Reflection on teaching in mathematics mobilized in lesson study

ABSTRACT
The article analyzes the reflection on teaching, mobilized in a lesson study that involved mathematics teachers. The research is based on Schön and Perrenoud's concept of reflection and is guided by the question, "What reflections on teaching do mathematics teachers mobilize in a lesson study context?". To answer this question, we did an investigation in a lesson study context, structured in twelve meetings of approximately two and a half hours each, involving eight mathematics teachers from the final years of elementary school of the state public education network of Rio Grande do Sul, Brazil. The empirical material of the research consists of the researcher's field notes, the participants' logbooks, the transcripts of the sessions, and the interview carried out at the end of the lesson study. The qualitative and interpretative analysis showed that the reflection on teaching was characterized as retrospective and prospective, covering three themes: student learning, mathematics teaching, and teaching moral responsibility and ethics.

INTRODUCTION

Thinking and reflecting about the reality that surrounds us, about ourselves, our beliefs, actions, choices, and relationships, is generally an activity inherent to human beings. However, the reflective movements involved in the thinking process do not appear spontaneously. According to Vieira (2021), reflection comprises the mental act that leads to the development of new forms of knowledge, enriches established knowledge, and fosters awareness. In this perspective, the ability to think and reflect establishes the human being as a creative subject and not as a mere replicator of ideas and practices that come from outside, a fundamental principle for the concept of the reflective teacher proposed by Isabel Alarcão (2003).

The concept of reflection is highly valued in the context of discussions and research related to teacher development and teaching, a professional activity that circumscribes elements of a different nature, internal and external to the profession. The performance of teaching, due to the nature of this activity, constitutes an intermittent process of reflection that combines and balances the teacher's personal and professional identity in such a way that this professional has for him/herself and others an essence built by his/her performance in class (ALARCÃO, 2003). In this sense, the teacher is a professional that reflects in action and on the action, at the moment he/she concretizes it and afterward so that these reflective movements impregnate and feed each other (SCHÖN, 1983) through the dialectical movement of teaching and learning, managing relationships, time, knowing and experiences.

Our readings and discussions on reflection and especially our involvement with teacher education in research and implementation of formative actions for basic education teachers brought us closer to an approach to a teacher professional development called lesson study, which is based on collaboration and reflection. Originally from Japan, the lesson study was defined as a way to prepare the teacher to develop the practice (ISODA, 2007; LEWIS, 2000; LEWIS et al., 2013; RICHIT, PONTE, 2020), aiming to modify the mathematics pedagogical practices (RINCÓN; FIORENTINI, 2017) and promote student learning (LIM et al., 2016). The dissemination of this approach in Western countries began in the 1990s through published research on the implementation of the approach in the United States (RICHIT; TOMKELSKI, 2020).

Considering those aspects, several questions arose about the reflection supporting and guiding lesson studies. Specifically, we centered on the following: What reflections on teaching mathematics teachers mobilize in a lesson study context? This research question and others that emerge in our academic and professional experiences guide our quest to understand the reflective process engendered in lesson studies.

With this in mind, we focused on analyzing the reflection mobilized in a lesson study, examining an experience promoted with mathematics teachers from the public school system in Rio Grande do Sul. Of an interpretative and qualitative nature (ERICKSON, 1986), the analysis is based on the empirical material constituted in the lesson study.

The research emerges from the need for teachers’ professional development, demanded by the Brazilian educational context. Moreover, it constitutes an
initiative to value training actions that seek to resignify professional practices and cultures in dialogue with social change. In this perspective, the lesson studies appear in this scenario as a possibility of intervention in the reality of teachers, providing them with conditions to rethink their practice, function, and professional identity. Finally, we believe that participation in professional development approaches focused on classroom practice, such as lesson studies, can allow the teacher to reflect on different aspects of teaching, promoting their personal and professional growth.

REFLECTION IN, ON, AND FOR TEACHING

Reflection, defined as “asking ourselves what is happening or what will happen, what can we do, what should we do […]” (PERRENOUD, 2002, p. 30), has been highlighted as an important principle of teacher development. Schön (1983) conceives reflection for teacher development because it generates knowledge based on practice. Thus, “more than just a process through which teachers look at experiences from their past practice, reviewing episodes, emotions, and events, reflection is also projected towards the practice to be developed […]” (VIEIRA, 2021, p.61). Corroborating this perspective, Selingardi and Menezes (2017, p.273) add that when faced with the challenges of everyday practice, the reflective teacher “performs a movement of reflection that consists of turning to the study of a theory in search of a solution. He/she returns to his/her practice, now altered in the light of the reflection on that theory”.

Donald Schön develops the concept of the reflective teacher from opposition to what he calls epistemology of technical rationality. This paradigm imposes on the professional a submissive and transmissive posture of scientifically consolidated and systematized knowing. From this perspective, it would be up to the professional to apply the knowledge acquired through the definition of adequate instruments and means to achieve an established purpose. According to Schön (1983), this understanding presupposes a hierarchy in the educational process, firstly, of knowledge – valuing a technical-scientific process; and submitting the practice to these principles.

The epistemology of practice comes into conflict with the development of new professions and specialized fields – notably from the 1960s onwards – that need qualified professionals to deal with foreseen situations, but who can rationally and effectively deal with new conflicting situations, in epistemic, ethical, and moral dimensions (SCHÖN, 1983). This movement reveals that we must overcome the dichotomy between practice and theory, and propose an epistemic horizontality that allows the teacher to promote new practices in the face of the challenges that appear in the professional context. From this discussion, Schön proposes the paradigm of epistemology in practice.

