the tool explained in the area for comments: "Yes, I knew what to do, but the visual tool makes you think about the graph better". The tool was not always needed (eleven students); these students knew already how to construct the graph (option d). No student selected options a) or e).

Half of the students of FS (10th grade, pre university stream) felt supported by the tool (Fig. 3B); the responses are similar to DS's class but, in the case of FS the options a) and e) are selected by two students. There is also a higher percentage of students that didn't need the tool as they knew how to construct the graph (48%).

Fig. 3C shows that the majority of JV's students (bachelor mathematics education) reported to be helped by the tool and, mostly to improve the graph by correcting some mistake (33%) None of the students selected options a), e) or f).

Finally, in Fig. 3D we see that the class of RJ (10th grade, vocational stream) presents a different pattern than the other three classes. There is a higher percentage of students who selected options a) or e), in total 57%. And, a lower percentage of students who chose options c) or d) in total 15%.

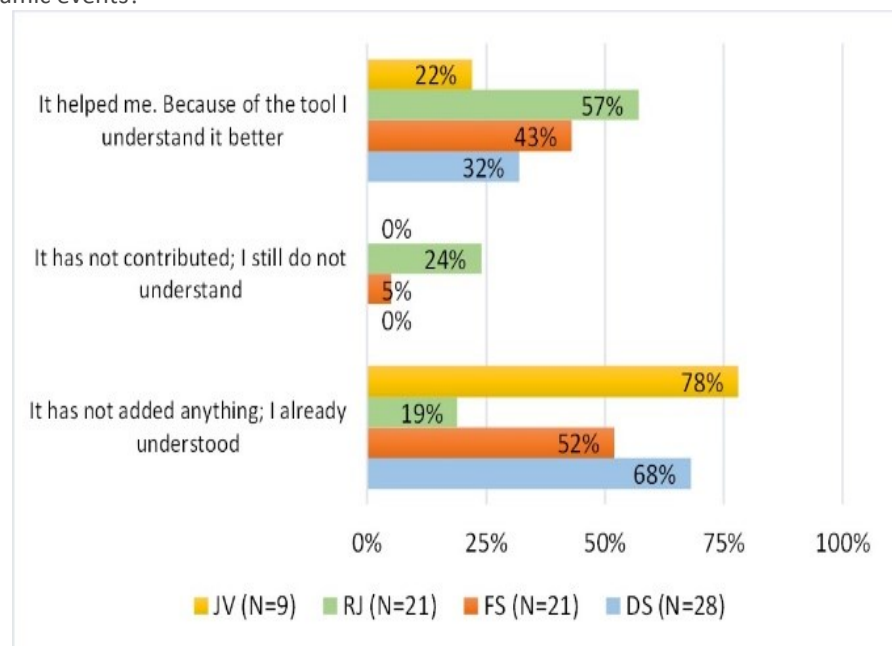
In summary, about half of students in all classes reported that the tool have helped them to create, to improve or correct a graph. These students however differ in their use of the support. Students at 10th and 11th grade with mathematics B reported to have already an idea about the shape of the graph and the tool help them to work it out and consolidate this idea. Students-teachers at the bachelor have a good idea about the graph and the tool helped them to correct some mistake. In these three classes the students who reported not having been helped by the tool already knew about the graph and the tool was not needed. This is different for the students at the 10th grade following vocational stream. Also the half felt support of the tool but they did not have previously any idea about how the graph would be or they had vague idea. The tool helped them in the construction of the graph and to improve their vague initial image. The majority of students in this class who reported not felt the tool support was because they don't understand the topic and only a few percentage reported to know about the graph and therefore the tool was not needed.

DID THE TOOL CONTRIBUTE TO UNDERSTAND GRAPHS BY DYNAMIC EVENTS?

