

Exploring equidecomposition with digital games: a technological proposal for teaching area concepts in basic education

ABSTRACT

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Geometry instruction in Basic Education, excessively focused on formula memorization, often creates a gap in the intuitive understanding of the concept of area. To address this challenge, the objective of this study is to present and analyze the pedagogical potential of a didactic proposal that utilizes interactive digital games to explore the fundamentals of equidecomposition—which consists of the process of decomposing a figure into finite parts and rearranging them to form another of the same area. The methodological approach was structured as a qualitative case study, conducted with 14 third-year high school students from a full-time state school in the municipality of Caucaia, Ceará, Brazil. The intervention consisted of presenting the theory of equidecomposition, followed by the implementation of a didactic sequence featuring two digital games focused on rearranging planar figures, in addition to data collection carried out via a questionnaire administered after the completion of the activities. The questionnaire results demonstrated that students assimilated the concept of area conservation, applying visual decomposition strategies to solve the proposed problems, rather than primarily resorting to formulas. We conclude that the integration of theory and virtual manipulation within digital games proved to be an effective strategy for the development of geometric reasoning, indicating that interactive technological tools are robust allies for engaging high school students in learning mathematical concepts.

KEYWORDS: Equidecomposition; Geometry Teaching; Educational Games; Educational Technologies; Educational Product.

Explorando a equidecomposição com jogos digitais: uma proposta tecnológica para o ensino de áreas na educação básica

RESUMO

O ensino de geometria na Educação Básica, focado excessivamente na memorização de fórmulas, frequentemente cria uma lacuna na compreensão intuitiva do conceito de área. Para enfrentar esse desafio, o objetivo deste estudo é apresentar e analisar o potencial pedagógico de uma proposta didática que utiliza jogos digitais interativos para explorar os fundamentos da equidecomposição, que consiste no processo de decompor uma figura em partes finitas e rearranjá-las para formar outra de mesma área. O percurso metodológico foi estruturado como um estudo de caso com abordagem qualitativa, desenvolvido com 14 estudantes do 3º ano do Ensino Médio de uma escola estadual de tempo integral no município de Caucaia, Ceará. A intervenção consistiu na apresentação da teoria da equidecomposição, seguida da aplicação de uma sequência didática com dois jogos digitais focados no rearranjo de figuras planas, além da coleta de dados realizada por meio de um questionário aplicado após a conclusão das atividades. Os resultados do questionário demonstraram que os alunos assimilaram o conceito de conservação de área, aplicando estratégias visuais de decomposição para resolver os problemas propostos, em vez de recorrerem primariamente a fórmulas. Conclui-se que a integração entre a teoria e a manipulação virtual nos jogos digitais revelou-se uma estratégia eficaz para o desenvolvimento do raciocínio geométrico, indicando que as ferramentas tecnológicas interativas são aliadas robustas para engajar alunos do Ensino Médio na aprendizagem de conceitos matemáticos.

PALAVRAS-CHAVE: Equidecomposição; Ensino de Geometria; Jogos Educacionais; Tecnologias Educacionais; Produto Educacional.

INTRODUCTION

Geometry instruction in Basic Education, specifically the concept of the area of planar figures, presents a recurring pedagogical challenge. Often, the classroom approach prioritizes memorization and the mechanical application of formulas, which can lead to a superficial understanding by students (Pavanello, 2010). This practice disconnects calculation from geometric intuition and conceptual comprehension, leading students to perceive geometry as a set of rules disconnected from physical reality. This learning gap is evidenced by students' difficulty in transitioning between different registers of semiotic representation – for example, from the visual register to the numerical register (Duval, 2009) – when solving problems that require more than the simple application of an algorithm.

In this context, the concept of equidecomposition emerges as a theoretical and practical tool of great potential to address this difficulty. Equidecomposition explores the possibility of transforming one geometric figure into another through cutting and rearranging, maintaining the total area measure. This notion, mathematically founded on the Wallace-Bolyai-Gerwien Theorem for planar polygons, offers a powerful intuitive basis for students to understand why figures of different shapes can possess the same area. Pedagogically, instead of simply accepting a formula, the student is invited to visualize and manipulate the figure's transformation, actively constructing the meaning of area (Caraça, 2000).

To transpose this concept to the contemporary classroom reality, Digital Information and Communication Technologies (DICT) present valuable resources. The use of interactive digital games allows for the simulation of cutting and rearranging in a dynamic and engaging way, overcoming the limitations of physical materials and increasing student motivation (Gee, 2007). Gamification and interactivity can facilitate the visualization of complex transformations and provide immediate feedback, which are essential elements for knowledge construction in High School. The integration of active technologies favors teaching models in which the student assumes a central role in their learning process (Moran, 2015).