Donald Schön considers that knowledge comes from an interaction between subject and world, i.e., the content of our thoughts is constituted and mediated by our intermittent action and interaction with the subjects and objects that surround us. In this way, the teacher’s knowledge develops and reveals itself in the heat and movement of action, characterizing two distinct but interconnected forms of reflection: reflection in action and on action (SCHÖN, 1983).
Philippe Perrenoud helps us to understand these forms of reflection. Reflect **in** action is a more immediate inference with respect to action. Here, we assume “[...] reflection about the situation, objectives, means, place, operations involved, provisional results, the foreseeable evolution of the action system” (PERRENOUD, 2002, p. 30). Reflection **in** action is the process of asking ourselves what happens, or what can happen, what we can or should do, what the best strategy is. The reflection **on** action, on the other hand, presupposes some distance. In this case, the action is taken as an object of reflection to make it critically intelligible. Nevertheless, Perrenoud recognizes that these two types of reflection do not differ in nature but in objects, therefore, being in constant relationship, in mutual influence. Thus, Schön conceives the reflective professional in a broader sense, without being restricted to a specific profession or training action.

Perrenoud (2002), when intending his discussions for the teacher’s reflection, considers that the reflection **in** action must be understood by its correspondence, i.e., reflectivity in the heat of action. This reflection is characterized by the circumstances and situations of interaction with the world, i.e., reflection emerges when the teacher is in an action that demands faster and more effective decisions; however, if it is possible to take any action that requires more time and complex decisions, then reflection occurs. Reflection **in** action suggests the apprehension of constitutive elements of learning, which are under analysis and criticism of reflection on action (PERRENOUD, 2002).

Reflection distanced from the action – on the action – occurs without the teacher being involved - or interacting with - the objects of this action. Reflection takes place on what has happened, what can happen, in the resumption of what has already been done, etc. Reflection **on** the action takes place in retrospective and prospective dimensions. The first occurs in the face of reflection after the activity, more immediately, whose operation falls on the understanding of what worked or not and seeks to prepare the professional for new challenges (PERRENOUD, 2002). Reflection **in** action usually appears while the teacher is planning a new activity or anticipating situations or problems. In this, the teacher relies on personal and professional experience to evaluate and decide on the most appropriate resources and methods for a planned activity (PERRENOUD, 2002).

Reflection **after** the action tends to permeate the professional’s routine beyond their activity, causing the objects of reflection to be prolonged in the time and places where the professional interacts - either in the sense of retrospective or prospective reflection or both in mutual interaction. The object of reflection refers to what is directly related to the teacher’s action “[...] during the educational act as context, methods, teaching purposes, knowledge and skills that students are developing, difficulties that arise, knowledge, and teacher weaknesses [...]” (SERRAZZINA, 2014, p. 1054).

The action, therefore, is not guided by spatial or temporal dimensions, revealing itself according to the implications and modifications that it enhances in the teacher’s practice. In this sense, this extension of reflection is constantly renewed since the teacher does not withdraw from the world to think about it (VIEIRA, 2021). On the contrary, the teacher is immersed in a specific reality, experiencing new situations, in such a way that the reflective movement is always updated according to the objects of interaction and reflection of the teacher, whether they are purely theoretical (knowledge of their area, for example), or
constituent parts of teaching, such as the relationship with students, learning issues, knowledge, values, among others (SERRAZZINA, 2014).

Yet, in addition to reflection on the action, Perrenoud affirms that the teacher's reflection is projected towards what he calls action schemes, at the individual level, and for a collective system of actions. This reflection on their action schemes results from the awareness of the repetitive character of some reactions and some sequences, i.e., the existence of scenarios that reproduce in similar situations. This “permanence is a source of identity, but also of dissatisfaction, causing the actor to become very suspicious, impulsive, shy, anxious, naive, slow, whimsical, or irritable” (PERRENOUD, 2002, p. 40).

Reflecting on action schemes consists of reflecting on the rational foundations of action, i.e., on the available information, knowledge, values, and methods that support teaching practice. In this reflection on the action scheme, we find the unconscious elements of practical actions and mental operations, which, in the end, are internalizations and theorizations of elements of practice (PERRENOUD, 2002). These action schemes can be better understood when we take them for their manifestation, which is revealed through habits, attitudes, values, ways of being and acting (interaction with the other and with the context). Making reflection on the action a movement of reading action schemes makes it possible for the teacher to visualize the collective systems of action to which it is integrated, intermittently produced from the relationship with other professionals.

Upon entering these action systems, the teacher adapts the action schemes to the collective system, which, in turn, impoverishes, enriches, and can promote changes in practice (PERRENOUD, 2002). The relationship between individual action schemes and collective action systems stimulates the link and forms of distinct collective relationships, which can appear in different dynamics and formative and non-formative or organizations, for example.

Reflection on action, therefore, introduces a reflection on the relationship, on how to create or maintain bonds with others (CIFALI, 1994), as well as on the dynamics of groups and organizations (PERRENOUD, 2002, p. 41).

Thus, reflection emerges from the concreteness of a singular action, allowing the teacher to find the underlying roots of the problematic action he/she faces. From the singularity of a situation, the teacher can, by projecting him/herself, visualize these systems of actions, and the constraints that, at given times, acted in an obscure and unconscious way, emerging in immediate and categorical decision making.