Question 8 is assesses if the tool contributed for the learning beyond the particular assignment. The results of these questions are presented in Fig. 4. The

majority of DS' students (68%) stated that the tool didn't contribute for their understanding of the subject as they already understood it; only 30% of the students considered that it provided understanding. No student stated that it didn't contribute at all and that the topic was not understood. Interesting is that a higher percentage of FS' students (43%) than DS reported that the tool helped to better understand the subject, in general. Only one student reported not understands the subject and not felt helped by the tool. The great majority of JV's students (78%) reported to already understand the subject; the rest (22%) considered that they understand better the subject with the tool. About the halve of RJ's students reported to have been helped by the tool. Halve of them (24%) didn't know how to construct the graph before the tool; 19%, had an idea about the graph and the tool helped to improve this idea and 5% felt support because the tool helped them to correct some mistake they made. A high percentage of students (43%) however. didn't felt support by the tool. In this sense the responses of this class are distinct from the other classes. The percentage of students lacking pre-knowledge is much higher than in the other classes (as we have noticed in students' responses to question 7).

Fig. 4 Q8 Has the tool contributed, in general, to your understanding of graphs by dynamic events?



REASONS PRESENTED BY THE STUDENTS WHO FELT HELPED BY THE TOOL

Question 9 was an open question in which students who were helped by the tool could explain their reasons. Table 2 summarize these results, the students of DS (12) pointed reasons as: drawing (4 students. e.g. "the slopes better drawing"); one sees the result (two students, e.g. "you see your results"); visualization (two students); providing self the explanation "by providing and explanation and explaining why it is" (one student), relevance to practice (one student); help to improve the form (two students).

Table 2 Q9 If the tool has helped you understand graphs in context assignments, explain how

Summary of the reasons presented by students	DS (N=12)	FS (N=7)	RJ (N=11)	JV (N=2)
drawing	4	-	-	-
one sees the result	2	1	-	1
visualizing	2	4	3	-
providing self the explanation	1	-	-	-
relevance to practice	1	-	-	-
help to improve the form	2	2	-	-
it clarifies	-	-	4	-
receiving explanation (from the teacher)	-	-	4	-
feedback	-	-	-	1
It helped but no reason is given	-	-	2	-

FS's students (7) provided also diverse reasons: visualizing (4 students), in particular the visualization of the functional relation: "At the animation, you saw at the bowl how the proportions between the amount of water and its height were". It clarifies (two students, e.g. "It looked clear"). And one student pointed seeing the results. This student explains in detail how the contrast between the figure he gets and the one he expects trigger him to keep trying. In his words: "Because if you got something that did not exactly look like what you thought, than you think about yourself "how should I do it to get what I wanted" such evaluations work well. And, in my case, it also makes me want to try out other lines and what's going on"

Thirteen students of RJ answered the question. They referred to receiving the explanation and some in particular, teachers' explanation with the tool (4 students, e.g. "by the teacher's explanation when the assignment ended"). Other students refer to the visualization (3) and clarification (2). Two students stated they were helped but they don't explain the reason (helped but no reason is given). The answers provided by this class are vague and gives much room for interpretation.

The students of JV (2) pointed the feedback and seeing the result at the end. One of the students explains: " because of the feedback is clear why I did something wrong ". The other student states: 'Because at the end he showed my drawn jar, I understood it better".

