Brazilian Journal of Instrumentation and Control
REVISTA BRASILEIRA DE INSTRUMENTAÇÃO E CONTROLE

Optimal Estimation of Mass Partition in a Centrifugal Separator by Metallurgical Balance

Sidney A. A. Viana

Abstract — **Mineral processing plants make use of several types of equipments for size separation (size classification) to segregate ore particles by size. A particular type of such equipments is the centrifugal separator, which is intended to receive an input stream of ore slurry to be partitioned into two output streams: a coarse and a fines one. The coarse stream contains most of the coarse solids particles of the slurry, whereas the fines stream contains most of the fine particles. Although a centrifugal separator intends to perform a physical segregation of the solids particles by their size, a chemical segregation also results, in such a way that the chemical content of the coarse and the fines streams are normally different from the content of the input stream. When evaluating the performance of the separation process, three fundamental aspects should be analyzed: 1) the size distribution of the solids particles in each stream; 2) the amount of solids mass from the input stream that goes to the coarse stream and to the fines stream, that is, the mass partition; and 3) the chemical content of each stream. This work presents the application of the Least Squares method of optimization to calculate the mass partition, based on the measured chemical content of the streams, and on the metallurgical balance equations of the separation process.**

Index Terms — **least squares, mass balance, mass partition, metallurgical balance, size separation, optimization.**

I. INTRODUCTION

BAUXITE ore is the basic raw material for the aluminium production chain [1], which has four major steps, normally implemented as separate industrial plants: 1) processing of the raw ore from the bauxite mines, to produce concentrated ore; 2) refining the concentrated bauxite ore through the Bayer process [2], to produce high-purity smelter-grade alumina; 3) electrolytic reduction of the smelter-grade alumina in electrolytic furnaces, to produce primary aluminium in the form of ingots, billets, or slabs; and 4) smelting and conformation of the primary aluminium, to generate aluminium products.

The main chemical compound of interest in the bauxite ore is the alumina (Al_2O_3), but some other chemicals are also considered to characterize the chemical profile of the ore: silica (SiO_2), iron dioxide (FeO_2), and titanium dioxide (TiO_2). The concentrations of chemicals in an ore can be measured by specific methods like X-ray mass spectrometry, atomic absorption spectroscopy, inductively coupled plasma spectrometry, or chemical reaction methods. Moreover, a certain amount of the content is characterized as "loss on ignition" (LoI), which represents the amount of moisture or impurities lost when the ore is ignited under specific conditions. For purposes of chemical content characterization, the LoI can be regarded as an additional chemical compound.

The Bayer process is the most widely used in industrial scale to produce high-purity alumina from bauxite ore. Only part of the total alumina in the ore, called *valuable alumina*, can be extracted by reagents in the Bayer process, to produce smelter-grade alumina. And conversely, a certain portion of the total silica in the ore, known as *reactive silica*, is a harmful compound because it competes with the valuable alumina by the reagents, thus impairing the extraction of this latter.

Due to this fact, bauxite ore plants are required to produce, as input for the Bayer process plants, concentrated ore with higher content of valuable alumina and lower content of reactive silica. This requirement should be extended to all mineral processing equipments in an ore concentration plant. Another important requirement is that the equipments should be able to convert most of the feeding (input) ore into product (output), thus reducing the amount of reject (waste) material. The ratio between the product and the feeding masses is referred as massic recovery, and is a measure of equipment performance.

In this context, this article describes the determination of the mass partition performed by a centrifugal separator [3]. The main contribution of this work is to demonstrate how to determine the the massic recovery of a mineral processing equipment from chemical compound measurements of the streams, when it is not feasible or impractical to directly measure the mass of ore in the streams. The article is organized as follows: Section II presents the material processing performed by a centrifugal separator with three streams (feeding, product, and reject). Section III the metallurgical

S. A. A. Viana is a Senior Member of the IEEE – The Institute of Electrical and Electronics Engineers. He is currently with the Ferrous Automation Engineering Department of VALE, Belo Horizonte, MG, Brazil (e-mail: sidney.viana@vale.com).

balance problem relating the mass partition and the chemical compound measurements of the streams. Section IV describes the solution of the metallurgical balance problem using an optimization method, in order to compute the mass partition performed by the centrifugal separator. Finally, Section V discuss the results obtained.