Considering the potential of articulating equidecomposition with technological tools, this article aims to present and analyze a didactic proposal based on on-line games, developed for teaching the areas of planar figures to High School students. The text is structured as follows: initially, the theoretical foundation that articulates the teaching of areas with the use of games in education is discussed; next, the methodological trajectory of the research is described, detailing the case study and the sequence of applied activities; finally, the results obtained with the implementation of the proposal are presented and discussed, evaluating its impact on student learning.

TEACHING AREAS IN BASIC EDUCATION

Teaching the concept of the area of planar figures is a pillar of geometry in Basic Education. However, investigations in Mathematics Education (Pavanello, 2010) and the analysis of current pedagogical practice indicate a predominance of teaching focused on the memorization and mechanical application of formulas. This procedural approach, although capable of generating correct answers in

standardized questions, frequently fails to build conceptual knowledge about what the area measure represents, resulting in the recurring confusion between area and perimeter (Baturó & Nason, 1996).

The relevance of this content is constantly reaffirmed in large-scale external evaluations. In the National High School Exam (ENEM) and in various university entrance exams, questions involving area calculation are frequent, often requiring the candidate's ability to interpret interdisciplinary contexts and apply geometric knowledge to complex problem situations. However, preparation for these exams can paradoxically reinforce formulaic teaching if the focus is solely on the rapid resolution of standard items. Conversely, scientific competitions such as the Brazilian Public School Mathematics Olympiad (OBMEP) frequently propose problems that demand deeper reasoning, in which the decomposition and rearrangement of figures are more efficient resolution strategies than the direct use of formulas.

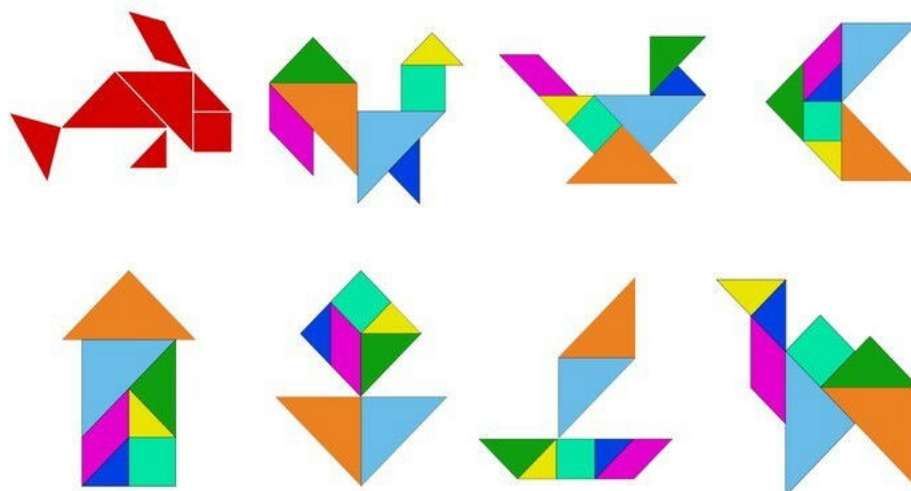
This discrepancy between the need for conceptual understanding (evidenced in olympiads and complex ENEM problems) and the practice of memorization is a central point of tension. The National Common Curricular Base (BNCC) seeks to mediate this tension by guiding the development of skills that involve the "decomposition and/or composition of figures" (Brasil, 2017). This signals a paradigm shift, aligning the curricular guideline with the need to overcome the purely algebraic approach. Therefore, it is fundamental that classroom practices incorporate methodologies that allow for visualization and manipulation, justifying the exploration of equidecomposition as a robust didactic tool.

EQUIDECOMPOSITION AS A PEDAGOGICAL TOOL

Equidecomposition is a geometric concept that formalizes the intuitive idea of cutting a figure and rearranging its parts to form another figure, without loss of material, as shown in Figure 1. The fundamental underlying principle is area conservation: although the figure's perimeter and shape may change drastically during rearrangement, the total surface measure remains invariable (Caraça, 2000). Pedagogically, this concept is powerful because it allows area to be decoupled from the specific shape of the figure, focusing on the amount of two-dimensional space occupied and serving as a bridge between visual intuition and mathematical rigor.

Figure 1

Examples of some Tangram shapes



Source: Wikipédia (2017).

The primary pedagogical virtue of equidecomposition lies in its ability to directly oppose the mechanical learning discussed in the previous topic. While the traditional approach encourages the student to seek a ready-made algorithm, equidecomposition forces the development of visual-spatial reasoning. By decomposing and rearranging figures, the student is led to question why the area is conserved, constructing a conceptual understanding of the measure. This approach transforms area calculation from a numerical substitution exercise into a geometric investigation activity.

This development of reasoning is crucial for facing the challenges proposed by external evaluations, such as ENEM and OBMEP. As previously noted, many questions in these exams, especially olympiad questions, are deliberately designed to be complex if solved by traditional algebraic methods but have elegant and quick solutions when approached through figure decomposition. The equidecomposition technique, therefore, transcends theoretical curiosity and becomes a practical heuristic tool, equipping students with non-standard problem-solving strategies that are highly valued in these assessments.