In short, teacher reflection proposed by Perrenoud takes place in the heat of action, in action, on action, on action schemes and systems of collective action. This reflective practice takes place in two processes, of retrospection and prospection, which gives feedback while being concretized. However, it is not about compartmentalized reflective processes but a proposal to elucidate some reflective processes engendered in teaching practice. The objects of this reflection vary according to their moments and intensity, as well as the individual and collective condition. In the following sections, we will examine reflection in lesson study, seeking to draw attention to fundamental aspects that mobilize teachers in decision-making in this process.
LESSON STUDY

Lesson study is a professional development approach widely practiced in Japan, which spread to several countries as works published in English began to appear (LEWIS, 2002; LEWIS & PERRY, 2017; RICHIT, 2020). Research findings suggest that lesson study impact teachers' knowledge, professional communities, teaching practices, and student learning (HART, ALSTON, & MURATA, 2011; LEWIS; PERRY, 2014; LEWIS, PERRY, & HURD, 2009; OLSON, WHITE, & SPARROW, 2011).

Lesson study, which usually begins with the identification of a relevant problem in the students' learning (PONTE et al., 2014) or with a teacher's teaching difficulty, is a very particular development dynamic, structured in well-defined steps (LEWIS; HURD, 2011) and supported by two basic principles: collaboration and reflection (LEWIS, 2002; MURATA, 2011; PONTE et al. 2014; RICHIT, 2020; RICHIT, 2021).

Although the literature in the area reveals some variations in the structure of the class study (RICHIT, 2020), we consider in our work the model proposed by Lewis (2002), for which this approach is organized around four central moments in that teachers work collaboratively in the following actions: formulating the objectives, planning, teaching, debriefing, and reviewing the class. In the formulation of objectives for student learning and a development, a topic or theme from the curriculum is selected; in planning the investigation class, the class is carefully prepared to achieve the defined objectives; in implementing/teaching the investigation class, the team that participates in the lesson study observes and collects information about the students' actions throughout the class, with notes or audio or video recordings; and debriefing on aspects related to student learning and development observed and recorded during the class can contribute to improving it (RICHIT; PONTE, 2019; RICHIT; TOMASI; MELO, 2021).

Concerning each of the stages of the lesson study, we highlight some characteristics. In setting objectives for the investigation class, there is great concern with the students' needs and difficulties regarding learning the mathematics topic chosen for the lesson study. The planning process, which is developed around the development of the investigation class from the previously defined objectives, presupposes a judicious and reflective collaborative work in which the students' ways of thinking, their strategies for solving the proposed tasks, their difficulties, what they will say during class activities, etc. should be predicted. During the investigation class, one of the group members implements the class planned for a class of students while the other members, including the team that coordinates the process, observe and register the students' actions. The post-class reflection stage, i.e., when the group meets to discuss and reflect on what was recorded on video and observed by the other members, also contributes to professional self-criticism (RICHIT, PONTE, e TOMKELSKI, 2019). This investigation class, after being redefined, if desired, can be taught again to another class of students, repeating the entire process (PONTE et al., 2014).

In a work that seeks to explain the different moments of class study, Mônica Baptista highlights the specificities of the implementation of the investigation class, which is taught by one of the teachers of the group, and of the observation of this class, carried out by the other teachers and whose focus is the student,
especially on their learning and difficulties and the strategies they use to solve the task.

Unlike many other observational processes, which focus on teacher performance, the focus here is on the students. This analysis can lead to the reformulation of the lesson plan, changing the strategy to be followed, the materials to use, the tasks to propose, the questions to ask the students, etc. (BAPTISTA et al., 2014, p. 3).

The lesson study dynamic is conditioned by its basic principles (collaboration and reflection) that support and guide each stage. Collaboration is the way of interacting in

... teachers’ meetings that are based on shared responsibility for the work of teaching (interdependence), on collective conceptions of autonomy, on support for teachers’ initiatives and leadership with regard to professional practice and on group affiliation, having professional work as a basis (LITTLE, 1990, p. 519 apud QUARESMA; PONTE, 2019, p. 4).

Collaboration requires that participants get engaged; it stimulates dialogue and negotiation and the sharing of experiences, teaching resources, and ideas. However, it also requires spaces for participants to build relationships, which is fundamental for the training sequence.

The reflection materialized in a lesson study enhances the re-signification of relationships, favoring teachers with a critical and reflective attitude of intermittent re-reading of their selves, practice, knowledge, and values that support their practice and the teaching purposes.

Lesson study provides plenty of time and opportunities for teachers to reflect on their teaching practice and student learning. The knowledge gained from and for the reflective practice should be shared in some format with the larger teaching and educational communities (MURATA, 2011, p. 10).

Reflection, in this sense, is an intentional, conscious, and dynamic act in terms of professional experience, but also of the knowledge accumulated over the years of professional activity. Examining the lesson study in the Japanese experience, Catherine Lewis underscores the reflective nature of the approach.

... identifying one’s shortcomings and soliciting and gracefully accepting criticism may be ways of showing competence, not failures to be avoided. Nor is critique typically focused on a single individual; collaborative planning of research lessons means that criticism is generally shared with several colleagues (LEWIS, 2000, p. 26-27).

Reflection can be individual or collective and, therefore, is not restricted to the teacher. It is a process that extends to colleagues, teacher educators, the training activity, students, school, and education, materializing shared reflection (RICHIT, PONTE, & TOMASI, 2021). In summary, given the specificities in the development of the lesson study, several aspects of professional practice in mathematics are placed at the center of the processes of discussion and reflection, from which it becomes possible to overcome beliefs, review concepts, and modify classroom practices. At the end of this cycle, a process of professional development should be constituted, leading to the knowledge of students’
learning and promoting reflection and re-signification of the practice and professional culture of which they are a part (RICHIT; PONTE, 2020).