II. Processing of Bauxite Ore in a Centrifugal Separator

Mineral processing plants make use of several types of size classification (size separation) equipments to segregate ore particles, either in the form of bulk solids or slurries, by size [4]. Size separation is needed due to several reasons like: 1) to feed coarse particles back to breakage processes; 2) to meet product size requirements; or 3) to segregate different mineral species in the ore to meet chemical content requirements. For the processing of ore slurries, a particular type of classification

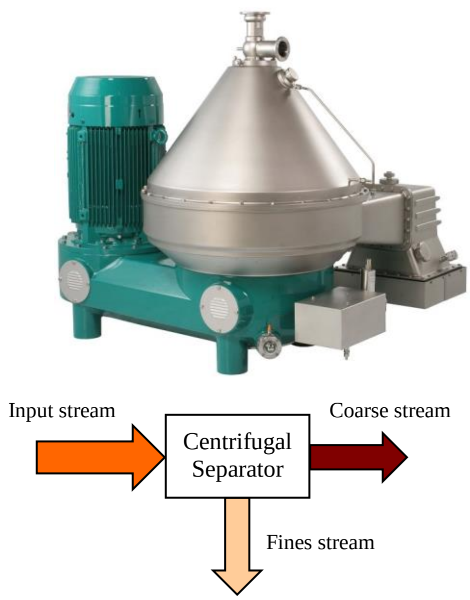

Fig. 1. A centrifugal separator for industrial applications [3] and its block diagram.

equipment is the centrifugal separator [3], which is intended to divide an *input stream* of ore slurry into two output streams: a *coarse stream* and a *fines stream*. The coarse stream contains most of the coarse solids particles of the slurry, whereas the fines stream contains most of the fine particles. Fig. 1 shows an example of centrifugal separator for industrial applications, and a block diagram of its separation process.

The size separation process performed by a centrifugal separator (indeed, by any type of separation equipment) is not perfect, so that it will exist a certain amount of fine particles in the coarse stream, as well as a certain (although very much lower) amount of coarse particles in the fines stream, as illustrated in Fig. 2. The main goal of a centrifugal separator is to produce a coarse stream with considerably less fine particles than the input stream. The size distribution of the particles in the coarse and the fines stream will depend not only on the separation performance of the equipment, but also on the size

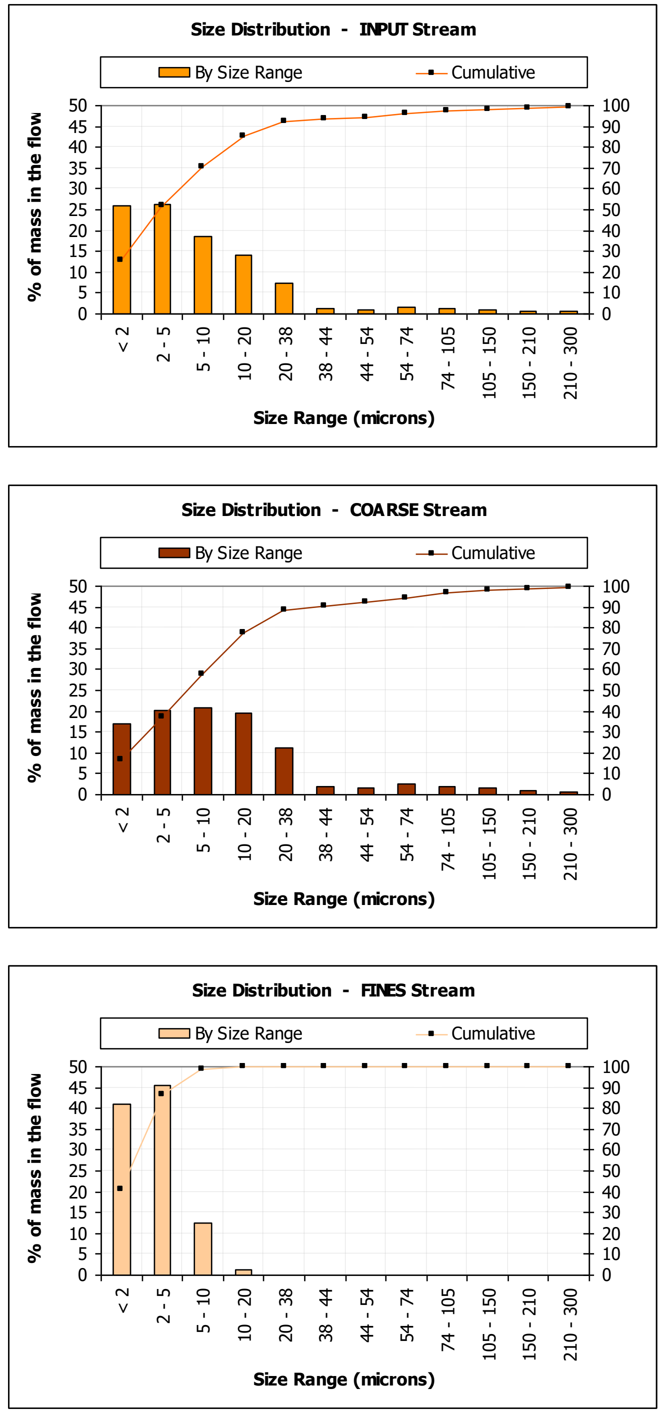

Fig. 2. Particle size distribution of each stream for the centrifugal separator.

distribution of the input stream.