The mathematical foundation for the use of equidecomposition in planar polygons is the Wallace-Bolyai-Gerwien Theorem. This theorem establishes that any two simple polygons are equidecomposable if, and only if, they possess the same area. Given the importance of this result in validating the didactic activities proposed in this study, we will present the central elements of its logical construction below. The structure of the proof will be organized as follows: first, the auxiliary lemmas and theorems that ground polygonal decomposition will be stated, such as the principle of polygon triangulation and the theorems that guarantee the transformation of a triangle into a rectangle of the same area.

After establishing these bases, the Wallace-Bolyai-Gerwien Theorem will be formally stated, and its proof will be outlined. The demonstrative process illustrates how, using the transitivity of equidecomposition, it is possible to prove that any polygon can be transformed into a square of equivalent area. Although

the complete replication of this formal proof is not the ultimate goal of teaching in Basic Education, the understanding of its existence and logic offers the teacher the theoretical security to apply practical activities. Tools such as the Tangram and the digital games explored in this research function as concrete representations of this abstract theorem, allowing students to build an intuitive understanding that the total area is the sum of the areas of the parts, regardless of how they are organized on the plane.

WALLACE-BOLYAI-GERWIEN THEOREM

To formalize the discussion, it is important to establish the definition of equidecomposition based on the literature. Equidecomposable figures are defined as those that "can be decomposable, that is, cut, into respectively equal parts" (Souza, 1973, p. 15). Analogously, Boltianski (1981, p. 9) states that two figures are equidecomposable if, by cutting one of them in a certain way into a finite number of parts, it is possible to compose the other figure with these parts.

The process of rearranging the pieces originating from the decomposition of a figure fundamentally utilizes isometries in the plane, such as translations and rotations (Lima, 2007). Figure 2 illustrates a practical example of this concept, showing the equidecomposition between polygons of distinct shapes that conserve the same area.

Figure 2

Equidecomposable square and triangle.



Source: Wikipédia (2017).

The Wallace-Bolyai-Gerwien Theorem rigorously establishes the condition for this transformation to be possible: two planar figures that have the same area (equivalent) are equidecomposable. To prove this central theorem, it is necessary to utilize the transitivity property of equidecomposition and demonstrate a set of preliminary results. In the following subsections, the proof will be presented, which is based on Theorem 1 and Lemmas 1, 2, and 3. Detailed proofs for these results can be explored in depth in Nascimento (2025).

Theorem 1: Every polygon with n sides, $n \geq 4$, can be decomposed into $(n - 2)$ juxtaposed triangles whose vertices are vertices of the polygon.

Lemma 1: Every triangle is equidecomposable to a rectangle.

Lemma 2: Two rectangles that have equal areas are equidecomposable.

Lemma 3: Every polygon is equidecomposable to a rectangle.

Wallace-Bolyai-Gerwien Theorem: Two polygons that have equal areas are equidecomposable.

Proof: By Theorem 1, each of the polygons can be decomposed into triangles, which, according to Lemma 1, are equidecomposable to rectangles. The two rectangles obtained from the decomposition of the two polygons have equal areas and, therefore, by Lemma 2, are equidecomposable. Hence, by the transitivity of equidecomposition, the two polygons are equidecomposable.

HILBERT'S THIRD PROBLEM AND EQUIDECOMPOSITION IN THREE-DIMENSIONAL SPACE

The investigation into equidecomposition, although extremely productive in the two-dimensional plane with the Wallace-Bolyai-Gerwien Theorem, naturally prompted mathematicians to question whether analogous principles would apply in higher dimensions. It was a search for an elegant generalization: could two polyhedra of the same volume always be decomposed into a finite number of congruent pieces to form one another? This seemingly simple question became Hilbert's Third Problem, proposed by the renowned mathematician David Hilbert in his famous list of 23 unsolved problems presented at the International Congress of Mathematicians in 1900. This problem represented a fundamental challenge to geometry and measure theory at the time.

According to Guimarães (2011):

Legendre, in a note in his *Éléments de Géométrie* (1794), had already demonstrated that two symmetrical polyhedra are equidecomposable with parts that have the same orientation. Hill, a professor at the University of London, provided examples in 1895 of tetrahedra equidecomposable with a cube. Bricard, a mathematics professor in Paris, provided a condition in 1896 for two polyhedra of the same volume to be equidecomposable, but his proof was incomplete. Max Dehn, a student of Hilbert, proved in 1900, a few months after the Paris congress, that there are polyhedra with the same volume that are not equidecomposable. In particular, Dehn showed that a regular tetrahedron and a cube, both with the same volume, cannot be equidecomposable. Hadwiger, in the 1950s, simplified Dehn's proof, employing the so-called Dehn invariant, which is equal for equidecomposable polyhedra. Thus, the decomposition of a prism into three pyramids can give rise to pyramids of the same volume but with different Dehn invariants. In 1965, the Swiss mathematician J. P. Sydler finally managed to prove that Dehn's conditions are necessary and sufficient for two polyhedra of the same volume to be equidecomposable (Guimarães et al., 2011).