Although vast around the world, the literature on lesson studies presents few works that deal with reflection as an object of study. Still, the works by Lee (2008), Gutierrez (2015), Suh and Seshaiyer (2015), Pang and Marton (2017), Quaresma and Ponte (2019), Richit, Ponte, and Tomasi (2021), and Vieira (2021) seek to think, each one under a specific approach, the reflection engendered in lesson study.

Examining the professional learning promoted in lesson studies, Suh and Seshaiyer (2015) emphasize that reflection allows teachers to value it to implement a different practice in class, in the face of a collaborative analysis of interpretation of math teaching and learning. Thus, the reflective nature of the lesson study, especially the reflection on the investigation class, constitutes one of the main avenues of professional development for giving the teacher the opportunity to look at the practice of their peers, look at their own practice, and critically reflect on it (PONTE et al., 2016).

Although we do not have much research on the subject, there is a diversity of objects and problems to be investigated in the reflection enhanced by the lesson study. In this work, we analyze the nature of the reflection of teachers participating in a lesson study, and how this act of thinking about the planning of a lesson can promote a new practice from the investigation class, favoring their personal and professional growth, metamorphosing their practice, and valuing them professionally.

**METHODOLOGY**

The investigation follows a qualitative and interpretive approach (ERICKSON, 1986), from which we analyze the reflection on teaching mobilized in a lesson study that involved mathematics teachers of the final years of elementary school. The research was developed in the context of a lesson study, developed in the second half of 2019, involving eight teachers who teach mathematics in the public school system of Rio Grande do Sul: Adelle, Ellie, Filipa, Judy, Kadu, Marie, Maggie – fictitious names. The teachers were invited by e-mail sent to schools belonging to the 15th Regional Education Coordination (15ª Coordenadoria Regional de Educação - CRE) of Rio Grande do Sul, Brazil, from which, according to their interest and availability, they registered and joined the process. We did not need to select the candidates because our lesson study cycles involve a maximum of ten participants, and we received nine applications, of which eight teachers attended the activity. Finally, the work complies with ethical research guidelines, having been approved by the UFFS Ethics Committee (Process No. 28982819.3.0000.5564).

The empirical material of the research comprises the data produced in the meetings of the lesson study, the accomplishment of the investigation class, and the interview at the end of the process. This material consists of 30 hours of audio recordings of the meetings. Initially, the content of the recordings was heard, transcribed, and textualized. Next, the transcribed material was compared with the recordings, original transcription, and textualization to preserve the accuracy and meaning of the teacher’s speech.
This material was analyzed in the context of the interactions between the participants of the lesson study, based on the textual documentation registers of what was produced by the teachers throughout this process (teachers’ logbooks, a sketch of the versions of the task developed, and a sketch of the preliminary versions of the planning of the investigation class) (ERICKSON, 1986). From this process, through which essential aspects related to reflection on teaching were revealed, we created the categories of analysis. Finally, we proceeded to a detailed discussion of those categories based on the empirical evidence identified in the data (ERICKSON, 1986).

In summary, in the analysis process, we identified manifestations in the teachers’ speeches about the reflexive dynamics in the context of the lesson study and about the objects of teacher reflection, which evidence a reflection under a retrospective and a prospective perspective, respectively. And these reflection perspectives focus on three themes: student learning, mathematics teaching, and ethical and moral responsibility of teaching.

THE RESEARCH BACKGROUND

The lesson study was carried out from August through December 2019, structured in twelve sessions of 2.5 hours each. The meetings were held at the facilities of the 15th CRE, headquartered in the municipality of Erechim, in the north of Rio Grande do Sul, Brazil. The dynamic of the lesson study cycle and the activities carried out are systematized in the following chart.