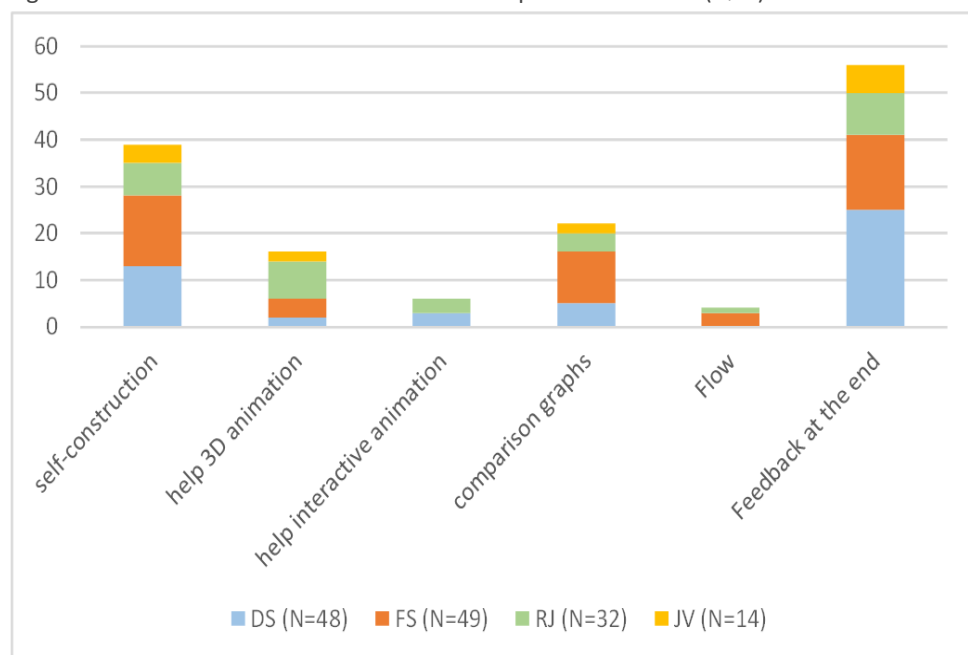
In summary, the self-thought reasons reported by the students who were helped by the tool varied much per class. The reasons "one seeing the results" and "visualizing" were pointed by students from three classes and 'help to improve the form' from students from two classes. The rest of the reasons were mentioned only in one of each class.

WHICH TOOL FEATURES HELPED THE STUDENTS MOST

Question 10 is similar to question 9 but it focus on the tool-features; it is a multiple choice and it allows for multiple answers and comments. The results are presented in the appendix tables (in Dutch) and in Fig. 5. In all classes with exception with FS the students answered this question, in FS one student didn't answer. Students could select more than one tool-feature.

In DS' class the tool feature that most helped students was to get the form of the jar at the end (89%) followed by drawing themselves the graph (46%). The other features were also mentioned by few students; no-one chose the flow. The students of FS and JV, as DS' students reported that most helping features were the feedback at the end (respectively 80% and 67%) and the self-construction (respectively 75%, 44%). In RJ's class the tool feature that most helped students was to get the form of the jar at the end (43%) followed the help 3D animations (38%) and then by self-drawing (33%). In all classes there were students who mentioned the 'comparison of the graph'. This percentage was particularly high by FS's class (55%). The help-features were not often mentioned; in particular the help-feature: interactive animation, which was not even mentioned in two of the classes.

Fig. 5 - Tool features that students found to help them the most (Q10)



In summary, the specific tool-features that students reported that helped them varied per class. But all classes reported that seeing the result of the form of the jar at the end and the self-construction graph were the most helping to them (with exception of the RJ class, in which a slightly higher percentage pointed the help 3D animation as more helpful than the self-construction). Also the comparison feature was considered by the four classes helpful and in particular by the FS's students. The help-features were not often mentioned.