Initial intuition might suggest an affirmative answer, since equidecomposition operates more practically in the 2D plane. However, the discussions on Hilbert's Third Problem revealed a complexity inherent to three-dimensional space that does not exist in two dimensions. The search for a solution required the development of new conceptual tools and a deeper understanding of the structure of solids. It was the young mathematician Max Dehn, Hilbert's own student, who surprisingly presented the negative solution to the problem just a few months after its enunciation.

Dehn demonstrated that, unlike what occurs with polygons, there are polyhedra of the same volume that cannot be decomposed into congruent parts to form one another. To do this, he introduced an invariant, known today as the Dehn Invariant. This invariant is a geometric property of a polyhedron that remains constant under equidecomposition. If two polyhedra possess equal volumes but their Dehn Invariants are different, then they cannot be equidecomposed. This discovery marked a turning point in geometry, revealing that two-dimensional intuition does not extend directly to three-dimensional space, and that the notion of "measure" (whether area or volume) has distinct characteristics in different dimensions.

The solution to Hilbert's Third Problem, by demarcating the boundaries of equidecomposition, reinforces the relevance of the Wallace-Bolyai-Gerwien Theorem for planar geometry. While the search for equidecomposition in three-dimensional space faces profound limitations, the theoretical guarantee in the two-dimensional plane is robust. For the teaching of geometry in Basic Education, this distinction is crucial: the activities of decomposing and rearranging areas, such as those proposed in this work, are mathematically valid and intuitive, offering a solid basis for understanding the concept of area, without the complexities and paradoxes that emerge when attempting to naively extend the concept to three-dimensional space.

From an educational standpoint, the negative response to Hilbert's Third Problem offers an important reflection on the limits of geometric intuition and the need for mathematical rigor. Although the mathematics behind the proof is advanced, the existence of such "paradoxes" and limitations in higher-dimensional geometry highlights the importance of properly consolidating concepts in more accessible dimensions. Equidecomposition in the plane, which possesses a robust theoretical guarantee (Wallace-Bolyai-Gerwien Theorem), becomes even more valuable as a didactic tool, as it allows students to build a solid foundation of understanding of area measurement without the conceptual pitfalls that would arise in a premature attempt to generalize to volume (Lima, 2007).

In this sense, the playful and manipulative approach assumes a central role. Activities that allow students to "cut and rearrange" planar figures—whether with concrete materials, such as the Tangram, or through digital games, like the ones explored in this work—are not mere pastimes. They represent a pedagogical path that respects the students' cognitive development, starting from the concrete and the experiential. Jean Piaget, through his theory of cognitive development, highlighted the importance of action upon objects for the construction of the logical and operative structures of spatial thought, which are fundamental for geometric understanding (Piaget & Inhelder, 1956). Such practices prepare the ground for the understanding of more abstract concepts, including the future appreciation of the limits of equidecomposition in other dimensions.

Thus, the contrast between the success of equidecomposition in the plane and its limitations in three-dimensional space serves as a backdrop that highlights the importance of teaching geometry in a contextualized and experimental manner. By prioritizing visualization and manipulation in planar equidecomposition activities, the educator not only facilitates the learning of areas but also implicitly introduces the idea that mathematics possesses nuances and depths that manifest differently across distinct dimensions. This approach contributes to a more critical

and less dogmatic mathematical education, in which the student learns to value both intuition and the need for formal validation—crucial elements for scientific thinking (D'Ambrosio, 1990).

DIDACTIC PROPOSAL

This section will present the methodological outline of the research and subsequently detail the developed educational product. The objective is to describe how equidecomposition was approached with High School students at a full-time state school in Ceará, utilizing interactive digital games as teaching and learning tools, culminating in data collection for the analysis of the proposal's effectiveness. It is important to emphasize that the proposal detailed here constitutes a validated educational product, the result of a master's thesis research linked to the Professional Master's Program in Mathematics in the National Network (PROFMAT).

Research in mathematics education is not restricted to the analysis of teaching and learning phenomena but also encompasses the production of materials and interventions aimed at qualifying pedagogical practice. As Ferreira Ortiz (2022) points out, the production dynamics in professional master's programs require structuring and validation stages so that the generated material transcends the academic environment. In this sense, the construction of an educational product assumes paramount relevance, as it offers concrete tools and structured methodologies that can be directly applied by other educators, presenting potential for replicability and adaptability to different educational contexts and effectively impacting the classroom.