<table>
<thead>
<tr>
<th>Meeting/Date</th>
<th>Activities</th>
<th>Duration</th>
</tr>
</thead>
</table>
| 1st 08/23/2019 | - Group presentation.  
- Motivations to participate in the lesson study.  
- Presentation of professional experiences in mathematics teaching, highlighting challenges, advances and perspectives.  
- Starting the reflections with the following question: What knowledge is essential for the professional practice of the teacher who teaches mathematics?  
- Application of a questionnaire to learn about teachers’ perspectives on education and professional development, and their expectations about the lesson study.  
- Brief historical and theoretical presentation of lesson studies.  
- Closing of the meeting. | 60 min  
30 min  
30 min  
30 min |
| 2nd 08/30/2019 | - Presentation on lesson study: definition, origin, structure, development dynamic, dissemination around the world, possibilities for the teacher’s professional development.  
- Organization of teachers in groups and definition of the topic to be addressed in the lesson study cycle.  
- Survey of students’ frequent difficulties in studying each topic.  
- Formulation of objectives for a class as a way to solve the difficulties pointed out.  
- Referrals to the next meeting and closing. | 40 min  
30 min  
30 min  
30 min  
20 min |
| 3rd 09/13/2019 | - Discussion and reflection on lesson study from reading articles on this topic.  
- Explanation of the stages of the lesson study. | 70 min  
40 min |
<table>
<thead>
<tr>
<th>Meeting/ Date</th>
<th>Activities</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>4th 09/27/2019</td>
<td>- Review of possible difficulties of students in the topic chosen by the groups and review of the objectives for the investigation class. - Guidelines for the next meeting.</td>
<td>30 min</td>
</tr>
<tr>
<td>5th 10/04/2019</td>
<td>- Debate on national curriculum guidelines for mathematics teaching in high school - Discussion on the distinctions between exercises, problems, and exploratory tasks. - Preparation of exploratory tasks for the investigation class. - Guidelines for the next meeting.</td>
<td>60 min</td>
</tr>
<tr>
<td>6th 10/18/2019</td>
<td>- Development of a sequence of exploratory tasks previously prepared by the teacher educators. - Discussion about the limits and possibilities of the proposed tasks. Notes on aspects to be improved in the tasks. - Guidelines for the next meeting.</td>
<td>80 min</td>
</tr>
<tr>
<td>7th 11/08/2019</td>
<td>- Continuing the development of a sequence of exploratory tasks previously prepared by the teacher educators. - Preparation of materials for the investigation class.</td>
<td>70 min</td>
</tr>
<tr>
<td>8th 11/08/2019</td>
<td>- Development of the investigation class between the two groups so that group 2 solved the task of group 1. Group 1 solved the task of group 2. - Discussion and modification of the deficit aspects presented. - Final structuring of each group’s investigation class. Class review under the guidance of the teacher educators.</td>
<td>60 min</td>
</tr>
<tr>
<td>9th 11/22/2019</td>
<td>- Review of exploratory tasks developed by the two groups. - Presentation of the investigation class prepared by the three groups and making notes and necessary modifications. - Adequacy of the aspects mentioned. - Finalization of the investigation class planning and guidelines for the next meeting.</td>
<td>20 min</td>
</tr>
<tr>
<td>10th 11/29/2019</td>
<td>- Development of the investigation class of Group 1. - Students’ assessment about the experience in which they were involved.</td>
<td>90 min</td>
</tr>
<tr>
<td>11th 12/13/2019</td>
<td>- Development of the investigation class of Group 2. - Students’ assessment on the experience developed.</td>
<td>90 min</td>
</tr>
<tr>
<td>12th 12/17/2019</td>
<td>- Reflection session on the investigation class developed by each of the two groups. - Assessment of the lesson study, taking into account its stages.</td>
<td>60 min</td>
</tr>
</tbody>
</table>

Source: Adapted from the planning of the Lesson Study (2019).

The mathematics curricular topic addressed in the lesson study was defined through negotiation between teachers about the students’ difficulties in understanding mathematical concepts, such as algebraic operations, polynomials, algebraic fractions, area and perimeter, factoring, and algebraic equations. Thus, considering the students’ frequent difficulties in understanding the meaning of area and perimeter and aiming to address a topic foreseen in the mathematics
curriculum, the group decided to study the concepts of area and perimeter of plane regions.

This topic was also chosen because it is possible to work with geometric figures, with mathematical properties and quantities, by handling measuring instruments and the use of different representations, such as drawings, graphs, tables, diagrams, and algebraic models. The negotiation of the topic and the decisions about the investigation class culminated in the organization of teachers into two groups to favor the interaction between peers and value all contributions. Both groups worked with such concepts and prepared similar classes, being distinguished only by the thematic context that introduced each class. We defined the 8th grade of elementary school for the investigation class because the teachers who were available to teach it, Marie and Kadu, taught in that school grade.

The investigation class planning process, as it is permeated by collaboration and reflection, especially when reviewing the activities developed at each meeting, favored reflection on different aspects of mathematics teaching. Then, the teachers exchanged impressions about the groups' work, revealing that despite the organization into two smaller groups, they could talk about the planning of the investigation class, about the task context and structure, about the students’ difficulties in this curricular topic (area and perimeter) and in the resolution of the task that was being prepared, about the potential of the tasks designed to help the students’ difficulties in the respective topic. The teachers chose to work with the Tangram$, given the potential of mathematical exploration provided by this material. The task sought to favor exploring the topic 'area and perimeter' in terms of strategies, relationships, concepts, representations, conclusions, and generalizations presented by the students.

Thus, the first group chose to work with the Tangram to compose a mosaic, while the second group chose to create beds for a garden. The investigation class was carried out in 8th-grade classes from two schools: the first, at the Escola Municipal de Ensino Fundamental Presidente Vargas, in the municipality of Gaurama; and the second, at the Instituto Estadual de Educação de Marcelino Ramos, headquartered in the city of Marcelino Ramos.

At the end of the process, teachers were confident and encouraged to experiment with other strategies and resources in classroom practices, such as exploratory and investigative tasks and other forms of professional interaction, by creating study groups of teachers in their schools.

**REFLECTIONS MOVEMENTS PERFORMED IN LESSON STUDY**

The analysis of the empirical material evidenced two perspectives of reflection in the discussions and interactions between the professors: retrospective and prospective. These perspectives involved three main aspects: student learning, mathematics teaching, and the ethical and moral responsibility of teaching.
REFLECTION ON STUDENT LEARNING

Reflection on student learning involved retrospective and prospective perspectives (PERRENOUD, 2002), as teachers considered students’ frequent difficulties in ‘area and perimeter’ and other topics, seeking to understand details highlighted in those difficulties, such as the distinction between them. The teachers also sought to identify the causes associated with the students’ difficulties, emphasizing that it is important that the teacher identifies them as a way of planning actions to solve them.

[The students] have a lot of difficulties with this subject. They cannot solve simple issues. They don’t understand the statement. That [student] I tell you about, he doesn’t know how to add one plus one. [In notable products] they don’t identify what they have to add, you know. In the others, they forget the letters (Marie, Sept. 2019).

Because they are entering school younger, and then they do not have the maturity at the age they reach the sixth, seventh, eighth grades to understand this issue of letters along with numbers. (Filipa, Sept. 2019).