Although a centrifugal separator intends to perform a *physical segregation* of the solids particles by their size, a *chemical segregation* also results, in such a way that the chemical contents of the coarse and the fines streams are normally different from the content of the input stream. This is because the coarse particles typically result from the hardest mineral species in the ore, whereas the fine particles result from the softest mineral species. The main mineral species that form our bauxite ore are the gibbsite and the kaolinite. The first is

DOI:
ISSN: 2318-4531

TABLE I
MEASURED CHEMICAL CONTENTS OF EACH STREAM IN A CENTRIFUGAL SEPARATOR, FOR NINE MEASUREMENT TESTS.

Test	Stream	Al_2O_3 %	SiO_2 %	FeO_2 %	TiO_2 %	LoI %	Residue %	Total %
T-01	Input	43,529	19,060	13,928	2,087	20,950	0,446	100,000
	Coarse	46,691	15,614	13,143	2,119	22,353	0,080	100,000
	Fines	37,089	28,686	16,149	2,093	13,162	2,821	100,000
T-02	Input	43,212	19,383	13,804	1,963	20,698	0,940	100,000
	Coarse	46,681	14,740	12,596	1,982	22,752	1,249	100,000
	Fines	37,650	28,117	16,032	2,063	12,844	3,294	100,000
T-03	Input	43,035	19,841	14,067	2,060	20,453	0,544	100,000
	Coarse	47,649	13,595	12,470	2,165	23,330	0,791	100,000
	Fines	38,322	27,594	15,931	2,050	16,942	-0,839	100,000
T-04	Input	42,530	19,103	13,865	2,002	20,495	2,005	100,000
	Coarse	47,300	15,705	13,096	2,155	22,416	-0,672	100,000
	Fines	36,754	27,650	16,119	2,052	16,482	0,943	100,000
T-05	Input	43,847	18,770	13,957	1,999	20,741	0,686	100,000
	Coarse	49,449	12,299	12,200	2,182	23,660	0,210	100,000
	Fines	40,017	26,106	15,990	2,044	17,714	-1,871	100,000
T-06	Input	50,065	6,431	12,745	1,465	26,192	3,102	100,000
	Coarse	53,743	5,322	11,219	1,487	27,138	1,091	100,000
	Fines	46,632	10,181	17,551	1,707	23,681	0,248	100,000
T-07	Input	51,279	6,580	13,012	1,495	26,449	1,185	100,000
	Coarse	53,190	6,025	12,108	1,491	26,722	0,464	100,000
	Fines	45,007	11,604	19,811	1,847	22,256	-0,525	100,000
T-08	Input	51,685	6,697	13,137	1,499	26,389	0,593	100,000
	Coarse	52,699	6,273	12,490	1,477	26,489	0,572	100,000
	Fines	41,489	13,546	21,831	1,949	20,713	0,472	100,000
T-09	Input	40,576	20,437	14,853	1,960	19,584	2,590	100,000
	Coarse	47,590	13,129	13,657	1,946	23,410	0,268	100,000
	Fines	40,270	23,345	15,782	2,048	18,463	0,092	100,000

hard and contains most of the valuable alumina, while the latter is soft and contains most of the reactive silica.

Table I shows the chemical contents of each stream in a centrifugal separator, measured by inductively coupled plasma spectrometry, for nine tests. Note that, for each stream, the sum of the concentrations of all its chemicals, including the LoI, does not result equal to the ideal value 100%. This is due to: 1) the existence of other disregarded chemicals with very small concentration values, which are not representative of the ore, and so were not taken into account in the content measurements; and 2) measurement errors from the plasma spectroscopy (even if those errors are very small, they will actually exist). The values that make the sum of the concentrations of all the chemicals exactly equal to 100% are the *residue* values.

III. THE METALLURGICAL BALANCE PROBLEM

Let φ denotes any chemical compound of interest in the bauxite ore, that is, $\varphi \in \{Al_2O_3 , SiO_2 , FeO_2 , TiO_2 , \text{“LoI”}\}$. Moreover, let m_I, m_C, and m_F denote the input, coarse, and fine solids mass, respectively; and c_I, c_C, and c_F denote the concentration of any chemical compound φ in the input, coarse, and fines stream.