The elaboration of an educational resource, such as a didactic sequence with digital games, requires a rigorous process of conception, development, and validation. This process must consider the specificities of the target audience, the learning objectives, the curricular guidelines, such as the BNCC, and the available technological infrastructure. Furthermore, clarity in the description of the material and activities is essential so that other teachers can understand its logic, adapt its application, and thus expand the reach of the original proposal. By making available a material that can be implemented by others, the research assumes a transformative character, fostering teaching autonomy and the dissemination of good practices.

The educational product detailed here is composed of a variety of activities, including interactive online games and practical cutting and rearranging activities. The choice to integrate such digital resources aligns with recent investigations in the field; Meredyk et al. (2020), for example, evidence that the strategic use of educational applications creates creative and playful environments that immensely favor the learning of geometry. As D'Ambrosio and Lopes (2014) point out, mathematics must be seen as a field of investigation, in which problem-solving and modeling are fundamental tools for knowledge construction. In this perspective, the present work aims to provide quality mathematical education, being accessible and promoting meaningful learning, with the intent of stimulating the development of essential competencies and skills, such as scientific, critical, and creative thinking, communication and argumentation, teamwork and

cooperation, digital culture, autonomy and proactivity, and finally, resilience and persistence.

For the target audience, composed of High School youth, the methodology using games proves particularly effective, as it stimulates logical reasoning, problem-solving, and creativity in a playful manner. According to Huizinga (2000), games are inherent activities to human culture, capable of deeply and meaningfully engaging individuals. Moreover, the interactive and practical nature of these activities facilitates the understanding of abstract concepts like equidecomposition, allowing students to visualize and manipulate the figures, experiencing in practice how area conservation manifests through transformations. This concrete-pictorial-abstract approach is defended by Bruner (1966) as essential for the construction of meanings in learning.

The proposed activities seek not only the mastery of the area content but also the enhancement of skills foreseen in the BNCC, such as computational thinking, argumentation capacity, and collaboration. It is believed that by engaging students in challenges that require the creative application of mathematical concepts, a significant contribution will be made to deeper and more enduring learning. For Vygotsky (1987), social interaction and practical activity are pillars of cognitive development, and it is in this sense that the proposed activities seek to promote collaborative learning and the collective construction of knowledge.

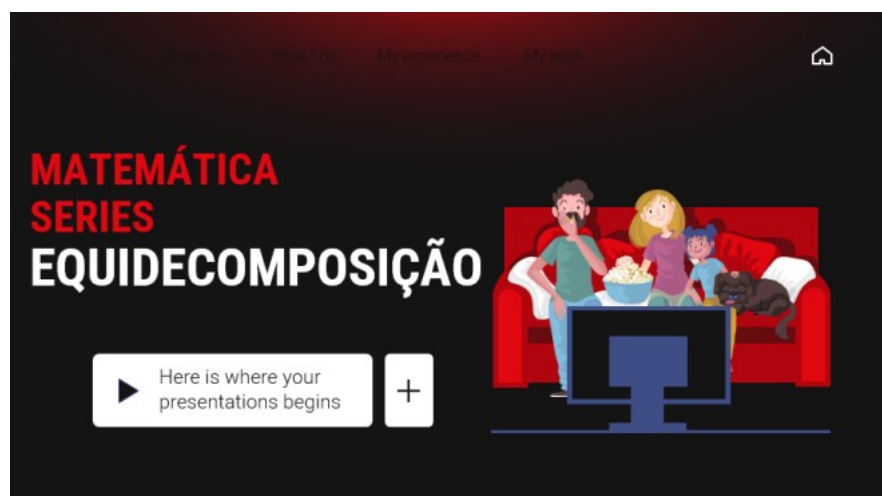
EDUCATIONAL PRODUCT PRESENTATION LECTURE

The introductory stage of the educational resource was the presentation of the project to the students. To this end, an initial presentation in a lecture format utilizing slides was chosen. This approach aimed to contextualize the theme of equidecomposition and the areas of planar figures, demonstrating the relevance of the subject and how it would be approached.

During the lecture, the aim was not only to expose the theoretical content but mainly to awaken the students' curiosity and interest. The slides were designed to be visually attractive and didactic, using as inspiration the template of an online streaming platform, *Netflix*. The central idea of equidecomposition was presented in an accessible way, explaining how the manipulation of geometric shapes can assist in the comprehension and calculation of areas in innovative ways. Figure 3 shows the layout of the presentation.

Figure 3

Lecture presentation layout



Source: Elaborated by the author (2025).

Figure 4

Presentation summary.



Source: Elaborated by the author (2025).

As noted in Figure 4, the lecture was carefully structured into four main topics. This division aimed to present the content progressively and didactically, ensuring that students could build knowledge in a solid and interactive manner.