[That's why] the teacher must be aware, next to them, he must understand everything the students do, if they are having any difficulties. [For that] the teacher needs a lot of time, a sequence and training for all this to happen. (Maggie, Oct. 2019).

Reflection on students’ learning and, above all, on their learning difficulties, from the perspective of the participants, constitutes a necessary action for the teacher, not only for its intrinsic value, but for the importance of thinking about their professional activity from a decentralized perspective; but for the moment itself, because it is a reflection about the daily action performed in mathematics teaching.

The teacher needs to understand how they solve [problems and] explore the answers, the errors, articulating their ideas with what they expect students to learn (Marie, Sept. 2019).

Marie's reflection on the teacher’s action is retrospective insofar as it moves away from the tensions of teaching practice and, with the help of her peers, she strives to understand the problems and contradictions of actions in the classroom, manifested in students' learning difficulties. Through this reflection on practice, the teacher focuses on overcoming challenges based on the incorporation of changes in teaching (prospective) since nothing is produced without attention or anticipation of moments of students' learning, their difficulties, ways of thinking and acting.

Thinking about these issues enables the teacher to externalize their impressions, expectations, and professional knowledge, reflecting on the values and action schemes rooted in their practice, especially about the strategies adopted in the teaching of mathematics, the ways of acting legitimized in this field, to the language of teaching, lesson planning, the relationship with colleagues and students, and the necessary changes in the classroom.

This movement from retrospective to prospective reflection also happened in the context of anticipating students' difficulties in solving the task proposed for the
investigation class, which required them to plan actions and strategies to help students overcome possible obstacles and, thus, solve the task.

In this sense, reflection about the action, while distances the teacher from the classroom, gives them the opportunity to explore reflection in action, brings it closer again to the actual practice (VIEIRA, 2021), such as the complementarity between the planning and implementation stages of the investigation class in a lesson study. In the face of this movement of reflecting in action and about action, listening to students' manifestations and understanding their difficulties related to learning, attitudes, expectations, conjectures, and conclusions, the teacher can understand the limits and possibilities of teaching action and, if necessary, seek ways to change it. When experiencing shared reflection in a lesson study (RICHIT, PONTE & TOMASI, 2021), participants can examine, resignify and modify action systems, bringing them closer and adapting them to collective action schemes, enhancing, thus, changes in their action (PERRENOUD, 2002).

Therefore, the reflection promoted in lesson study enhances relationships between peers, provides teachers with a critical and reflective attitude of intermittent re-reading of their selves, their practice, the knowledge, and values that support and guide their practice, as well as the purposes of teaching (MURATA, 2011). And those reflection movements allow teachers to grow with practice and the meanings they attribute to this practice.

**REFLECTION ON MATHEMATICS TEACHING**

The analysis showed the participants' reflections on the knowledge, strategies, and resources necessary for classroom mathematics teaching, which were objects of analysis and discussion during the lesson study. At the same time, those aspects supported and guided the proposition of the objectives for the planning and execution of the investigation class, the understanding of the students' learning, and the definition of strategies to settle the students' difficulties in solving the task. The dynamic, knowledge, and basic strategies for mathematics teaching materialized in the investigation class (3rd stage of the lesson study) were confronted with the teachers' previous perspectives, which allowed them to resignify their knowledge, strategies, and ways of conceiving teaching in math.

[The teacher often limits the explanation], but [the students] go further. And then we don't even have an answer for that, because it's limited. You think it's that and they are limit, and you have no answer for that. I think [the teacher] has to be prepared to have more things, because sometimes you are surprised with the children because they go much further than we think they would. (Filipa, Nov. 2019).

This tension that characterizes a phenomenon inherent to teaching can be seen as the interaction between individual and collective action systems. In other words, as the reflection process progresses, dialogue is established, ideas and experiences are shared, thinking and acting are assumed as a collective movement, new knowledge, strategies, and values are incorporated into the collective experience of the participants in this process.
At each meeting, things change in us, whether reflections, [knowledge]. Each one will start to make their changes, changes will start to happen [...], because they are part of this evolution (Mateus, Aug. 2019).

The shared reflection on the different impressions and perspectives of the participants develops and feeds in two ways. In the first one, the contrast between different experiences, points of view, and propositions is verified. When confronted with different knowledge, experiences, and resources related to mathematics teaching, the teacher notices him/herself and their practice, seeking to understand them from the perspective of the experience analyzed (derived from the practice of the other) and, then, by their own experience. Then, after examining elements of their individual action, he/she develops and shares with the group new understandings, which are impregnated with new meanings for their action.

[This experience in lesson study gave me the opportunity] to exchange ideas as a group on how to work on specific content, which, depending on its type, is very complicated for us to work with. We always follow that traditional model, whether we like it or not. [There was] an exchange of ideas on how we develop, how we work the topics with the students (Ellie, Dec. 2019).

The dynamic of the lesson study, which incorporates new teaching elements and questions culturally established aspects, takes on a collective meaning in the face of the questions discussed, in such a way that it fosters the consolidation of a collective thinking and acting, built on shared and traded principles and values. In this sense, the lesson study problematizes and values teachers’ experiences and knowledge, taking them as a starting point to mobilize reflection on mathematics teaching, highlighting the elements that interfere in this process and, above all, seeking to guide teaching towards student learning (HART, ALSTON, & MURATA, 2011; PONTE et al., 2016; LEE, 2008; RICHIT, PONTE, & TOMASI, 2021).