The performance of a mineral processing equipment is often characterized by its *metallurgical recovery*, which is a measure of how much a chemical compound pass from the input stream to an output stream. As an example, the metallurgical recovery of alumina to the coarse stream is given by:

$$r_{C(Al)} = \frac{c_{C(Al)}}{c_{I(Al)}} \times \frac{m_{C(Al)}}{m_{I(Al)}} \tag{1}$$

In a generic form, the metallurgical recoveries of any chemical compound φ, for the coarse and the fines stream are, respectively:

$$r_{C(\varphi)} = \frac{c_{C(\varphi)}}{c_{I(\varphi)}} \times \frac{m_{C(\varphi)}}{m_{I(\varphi)}} \tag{2}$$

$$r_{F(\varphi)} = \frac{c_{F(\varphi)}}{c_{I(\varphi)}} \times \frac{m_{F(\varphi)}}{m_{I(\varphi)}} \tag{3}$$

As described in the previous section, although a centrifugal separator performs a physical segregation of the solids particles by their size, a chemical segregation also results. If each stream in a separator is characterized by its solids mass and its chemical content, a problem that arises is: *by knowing (measuring) the chemical content of the three streams, is it possible to determine the solids mass in each stream, allowing the calculation of the metallurgical recovery?* To answer this question, one needs to understand the material balance equations of the separation process. As the input mass m_I is partitioned by the centrifugal separator to the coarse solids mass m_C and the fine solids mass m_F, with no accumulation of mass inside of the equipment, the following *mass balance* relationship must be satisfied:

$$m_C + m_F = m_I \tag{4}$$

Moreover, the *metallurgical balance* equation for any chemical compound φ is:

DOI: ISSN: 2318-4531

$$c_{C(\varphi)}.m_C + c_{F(\varphi)}.m_F = c_{I(\varphi)}.m_I \tag{5}$$

Therefore, for all the chemicals of interest, the following set of equations is derived from (4) and (5):

$$\begin{cases} m_C + m_F - m_I = 0 \\ c_{C(Al)}.m_C + c_{F(Al)}.m_F - c_{I(Al)}.m_I = 0 \\ c_{C(Si)}.m_C + c_{F(Si)}.m_F - c_{I(Si)}.m_I = 0 \\ c_{C(Fe)}.m_C + c_{F(Fe)}.m_F - c_{I(Fe)}.m_I = 0 \\ c_{C(Ti)}.m_C + c_{F(Ti)}.m_F - c_{I(Ti)}.m_I = 0 \\ c_{C(LoI)}.m_C + c_{F(LoI)}.m_F - c_{I(LoI)}.m_I = 0 \end{cases} \tag{6}$$

The goal would be to solve (6) for $\{m_C, m_F, m_I\}$, by knowing the content values $\{c_{C(\varphi)}, c_{F(\varphi)}, c_{I(\varphi)}\}$ for all chemicals, and then calculate the metallurgical recovery by (1). Nevertheless, it's not possible to find an unique solution $\{m_C \neq 0, m_F \neq 0, m_I \neq 0\}$ for (6), because it is a homogeneous set of equations and the chemical content values have some measurement errors, so that the only possible solution would be the trivial solution $\{m_C = 0, m_F = 0, m_I = 0\}$. Clearly, if $\{\hat{m}_C, \hat{m}_F, \hat{m}_I\}$ is a solution of the set, then $\{k.\hat{m}_C, k.\hat{m}_F, k.\hat{m}_I\}$, where k is a positive real number, is also a solution. In other words, there will be infinite solutions, and any particular solution will be found only if one of the three masses $\hat{m}_C$, $\hat{m}_F$ or $\hat{m}_I$ is specified (measured), allowing to solve the set of equations for the two others. Fortunately, the terms m_C/m_I and m_F/m_I in (2) and (3) make unnecessary to know the actual values of these masses, allowing to work in percent basis, by setting m_I = 100% and then solving the set of equations for $\{m_C, m_F\}$. By this way, equation (6) can be rewritten as:

$$\begin{cases} m_C + m_F = 100 \\ c_{C(Al)}.m_C + c_{F(Al)}.m_F = 100.c_{I(Al)} \\ c_{C(Si)}.m_C + c_{F(Si)}.m_F = 100.c_{I(Si)} \\ c_{C(Fe)}.m_C + c_{F(Fe)}.m_F = 100.c_{I(Fe)} \\ c_{C(Ti)}.m_C + c_{F(Ti)}.m_F = 100.c_{I(Ti)} \\ c_{C(LoI)}.m_C + c_{F(LoI)}.m_F = 100.c_{I(LoI)} \end{cases} \tag{7}$$