The first topic of the lecture was dedicated to presenting the central theme: equidecomposition in the plane and in space, explaining the fundamental concepts and demonstrating how figures with equal areas can be decomposed and rearranged. This introduction sought not only to define what equidecomposition is but also to show its relevance in the study of geometry, awakening the students' curiosity to the possibilities this idea offers.

Next, the second topic addressed the Tangram and its applications. This ancient Chinese puzzle served as a practical and playful tool to illustrate the principles of equidecomposition. The history of the Tangram was presented, along with its seven pieces and how they can be used to form an infinity of figures, always maintaining the total area. Its applications were discussed not only in mathematics but also in art and in the development of spatial reasoning. In addition to the theoretical explanation, interaction with the Tangram was enhanced through online games, allowing students to explore its possibilities dynamically and enjoyably.

The third topic followed a similar line, focusing on the *Stomachion* and its applications. Less known compared to the Tangram, the *Stomachion*, an ancient Greek puzzle attributed to Archimedes, offers even more complex challenges in equidecomposition. Its 14 pieces and the various forms that can be created were explored, deepening the concept of decomposition and recomposition of figures. As with the Tangram, the exploration of the *Stomachion* was enriched by online games, providing an interactive and immersive experience in manipulating the pieces and understanding area conservation.

Finally, the fourth and last topic of the lecture culminated in a practical cutting and rearranging activity. This part was designed to allow students to directly experience the presented concepts. Using simple figures, the students were able to decompose and recompose shapes with scissors and paper, concretely visualizing the invariance of the area. This activity served as a bridge between the theory exposed on the slides and practical experience, solidifying the understanding of equidecomposition.

Figure 5 shows the theoretical part on planar equidecomposition that the students were able to learn about, and, after questions raised during the lecture, the slide in Figure 6 was presented, which answered doubts about three-dimensional equidecomposition.

Figure 5

Planar equidecomposition



Source: Elaborated by the author (2025).

Figure 6

Three-dimensional equidecomposition



Source: Elaborated by the author (2025).

LEARNING ABOUT EQUIDECOMPOSITION WITH THE TANGRAM

Following the theoretical exposition on the subject, the activities proceeded to the exposition and practice concerning two puzzles that address equidecomposition. It began with the Tangram, which proved to be an invaluable tool for understanding the topic. By manipulating the seven geometric pieces of the puzzle, students not only visualized the concept of decomposing and recomposing figures but also experienced the property that the total area remains invariant, regardless of the pieces' configuration. The importance of the Tangram resided in its ability to transform an abstract concept into a tactile and visually stimulating activity. The practice of the game was applied through an online game on the *Divertudo* website, which consists of several figures to be constructed with the Tangram pieces. In each game, there are three figures, and the layout for choosing which category to play can be seen in Figure 7.

Figure 7

Tangram interface on the Divertudo website.

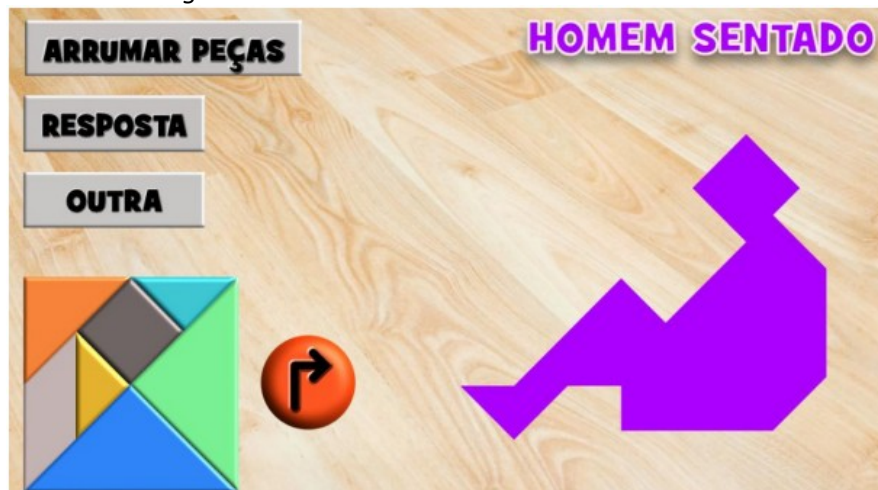


Source: *Divertudo*, 2025.

The students were free to choose which game and which figure they would assemble. Figure 8 shows an example of game 10, in which the student had to assemble a seated man.

Figure 8

Seated man in game 10.

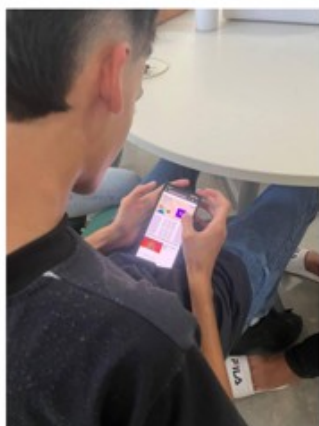


Source: *Divertudo*, 2025.