WHAT DID STUDENTS MOST LIKE ABOUT THE APPLICATION

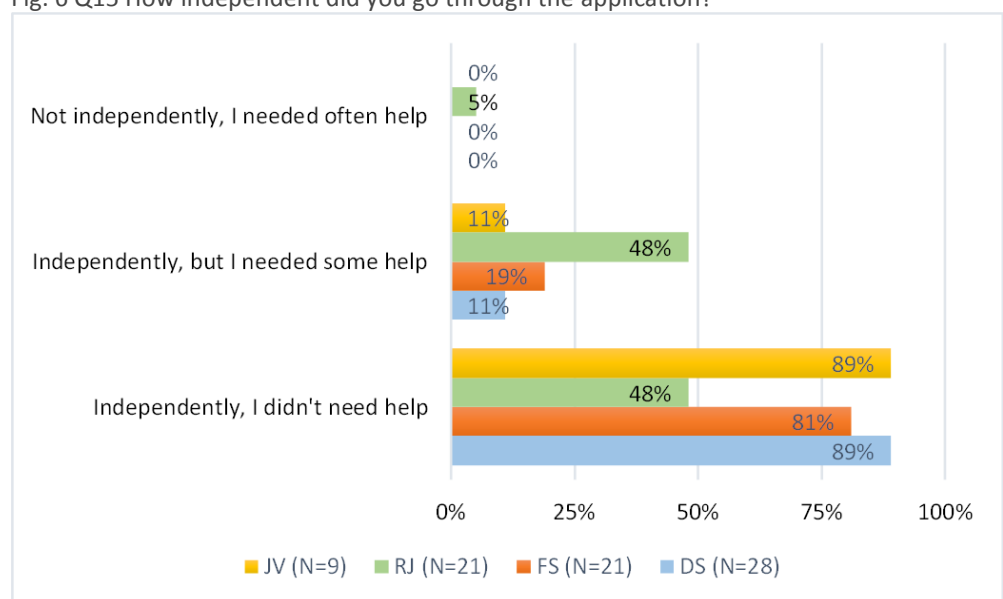
Question 14 assess how the tool is attractive for the students. The aspects more attractive to DS and FS's students were self-construction (respectively 50%

and 40%), the form of the jar at the end (respectively 68% and 76%) and feedback about your work (respectively 46% and 57%). These three features scored high also in the classes of RJ and JV but in the classes there was more dispersion in the answers. In the class of RJ the value of feedback about your work was as valued as the help 3D animation, the 3D experience and flow (29%). In the class of JV the self-construction and the feedback about your work was as valued as the help 3D animation (33%). The independent work was not often mentioned.

STUDENTS' SELF-REPORTED ABILITY TO WORK INDEPENDENTLY WITH THE TOOL

Fig. 6 regard to students' responses to questions Q15 and assesses the extension at which they were able to work independently with it.

Fig. 6 Q15 How independent did you go through the application?



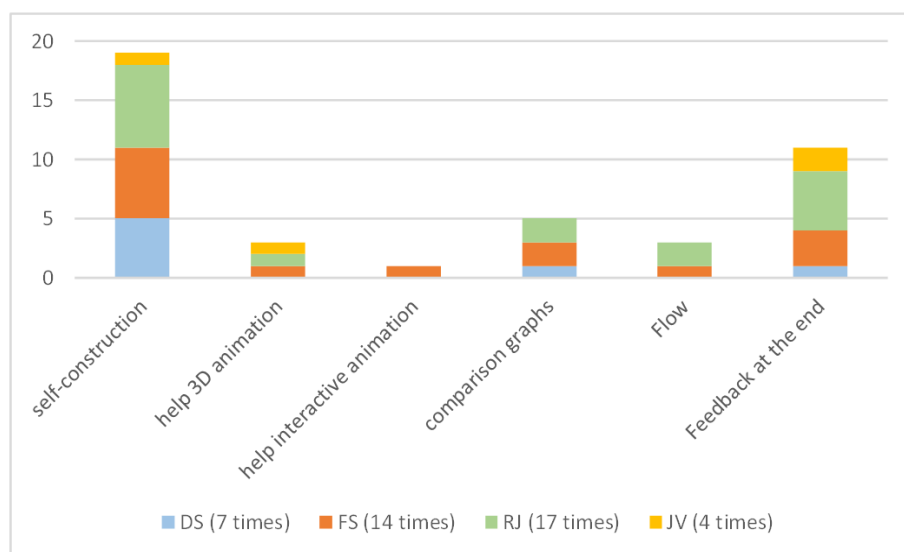
All students in the four classes (with exception of one student in RJ's class) reported that they could work independently with the tool (Q14). In the classes of DS, FS and JV a great percentage of the students (81%-89%) reported that they haven't needed help at all and a small percentage reported that they felt the need of some help (11%-19%). In the case of RJ class about the half (48%) didn't need help and the other half (48%) needed some help.