Classroom study enabled teachers to value reflection as a context to problematize mathematics teaching from a different practice (the investigation class) (RICHIT, PONTE, & TOMKELSKI, 2019), followed by collaborative analysis on students’ learning (SUH; SESHAIYER, 2015). Thus, the reflection encouraged in a lesson study, permeating all stages of this process, is an important means of professional development as it allows the teacher to notice both their peers’ and their own practice (PONTE et al., 2016), confronting and resignifying them in the face of the specificities of the mathematics teaching context.

The reflection on mathematics teaching, encompassing the knowledge, strategies, and values inherent to this process and the implications of this teaching in the students' learning, a central focus of the lesson study, characterized the dialectical relationship between retrospective and prospective. In other words, the reflection on action allowed the participants to critically analyze their practice and the practice of peers, mobilizing constitutive elements for student learning, resignified through reflection in action (PERRENOUD, 2002) materialized in the investigation class.

In this sense, reflection is a way of developing teachers because it generates knowledge based on practice (SHÖN, 1983), enhancing new practices, knowledge, values, and objectives. Reflection, therefore, is the process by which teachers examine their past experiences, examine their constitutive and intervening aspects, and project themselves for future practices (VIEIRA, 2021). However, due
to the intermittent nature of the reflection promoted in lesson study and the social and dialectical context in which it takes place, the reflection experienced by the participants within the scope of this approach becomes a movement of shared reflection (RICHIT; PONTE; TOMASI, 2021) between retrospection and prospection.

**REFLECTION ON THE ETHICAL AND MORAL RESPONSIBILITY OF TEACHING**

Although the moral and ethical responsibility of teaching is a relatively new object of reflection within the scope of lesson study and, apparently, disconnected from the focus of this approach because it is focused on student learning, it is an important dimension of professional teacher development.

The moral elements, in the form of intentions and objectives of the teaching action and the ethics of teaching took place in the lesson study in the face of discussions about the curricular and normative documents that guide teaching in Brazil, the National Common Curricular Base - BNCC. The reflection on the mathematics teaching recommended in this document, considering the social and economic discrepancies in the country, provided the opportunity for teachers to share successful experiences due to the teacher's commitment to their mission. Reflection, a fundamental dimension in the lesson study, proved to be a mobilizing device for discussion and reflection on teaching, on the implementation of the school's role, and the impact of the teacher's action and commitment on students' learning and training.

I once attended a class [...] at a school [...] It was quite simple. The classroom had a rough floor. All very simple, small windows, a curtain, a lamp in the center of the room. The room was kind of dark. And we went to work with a fourth-grade class on geometric solids. So, the students taught us a lesson, talking about relationships between faces, between edges, between vertices. They explained that such a figure has so many faces, that the face is quadrangular, it is triangular. And I kept asking myself: How do these kids know all this? So I asked the teacher how she worked with them and she described her practice. I concluded that the teacher's commitment made all the difference in the students' development (Mel, Aug. 2019).

[...] we come to the conclusion that those people see education as the only way to change, to want to change their lives, you know. This vision must come from the family, the teacher, the school, society, everyone. (Kadu, Aug. 2019).

The shared report enabled them to reflect on the curricular guidelines and their impacts on the teaching and students' learning, covering different aspects, such as the social function of the school, the implementation of teaching in the face of curricular guidelines, the paradigms established by official guidelines, the implications and ethical dilemmas of teaching on students' education, the valuation of the profession, the school material conditions, students' social reality, the reproduction of political narratives in the classroom, among others. The discussion on the BNCC guidelines for mathematics teaching, illustrated by the following excerpt, highlights some of those aspects:

**Judy:** Do you remember what happened at the first meeting of the BNCC? They sent some people to work with us. They were lost. I came to think that they put someone there who knows absolutely nothing about the classroom. They had never been in a classroom.
Filipa: Exactly. [People] who don’t understand anything about education, who throw things at us, and [say]: do it.

Mel: Some of the specialists participate, but a lot of people were placed there like that because someone referred them, a friend who referred them.

Judy: [...] but then why they cannot solve any of this, if they have those specialists? Because it’s a political game. It’s stalling.

Filipa: [That’s why] we end up being devalued. [...] We are a belittled class, it starts up there, it’s not here, then it’s the parents, the students. [...] some people call us bums. They say: What does this troop of bums want, all the time wanting to go on strike?

Ellie: Society demands that the teacher fulfills their duty, that they educate children and prepare them for the world. Yes, this is our commitment, we know. But, do those changes really want to change?

Judy: And I think that those changes, those things that are thrown at the school and that the teacher is obliged to comply with, like the BNCC and other novelties, are not really interested in improving. It’s just to say there’s been a change. Because in fact, it won’t change like that, out of obligation. It must change through the teacher and his/her commitment (Discussion held in August 2019).

The excerpt points out the teachers’ critical attitude to teaching forged in the unfolding of those documents. This reflection allows them to understand that teaching is not at the margin of external and internal school issues. On the contrary, the crossings of social, political, and ethical issues are impregnated in the teachers’ professional culture, which reflects in their expectations and actions.

As it is situated at the intersection between the pragmatic (ways of acting) and identity (senses and ways of being in the world) dimensions, this reflection movement is projected to the teacher’s professional and personal identity (PERRENOUD, 2002). This perspective spreads through the teacher’s participation in contexts other than the lesson study (RICHIT, 2020), but impregnated with elements necessary for reflection on practice and, perhaps, for proposing changes in practice, even if they are challenging to the first sight.