The activity generated notable engagement from the students. They demonstrated great enthusiasm for the various shapes they could assemble, exploring the creative possibilities of the game. The participatory and effective manner in which they dedicated themselves to the task was evident. Furthermore, the activity stimulated cooperation and group work. Students frequently shared ideas, discussed strategies to form the proposed figures, and helped one another in solving the challenges, reinforcing the idea that mathematics can be learned collaboratively and enjoyably. Figures 9 and 10 show the process of the students assembling the figures.

Figure 9

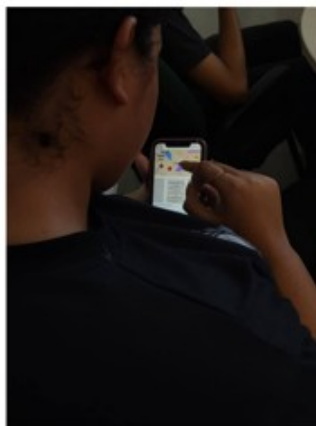
Student assembling a Tangram figure.



Source: Elaborated by the author.

Figure 10

Student assembling a Tangram figure.



Source: Elaborated by the author.

LEARNING ABOUT EQUIDECOMPOSITION WITH THE STOMACHION

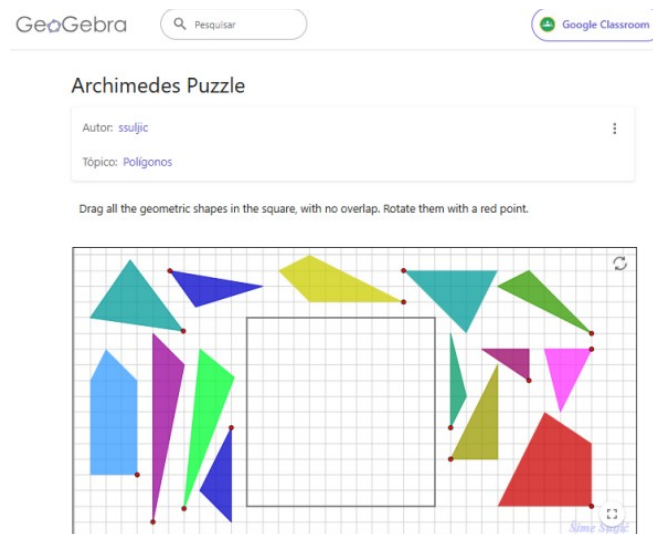
If the Tangram introduced students to the universe of equidecomposition in an accessible way, the *Stomachion* represented a step further in exploring this concept. This ancient puzzle, with its 14 distinct pieces, provided a more complex and in-depth challenge, broadening the students' understanding of how varied figures can be constructed from a fixed set of polygons while maintaining the original area.

The interaction with the *Stomachion*, both in discussions during the lecture and through online games, demonstrated the students' ability to face more elaborate problems. Although the larger number of pieces and the more irregular shapes of the *Stomachion* presented a superior challenge, the students dedicated themselves with the same curiosity and perseverance. This activity was crucial in solidifying the idea that equidecomposition is not restricted to simple shapes or a limited number of pieces. It was observed how experimentation with the *Stomachion* encouraged strategic thinking and persistence in the search for solutions—essential skills not only for mathematics but for various areas of knowledge.

The *Stomachion* assembly activity was carried out through the *GeoGebra* platform, utilizing the game *Archimedes Puzzle*, as shown in Figure 11.

Figure 11

Archimedes Puzzle game interface



Source: *GeoGebra* (2025).

CONCLUSION OF THE ACTIVITIES

To consolidate learning and extend the experience beyond the lecture environment, it was considered crucial that the students take the reflection on equidecomposition home. It is believed that the assimilation of complex mathematical concepts is more effective when students have the opportunity to revisit and manipulate ideas in their own time and space. Thus, learning is not

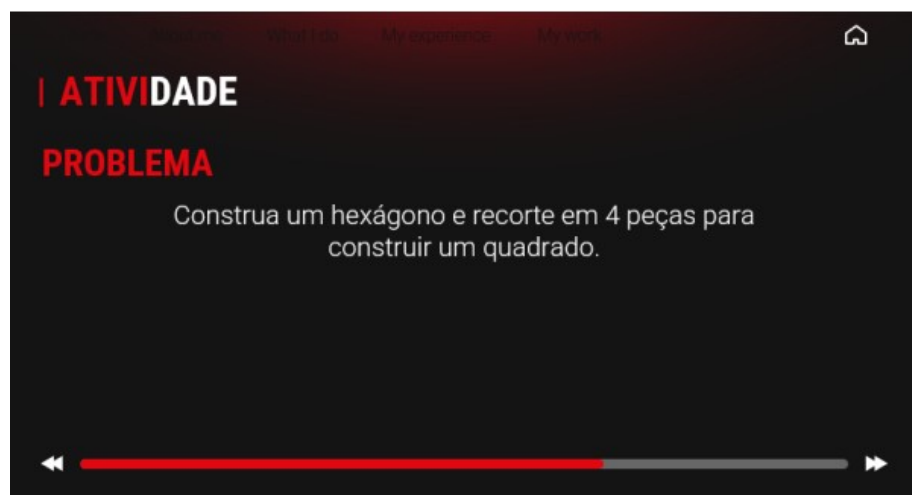
restricted to the moment of the class but becomes a continuous and more meaningful process.