or in matrix form:

$$\begin{bmatrix} 1 & 1 \\ c_{C(Al)} & c_{F(Al)} \\ c_{C(Si)} & c_{F(Si)} \\ c_{C(Fe)} & c_{F(Fe)} \\ c_{C(Ti)} & c_{F(Ti)} \\ c_{C(LoI)} & c_{F(LoI)} \end{bmatrix}_{N\times 2} \times \begin{bmatrix} m_C \\ m_F \end{bmatrix}_{2\times 1} = \begin{bmatrix} 100 \\ 100.c_{I(Al)} \\ 100.c_{I(Si)} \\ 100.c_{I(Fe)} \\ 100.c_{I(Ti)} \\ 100.c_{I(LoI)} \end{bmatrix}_{N\times 1}$$

$$\mathbf{C} \times \mathbf{M} = \mathbf{K} \tag{8}$$

where **C** is the matrix of output concentrations, **M** is the vector of output masses (to be determined), **K** is the vector of input concentrations, N = 6 is the number of equations in (7), and m_C and m_F are in percent basis.

Now, the goal is to solve (8) to find $\mathbf{M} = [m_C \;\; m_F]^T$.

IV. Solving the Metallurgical Balance Problem by Optimization

Due to the fact that the chemical content values $\{c_{C(\varphi)}, c_{F(\varphi)}, c_{I(\varphi)}\}$ in (8) have some measurement errors, and that **C** is not a square matrix, the best way to solve (8) for **M** is by using an optimization method, like the Least Squares method [4;5;6;7]. This is explained in the following.

Let $\hat{\mathbf{M}}$ denotes an estimate of **M**, and $\hat{\mathbf{K}} = \mathbf{C} \times \hat{\mathbf{M}}$ the corresponding estimate of **K**, according to (8). The estimation error is given by:

$$\mathbf{E} = \hat{\mathbf{K}} - \mathbf{K} = \mathbf{C}\hat{\mathbf{M}} - \mathbf{K} \tag{9}$$

Each component e_i in vector **E** is the error between the estimated value $\hat{k}_i$ in $\hat{\mathbf{K}}$ and its corresponding measurement k_i in **K**. As an indicator of the quality of the estimate $\hat{\mathbf{M}}$, one considers the mean squared error, denoted by J:

$$\begin{aligned} J &= \frac{1}{N}\sum_{i=1}^{N} e_i^2 = \frac{1}{N}\|\mathbf{E}\|^2 = \frac{1}{N}\mathbf{E}^T\mathbf{E} \\ J &= \frac{1}{N}(\mathbf{C}\hat{\mathbf{M}} - \mathbf{K})^T(\mathbf{C}\hat{\mathbf{M}} - \mathbf{K}) = \frac{1}{N}(\hat{\mathbf{M}}^T\mathbf{C}^T - \mathbf{K}^T)(\mathbf{C}\hat{\mathbf{M}} - \mathbf{K}) \\ J &= \frac{1}{N}\left(\hat{\mathbf{M}}^T\mathbf{C}^T\mathbf{C}\hat{\mathbf{M}} - \hat{\mathbf{M}}^T\mathbf{C}^T\mathbf{K} - \mathbf{K}^T\mathbf{C}\hat{\mathbf{M}} + \mathbf{K}^T\mathbf{K}\right) \end{aligned} \tag{10}$$

The principle of Least Squares states that $\hat{\mathbf{M}}$ will be the best or optimal estimate if it minimizes the error indicator J, that is:

$$\begin{aligned} \frac{\partial J}{\partial \hat{\mathbf{M}}} &= \frac{1}{N}\left(2\mathbf{C}^T\mathbf{C}\hat{\mathbf{M}} - 2\mathbf{C}^T\mathbf{K}\right) = \mathbf{0} \\ \frac{\partial^2 J}{\partial \hat{\mathbf{M}}^2} &= \frac{1}{N}\left(2\mathbf{C}^T\mathbf{C}\right) \geq \mathbf{0} \end{aligned} \tag{11}$$

Therefore, the optimal solution for (10) is given by:

$$\hat{\mathbf{M}} = \left(\mathbf{C}^T\mathbf{C}\right)^{-1}\mathbf{C}^T\mathbf{K} \tag{12}$$

The solution $\hat{\mathbf{M}}$ in (12) will only exist if $\mathbf{C}^T\mathbf{C}$ is non-singular (invertible). Normally, the measurement errors in the chemical content values will cause $\mathbf{C}^T\mathbf{C}$ to be non-singular. Equation (11) means that matrix $\mathbf{C}^T\mathbf{C}$ must be non-negative definite.