TOOL FEATURES THAT NEEDED HELP

In Q16 students could specify in which features of the tool they needed help. Twelve answers were given in DS class; almost the half (5 times) expressed need for help by the construction of the graph; help by the comparison of graphs (one time) and by the feedback at the end (one time) the rest (5 times) expressed no need for help. Nineteen students in FS class provided twenty three answers to the question; about one third regards difficulty with the construction of the graph (6 times) and the feedback at the end (3 times); the other features were also mentioned one or two times. Almost the half (9 answers) reported no need for

help. In the class of RJ, eighteen students answered the question and produced twenty answers. As the other two classes it was mentioned the difficulty with the construction of the graph (7 times) and the feedback at the end (5 times). The other features were also mentioned one or two times. In contrast with the other classes need for help was less mentioned (17%). In class of JV eight answers were produced by eight students. The halve of these answers referred no need for help; need for help by the construction of the graph was once mentioned; by the feedback at the end (two times) and one time by the 3D animation.

Fig. 7 - Q16 Specify which part you needed help



SUGGESTIONS FROM STUDENTS FOR IMPROVING THE TOOL

Finally, students' responses to question 17 provided suggestions to improve the tool and its use. A total of 51 suggestions were provided by the students of the four classes, which were bundled in main categories (Table 3). A few number of students stated that the tool doesn't need to be improved and some students said that they don't have suggestions. For instance one student stated:

"I found the tool easy to understand, so I have no concrete suggestions. I found it very clear, easy to understand if you do not know how to see the graph. I have no suggestions for this part."

Another student stated also that the tool is well as it is; this student provides also an interesting remark about the length of the tool that we should consider for further development of the tool:

"I think so well. It's not too long, so the" players "do not lose their attention either"

Most suggestions regards the drawing of the graph (19 suggestions in total). Some students stated that the graph was difficult to draw with some devices. We took this as an implicit suggestion to improve this feature of the tool (4). Other suggestions (15) are more concrete and provide directions. Some students proposed making available options in the tool that makes possible to draw piece

of graph - a straight line or a curve. Some students refer to specifically to the difficulty in drawing with the device (mostly the mouse

Table 3 Students' suggestions for improving the tool and its use in the class (Q17)

Tool features (suggestions)	Example
self- construction (19) Improve the drawing of the graph but no concrete directions (4) less unevenness in the graph (15) Some options are proposed by the students as making available options in the tool that makes possible to draw piece of graph - a straight line or a curve. Other students refer to the difficulty in drawing without providing direction for improvement. Some students refer to specifically to the difficulty in drawing with the device (mostly the mouse)	"Drawing the graph is very difficult with the mouse"(JV) "Let the 'players' draw the graph themselves, but make sure the graphs drawn have less unevenness."(DS); "That there are standard types of lines and that you have to choose it and then deform yourself"(DS); "erase some of the chart instead of erasing the entire graph"(FS); "A drawing pallet to help drawing lines because now the figure in the end does not look beautiful."(DS); "Option to self-fill in points in order to make it more accurate" (FS)
Feedback (5) Want to confirm whether the graph is correct or not (1) Get real-time feedback from the application; The feedback is only coming to an end; Advancing the shape of the fast could be an option (or an alternative (4)	"I could not see if my graph was approved or not. I could only judge this myself" (FS) "Option to see the jar within the program in real time while drawing the graph" (DS); "I'd rather have seen earlier my drawing from the graph" (DS)
instruction (4) Better explanation from the teacher or on the screen (3) provide an example (1)	"explain how it works" (FS) "give an example of a graph by other jar" (FS)
flow (2) go through the tool again without logging once more (1) put the round jar- assignment and the cylinder assignment in the same page and after each other (to facilitate the comparison) (1)	"at the end by the jars you can simply return to the previous menu"(RJ) "by putting the assignments above each other one can make the comparison right away" (FS)
help interactive animation (1) the interface from Help interactive animation must be improved	"I did not fully understand what was meant by the variable colors in the interactive section, but this may be due to an incorrect view." (DS)
Virtual Reality (2) VR-technology should work better	"The VR part does not work very intuitively: the camera moves around, looking upwards, the camera moves downwards"(DS)
other (6) varied suggestions technical (3) textual (3)	"Some graphs looked as good as the correct graph but they were considered to be wrong" (FS); "in the mobiel is everyting too small" (FS) " change jar in jar"(JV); the x and the y -asses are not defined" (DS); "x en y as staan niet "gedefinieerd"(DS)
expand the tool (10) to expand it in general no clear directions (2) to extend the tool in difficulty level (1) To expand it with more exercises (3), in	"it should be expanded"(DS) "more challenging"(RJ); A jar with angular shapes to elicit more thinking" (DS); "at the end more types of jars"(DS)