The reflective posture focused on the ethical and moral responsibility of teaching, which is developed in lesson study due to its reflective nature, contributes to imbuing the teacher with a posture of scrutiny of his/her practice in the face of the context in which it is implemented and the phenomena that interfere with it. This state of epistemic alert gives rise to fundamental elements in the teacher’s practice for ruptures and new identity and political agreements that concern him as a professional and subject in the world (SCHÖN, 1983).

We consider, therefore, that the reflection on the ethical and moral responsibility of teaching is a two-way movement between retrospection and prospection, mobilizing teachers to remain on constant epistemic alert. In other words, although far from the dynamic and discussions promoted in the lesson study, it remains analyzing and reflecting on the content, attitudes, predictions, and results, both unexpected or foreseen in the process. Therefore, getting involved in the dynamics of the lesson study allowed participants to reflect on the systems of individual and collective action – in this case, on the ethical and moral elements that permeate teaching.
CONCLUSIONS

The analysis focused on the theme of reflection on teaching forged in lesson studies, covering three aspects: student learning, mathematics teaching, and the reflection projected toward the moral and ethical responsibility of the activity.

These objects of reflection are understood as structuring elements of teaching and, therefore, definers of the teacher's professional identity and can be influenced by elements from the inside or the outside of the profession. Therefore, we consider that such objects of reflection complement each other, simultaneously stimulating reflective practice in equal measure, triggering changes in ideas, dispositions, and propositions established individually and collectively.

In this perspective, the teachers highlighted the contributions of the lesson study to their personal and professional growth, as highlighted by Filipa and Mateus, as they could reflect on basic aspects of teaching, transcending the personal to the collective dimension. Regarding reflection on students' learning, enhanced by the observation of students' actions in carrying out the task, the lesson study provided reflection on more general aspects, such as students' difficulties in interacting with conceptual, operative, and cognitive elements, such as representations and generalizations presented by students. By moving among these forms of reflection, teachers developed understandings of how to address some learning difficulties through changes in practice.

The reflection on teaching, done mainly in the stage of planning the investigation class and during its accomplishment, characterized the transition from retrospective to prospective reflection, allowing teachers to understand how their action (teaching) can potentiate or mark out the students' development of the content or knowledge under study. The reflection on the moral and ethical responsibility of teaching characterized the process of reflection for teaching, insofar as the lesson study led them to think beyond the classroom work, projecting their action for the future through the realization of their social role through transformative teaching.

We emphasize, however, that reflective movements and objects of reflection in lesson studies are still an unexplored field in mathematics education and education, so that there are many open questions, such as the nature of the reflection encouraged in the lesson studies, or even how the reflection on structuring elements of the teacher’s professionalism emerges in practice.
A reflexão sobre docência em matemática mobilizada em estudos de aula

ABSTRACT

O artigo se dedica a analisar a reflexão sobre a docência, mobilizada em um estudo de aula que envolveu professores de matemática. A pesquisa se baseia no conceito de reflexão de Schön e Perrenoud e é orientada pela questão “Quais reflexões sobre a docência são mobilizadas por professores de matemática no contexto de um estudo de aula?”. Para tanto, realizamos uma investigação no contexto de um estudo de aula, estruturado em 12 encontros de aproximadamente duas horas e meia cada, que envolveu oito professores de Matemática dos anos finais do ensino fundamental, pertencentes à rede pública estadual de ensino do estado do Rio Grande do Sul. O material empírico da pesquisa constitui-se das notas de campo do pesquisador, dos diários de bordo dos participantes, das transcrições das sessões e entrevista realizada ao final do estudo de aula. A análise, qualitativa e interpretativa, evidenciou que a reflexão sobre a docência se caracterizou retrospectiva e prospectiva, abrangendo três temas: aprendizagem dos alunos, ensino da matemática, responsabilidade moral e ética da docência.

ACKNOWLEDGMENTS

Our special thanks to the teachers involved in this lesson study, who contributed to the realization of this investigation. We thank the Universidade Federal da Fronteira Sul and CNPq for the financial support to the research (Process: 305476/2020-3).

NOTES

1. Those collective action systems can be understood by the conflicting and tensioning character of different action schemes. In those systems, there is an encounter and dispute for space between knowing, practices, and individuals' experiences, which seek to adapt to each other in a stable and harmonious relationship. Thus, this meeting has the ability to encourage different forms of relationship and interaction between subjects. If we think about teaching activity, examples of collective action systems can be found in relationships with students, with colleagues, with the administration, etc. In short, it is possible to think of this dimension in the sense of a professional culture that involves teachers from their initial training, specialization, performance, and career completion.

2. Beyond a purely rationalized act, Perrenoud suggests that those reflections can be triggered by aspects that are in or out of the subject. Thus, uncertainties, lack of meaning, search for identity, conflicts, management and relationship problems, students' learning difficulties, demotivation, and devaluation can be some of the conditioning factors for this reflexive practice.

3. Tangram is a geometric puzzle of Chinese origin. Its origin is speculated between the 10th and 13th centuries, during the Song dynasty. It consists of seven pieces, namely: two large triangles, two small triangles, a medium triangle, a square, and a parallelogram.

REFERENCES


ISODA, M. Una breve historia del Estudio de Clases de Matemáticas en Japón Dónde comenzó el Estudio de Clases y qué tan lejos ha llegado. In: ISODA, M.;


**Received:** 02 nov. 2021  
**Approved:** 15 dec. 2021  
**DOI:** 10.3895/actio.v7n1.14886  
**Correspondence:** Adriana Richit  
Avenida Sete de Setembro, n. 222, Apto 92, Centro, Erechim, Rio Grande do Sul, Brasil.  
**Copyright:** This article is licensed under the terms of the Creative Commons-Attribution 4.0 International License.