If one wants that the solution $\hat{\mathbf{M}}$ makes the first equation in (7) to be exact up to some degree of accuracy, one can consider an extension of the basic Least Squares, called Weighted Least Squares [4,5], which includes a diagonal matrix **W** whose diagonal elements are weights for each individual equation in (7). In this case, the optimal solution is given by:

DOI: ISSN: 2318-4531

$$\hat{\mathbf{M}} = \left(\mathbf{C}^{\mathrm{T}}\mathbf{W}\mathbf{C}\right)^{-1}\mathbf{C}^{\mathrm{T}}\mathbf{W}\mathbf{K}$$
$$\mathbf{W} = \left(w_{ij}\right)_{N\times N} \quad | \quad w_{ij} = 0 \text{ for } i \neq j;\; w_{ij} = 1 \text{ for } i = j;\; w_{11} = \lambda \qquad (13)$$

where λ should be a high value empirically chosen to make the first equation in (7) as exact as desired. Typical values for λ are $10^5 \sim 10^9$. Equation (12) is a particular case of (13), when **W** is an identity matrix, that is, when all individual equations have the same weight, equal to the unity.

Although the Least Squares method is simple, it is very sensible to the presence of *outliers* in the measured data (the chemical content values). Outliers are data values containing high measurement errors, which will impair the accuracy of the solution $\hat{\mathbf{M}}$ given by (13). Therefore, it's important to assure that the measured chemical content values have a good accuracy. A simple – and limited – way to check the accuracy of the chemical content values is by the magnitude of their residues (Table I). Although a low magnitude of a residue does not mean the absence of high errors, if a residue has a significant magnitude, then at least one chemical content value has a high error. In this case, a more accurate measurement of that chemical content values should be performed.

TABLE II
INFLUENCE OF λ ON THE TOTAL MASS, FOR THE DATA FROM TEST T-01.

λ	Coarse Mass (%)	Fines Mass (%)	**Total Mass (%)**
1	74,636	24,960	**99,596**
100	74,633	24,977	**99,610**
1.000	74,619	25,088	**99,707**
1×10^5	74,576	25,413	**99,989**
1×10^7	74,575	25,425	**100,000**
1×10^8	74,575	25,425	**100,000**
1×10^9	74,575	25,425	**100,000**

V. RESULTS

The computational implementation of the Least Squares method to evaluate (13) was done in VBA™ (Visual Basic for Applications), for use in MS-Excel™.

Firstly, it was investigated the influence of λ on the solution $\hat{\mathbf{M}}$, according to (13). Several values for $w_{11} = \lambda$ were tried, in order to satisfy the first equation in (7). This was achieved for $w_{11} = 10^8$, or higher. Table II shows some results, for the data from test T-01 in Table I. Note that when the total mass reaches the ideal value 100%, the coarse and the fine solids masses remain unchanged even if λ is further increased.

Upon verified the influence of λ on the results, it was fixed to $\lambda = 10^8$. Then, the estimate of $\hat{\mathbf{M}}$ and its respective error indicator J were calculated for all the measurement tests, as shown in Table III. The calculated masses of ore in the coarse and fines streams are expressed in percent of the feeding stream, assumed as 100% according to (7). The estimates of $\hat{\mathbf{M}}$ were then used to calculate the metallurgical recoveries of alumina and silica, according to (2) and (3).

To better understand why the calculated masses in Table III are the "best" or "optimal" estimates for $\hat{\mathbf{M}}$, Fig. 3 shows the error indicator J as a function of the masses m_C and m_F, for Test T-01. The shape of J depends on matrix **C** and vector **K**, according to equations (9) and (10), and it clearly indicates that J has a *global minimum*. This minimum is indicated by a dot in Fig. 4, which is just the optimal estimate for $\hat{\mathbf{M}}$, as stated by the principle of Least Squares. Moreover, the straight diagonal gray line in Fig. 4 represents the constraint $m_G + m_F = 100$, which is the first equation in (7). The optimal estimate will stay on that line whenever it meets this constraint, by a proper choice of $w_{11} = \lambda$.

As the quality of the results (estimates of the coarse and the fine solids mass) are influenced by the quality of the data (measured chemical content values), and considering that the

TABLE III
ESTIMATES OF THE COARSE AND FINE SOLIDS MASSES, ERROR INDICATOR J, AND METALLURGICAL RECOVERIES OF ALUMINA AND SILICA.