particular to consider more variety of jars-shapes (4)	"more exercises" (FS)
use of the tool inside and outside the class (3)	"to be used as housework ; is modern and nicer"(FS);
ideas to use the tool inside and outside the classroom included housework (1), as game to exercise mathematics (1) and to be often used in the lesson (1)	"as a game; for instance to practice mathematics"(DS); "to be used often in lesson; it should be nice"(RJ)

DISCUSSION AND IMPLICATIONS

STUDENTS SELF-REPORTED LEARNING WITH THE TOOL

Students' responses in sub-section 5.1-5.4 provided information about how they think to have learned with the tool. First, in general, the majority felt helped by the tool but, differ in their use of the support.

- Students in advances classes 11th grade with mathematics B and bachelor reported to have already an idea about the shape of the graph and the tool help them to work it out and consolidate this idea or to correct some mistake. This was also the case of many students in 10th grade following pre-univ stream. In these three classes the students who reported not having been helped by the tool already knew about the graph and the tool was not needed.
- Students from the 10th grade following vocational stream (about the half) report o they did not have previously any idea about how the graph would be or they had vague idea. The tool helped them in the construction of the graph and to improve their vague initial image. The majority of students in this class who reported not felt the tool support was because they don't understand the topic and only a few percentage reported to know about the graph and therefore the tool was not needed.

These results points that the tool offers different support for students who already have some knowledge about graphs from dynamic events than from students who don't. In the last case it can happen that the tool doesn't help at all. As we have seen by the students responses from the class RJ. However, the fact that the tool doesn't help to accomplish the assignment still doesn't mean that it doesn't add understanding to the subject. Students pre-knowledge showed therefore to be a crucial factor in the way students learn with the tool. We should be careful however with this interpretation as the results of one class cannot be generalized; other factors (besides coincidence) can play a role in the difference: the teacher, the lesson duration, RJ was also the only teacher who did not use work from his students in the final discussion. Anyway; there was a group of students who needed support and it were not helped with the tool. Understand why and what can be improved in the tool are important aspects to take for further investigation.

Secondly, the reasons pointed by the students with regard to the learning with the tool were much varied. The most common reasons were:

- "one seeing the results" and "visualizing" , which were pointed by students from three classes and 'help to improve the form' , which was pointed by students from two classes.
- The rest of the reasons were mentioned only in one of each class, which suggests that the tool elicits a diversified range of experiences.

According to a socio-constructivists perspective on learning students learn through trying from themselves and in interaction with others. In this view reasons that involve more learner participation (drawing, providing self-explanation, help to improve) can indicate more opportunity for learning than passive reasons (e.g. receiving explanation). If this is the case the students in the class of DS have been more supported by the tool than the students in the class RJ, although these ones were the ones who probably had more need of the tool. This is a critical aspect that we should consider seriously for the further development of the tool. There was a great variety of reasons presented by the students. We designed the tool to allow students to work at their own level and therefore the support provided by the tool will be much influenced by their background. A limitation of these results is the summary of the reasons in categories. These reasons were not always clear and we made some inferences about what the students wanted to say. Specially in the class of RJ the answers were much vague. It is possible that some categories would drop off and some new would appear if we had opportunity to prompted the students to clarify their answer.