				Recovery to the Coarse Stream		Recovery to the Fines Stream	
Test	**Stream**	**Calculated Mass (%)**	**J**	**r_C Al_2O_3 (%)**	**r_C Si_2O_2 (%)**	**r_F Al_2O_3 (%)**	**r_F Si_2O_2 (%)**
T-01	Coarse Fines	**74,575** **25,425**	2.814,930	**79,992**	**61,092**	**21,664**	**38,266**
T-02	Coarse Fines	**68,166** **31,834**	3.452,206	**73,639**	**51,838**	**27,736**	**46,178**
T-03	Coarse Fines	**54,018** **45,982**	297,327	**59,810**	**37,013**	**40,946**	**63,950**
T-04	Coarse Fines	**64,919** **35,081**	3.795,958	**72,200**	**53,371**	**30,316**	**50,777**
T-05	Coarse Fines	**49,527** **50,473**	1.999,205	**55,855**	**32,453**	**46,064**	**70,199**
T-06	Coarse Fines	**64,781** **35,219**	4.638,557	**69,540**	**53,610**	**32,804**	**55,755**
T-07	Coarse Fines	**84,834** **15,166**	1.538,745	**87,996**	**77,679**	**13,311**	**26,745**
T-08	Coarse Fines	**92,958** **7,042**	305,100	**94,782**	**87,073**	**5,652**	**14,243**
T-09	Coarse Fines	**21,116** **78,883**	4.682,674	**24,767**	**13,566**	**78,289**	**90,108**

DOI: ISSN: 2318-4531

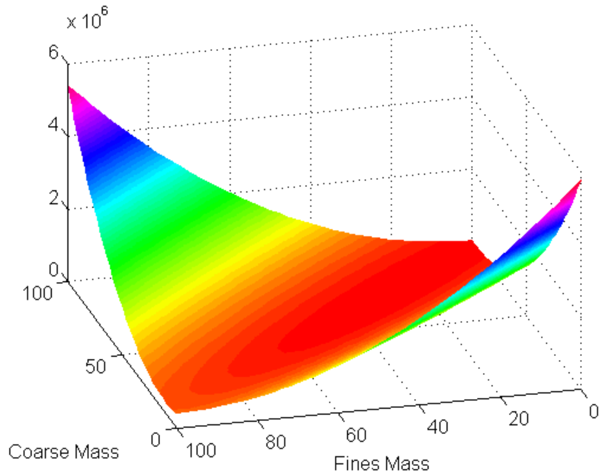

Fig. 3. Shape of the error indicator *J*, for the data from test T-01.

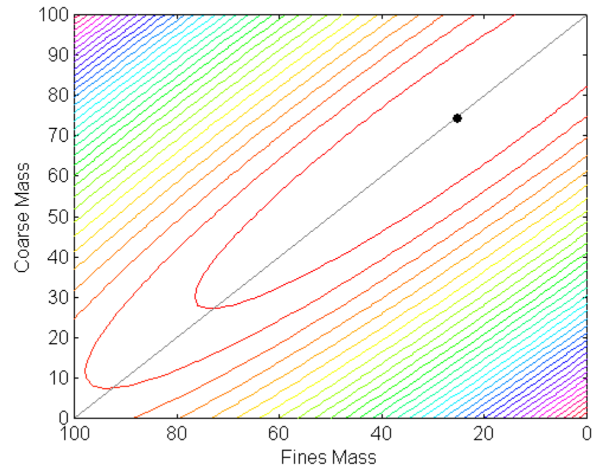

Fig. 4. Level curves of *J* with the point at which *J* is a minimum.

sum of the concentrations of all the chemicals is not equal to the ideal value 100% (Table I), the following question arose: *as the residue values cause the sum of the measured chemical content values to be exactly 100%, will the results be improved if the residue values are also considered in the computations?* To answer this question, one needs to include the following equation into (7):

$$c_{C(Residue)}.m_C + c_{F(Residue)}.m_F = 100.c_{I(Residue)} \quad (14)$$

This is like to consider the residues as an additional chemical compound that allows the sum of the measured chemical content values to be exactly 100%.

The results obtained for $\hat{\mathbf{M}}$ after including (14) in (7) are shown in Table IV, together with the results from Table III, for better comparison. The value of *J* increased significantly for almost all the tests, except for T-01 and T-08. The mean and the standard deviation of *J* for all the nine tests, indicate that the estimates of $\hat{\mathbf{M}}$ are normally impaired when the residue values are taken into account in the computations. Therefore, the final optimal estimates of $\mathbf{M} = [m_C\ m_F]^T$ are those calculated without the residues.

VI. Conclusion

This article presented the application of the Least Squares method to estimate optimal values of the mass partition performed by a centrifugal separator, according to its material balance equations, in order to evaluate its metallurgical

TABLE IV

Influence of the residue values on the estimates of $\hat{\mathbf{M}}$.

		Computation #1 (without residue values)		Computation #2 (with residue values)	
Test	**Stream**	**Calculated Mass (%)**	**J**	**Calculated Mass (%)**	**J**
T-01	Coarse Fines	74,575 25,425	2.814,930	74,824 25,176	2.524,507
T-02	Coarse Fines	68,166 31,834	3.452,206	68,690 31,310	4.395,679
T-03	Coarse Fines	54,018 45,982	297,327	54,260 45,740	665,319
T-04	Coarse Fines	64,919 35,081	3.795,958	63,786 36,214	10.522,246
T-05	Coarse Fines	49,527 50,473	1.999,205	50,479 49,521	5.498,442
T-06	Coarse Fines	64,781 35,219	4.638,557	66,313 33,687	12.693,079
T-07	Coarse Fines	84,834 15,166	1.538,745	85,317 14,683	2.539,722
T-08	Coarse Fines	92,958 7,042	305,100	92,959 7,041	255,560
T-09	Coarse Fines	21,116 78,883	4.682,674	21,348 78,652	13.993,404
		mean std.dev	**2.613,856** **1.686,356**	mean std.dev	**5.898,662** **5.212,791**

DOI: ISSN: 2318-4531

recovery of alumina and silica. The methodology presented should be useful when it's not feasible or impractical to measure the actual mass values processed by the equipment. In this case, one can simply consider the masses as a percent value of the input stream, according to equation (7). The methodology can also be used for other types of equipments with similar processing characteristics.

Although the main result of interest is the mass partition estimate $\hat{\mathbf{M}}$, the error indicator J is also an important parameter, as it measures the quality of the estimate. Besides the usual formulation of the Least Squares method presented in this article, an alternative formulation based on Lagrange Multipliers can also be used to solve the optimization problem presented. The user can choose either method for use depending on his familiarity with the mathematical formulations, the complexity of the application problem, and the computational resources to implement the method.

Acknowledgment

The author would like to thank Mr. Clóvis Maurity (Geologist) and Mr. Gerson Oliveira (Process Engineer) for their insightful comments on Sections I and II, and for the discussions about the results.

References

[1] M. Authier-Martin, G. Forté, S. Ostap, and J. See. "The Mineralogy of Bauxite for Producing Smelter-Grade Alumina", *JOM Journal of the Minerals, Metals & Materials Society*, vol. 53, no. 12, pp. 36-40, Dec. 2001.

[2] D. Donaldson, and B. Raahauge (Editors). *Essential Readings in Light Metals - Volume 1: Alumina and Bauxite*. Hoboken, NJ: Wiley, 2013.

[3] GEA Mechanical Equipment. "Centrifuges from GEA Westfalia Separator for Industrial Minerals", GEA Westfalia Separator Group GmbH, Oelde, Germany, 2012.

[4] B. A. Wills, and T. J. Napier-Munn. *Wills' Mineral Processing Technology (Chapter 3: Metallurgical Accounting, Control and Simulation)*. Oxford, UK: Butterworth-Heinemann, 2006.

[5] E. Kreyszig, *Advanced Engineering Mathematics*. Hoboken, NJ: Wiley, 2011.

[6] G. J. Bierman. *Factorization Methods for Discrete Sequential Estimation*. New York, NY: Academic Press, 1977.

[7] H. W. Sorenson, "Least Squares Estimation: from Gauss to Kalman", *IEEE Spectrum Magazine*, vol. 7, no. 7, pp. 63-68, Jul. 1970.

[8] S. Narasimhan, and C. Jordache. *Data Reconciliation and Gross Error Detection: An Intelligent Use of Process Data*. Houston, TX: Gulf Publishing Company, 1999.

[9] J. M. Richardson, and R. D. Morrison (M. C. Fuerstenau, and K. N. Han, Editors). *Principles of Mineral Processing (Chapter 10: Metallurgical Balances and Efficiency)*. Littleton, CO: SME Society for Mining, Metallurgy, and Exploration, 2003.

[10] D. Hodouin, and M. D. Everell. "A Hierarchical Procedure for Adjustment and Material Balancing of Mineral Processes Data". *International Journal of Mineral Processing*, vol. 7, no. 2, pp. 91–116, 1980.

[11] G. Heyen, and B. Kalitventzeff. (2012, Oct.). Process Monitoring and Data Reconciliation. LASSC Laboratoire d'Analyse et Synthèse des Systèmes Chimiques, Liège, Belgium. [Online]. Available: http://www.lassc.ulg.ac.be/webCheng00/meca0468-1/Validation_review.pdf

DOI: ISSN: 2318-4531