Third, the specific tool-features that students reported that helped them varied per class. But all classes reported that:

- Seeing the result of the form of the jar at the end and the self-construction graph were the most helping to them (with exception of the RJ class, in which a slightly higher percentage pointed the help 3D animation as more helpful than the self-construction). Also the comparison feature was considered by the four classes helpful and in particular by the FS's students.

The help-features were not often mentioned. We find this surprising. From the questionnaire data is not possible to understand the reason behind this result. One possibility is that the students didn't consult the then help. While students were obligated to use the other features the help feature was not compulsory. Other possibility is that the help is not adequate for the students. Another possibility is that the students didn't understand how the help worked and therefore they could not use it properly. More insight on this matter will be provided from the analyses of the logbook data, lesson observations and interviews.

HOW STUDENTS' WORK WITH THE TOOL IN THE CLASS

Students' responses to questions Q13-Q17 provided information about aspects related with the usability of the tool by the students: attractive, working independently and need for help.

- The most attractive aspect of the tool to the students in all classes was the self-construction or the form of the jar at the end. Other features found

attractive by the students but with lower frequency were feedback about your work, help 3D animation and 3D experience.

- All students from the four classes with exception of one could work independently with the tool. Only eighteen students needed help at some point.
- The tool-features that requested more help were the self-construction (mentioned 18 times in all classes) and the feedback by the submission of the graph (mentioned 11 times).
- Students suggestions provided insightful ideas to improve the tool.

Other tool features were mentioned but with much less frequency. The help features were rarely mentioned in students responses and students' difficulty with the feature help interactive animation was mentioned only once. We find this intriguing because in the lesson observations we observed students struggling with this tool-feature. This is one of the aspects that we need to further investigate through, for instance interviews.

FINAL CONSIDERATIONS

With the increasing availability of technology it becomes more attractive for teachers and researchers to develop digital tools that can be used to support learning. The development of such tools, however, requires specific and varied knowledge from the designer: and about the user experience. In this study we have learned more about how students experienced and used IVM in the classroom. The findings suggest that the students can work independently with the tool in the classroom and without much help. The tool can create opportunities for students to produce and try to improve a mathematical representation of a dynamic event. However, we do not provide much information about the process of coming to generate the graph representations and verbal explanations and its transformation. This study invites further research on this matter. Our research also calls for an extension of the tool and improvement of some features, students provided insightful suggestions that can help us in this direction.

Matemática virtual interativa: um estudo exploratório na sala de aula

RESUMO

Matemática Virtual Interactiva (*Iteractive Virtual Math - IVM*) é uma ferramenta digital de visualização para apoiar a aprendizagem de gráficos de eventos dinâmicos no ensino secundário. Na versão protótipo, os alunos constroem um gráfico e tentam melhorá-lo e com o *feedback* da ferramenta. Numa experiência de investigação que envolveu quatro turmas de ensino secundário e terciário e seus professores de matemática, investigamos como os alunos usaram a ferramenta na sala de aula. Neste estudo, focamo-nos na experiência de aprendizagem dos alunos e nos resultados que possam fornecer orientações para o desenvolvimento futuro da ferramenta. Os dados analisados consistem em questionários e observações de sala de aula. Um dos resultados principais é que alunos, em diferentes níveis escolares, consideram a ferramenta útil para construir ou melhorar representações gráficas e contribui para a sua compreensão do tema. As características da ferramenta que mais ajudaram os alunos foram a construção dos gráficos e a obtenção de *feedback* sobre o seu próprio gráfico entregue no final. Outro resultado são que os alunos conseguem trabalhar de forma independente com a ferramenta e novo conhecimento sobre as características da ferramenta que são atraentes para os alunos ou que precisam ser melhoradas.

PALAVRAS CHAVE: Estudantes que aprendem. Virtual Interativo. Matemática. Sala de aula.

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