With this objective in mind, a practical cutting and rearranging activity was proposed to be carried out at home. This task not only reinforced the concepts explored with the Tangram and the *Stomachion* but also encouraged the students' autonomy in problem-solving. By repeating the experience of decomposing and recomposing figures, they were able to solidify their understanding of area invariance and improve their spatial skills. The expectation is that this extracurricular activity stimulates curiosity and critical thinking, encouraging them to perceive the mathematics present in various everyday situations.

Figure 12 shows the lecture slide with the activity that the students were to develop at home.

Figure 12

Slide with the proposed activity



Source: Elaborated by the author (2025).

REPORT ON THE IMPLEMENTATION OF THE EDUCATIONAL PRODUCT: A CASE STUDY

The application of the educational product was carried out in a public state school that operates on a full-time basis. The target audience consisted of third-year High School students, a group that, due to the school's modality, has an intense academic routine, with a workload of nine daily classes and between five to seven weekly mathematics classes. The institution has five third-year classes, totaling a universe of approximately 150 to 175 students, which initially suggested a vast field for the application of the didactic proposal.

However, the implementation faced a significant contextual challenge. The period chosen for the application coincided with the preparations for the school's June festivities (*Festas Juninas*), a cultural event of great relevance to the school community. Most of the student body was deeply involved in rehearsals and the organization of the presentations, which resulted in a severely reduced time

availability for extracurricular activities. Given this scenario, participation in the activity could not be generalized, making it necessary to extend a targeted invitation to students who demonstrated interest and whose schedules were compatible.

Consequently, adherence was limited by the described external factor. Out of the entire universe of third-year students, a group of 14 students volunteered to participate in the pedagogical intervention. Although the number was reduced compared to the total number of students in the grade, the formation of a smaller, voluntary group allowed for closer monitoring and a deeper qualitative analysis of the participants' interaction with the proposed educational product.

Additionally, the experience revealed a practical and infrastructural challenge that imposes itself on any educator wishing to replicate this approach. The application of the activities intrinsically depends on access to technology. The availability of a functional computer lab, or that students possess their own mobile devices such as cell phones or tablets, is indispensable. In both cases, stable internet access is a non-negotiable condition, since the games and interactive tools are accessed online—a requirement that may represent a barrier in different school contexts.

RESULTS

The didactic intervention phase, which comprised the introductory lecture and the activities with the Tangram and *Stomachion* online games, culminated in the collection of essential data to evaluate the impact of the educational product. The complete and detailed results of this research, including the full text of the applied questionnaire and the graphical representations of the findings, can be consulted in Nascimento (2025). This section is dedicated to the presentation and analysis of the main *insights* obtained through an online questionnaire, elaborated on Google Forms, and administered to the 14 third-year High School students after the conclusion of the activities. The questionnaire sought to capture the students' perceptions regarding the lecture, the games, and the relevance of equidecomposition for the understanding of areas, aiming to infer the effectiveness of the employed pedagogical strategies.

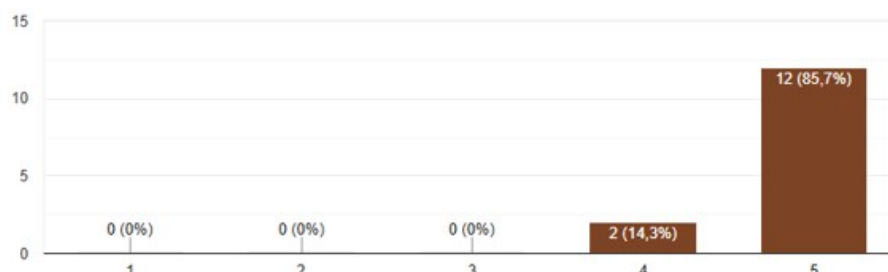
In these graphical representations, the horizontal axis utilizes a five-point Likert scale, in which the number 1 represents the minimum evaluation (totally dissatisfied or not very useful) and the number 5 indicates the maximum evaluation (totally satisfied or very useful/relevant). The vertical axis, in turn, represents the absolute number and the percentage of students who selected each response level. The vast majority of the participants attributed high scores (level 5) to the questions of satisfaction (85.7%) and utility (85.7%) regarding the exposed content. This result indicates that the initial contextualization was not only effective in engaging the class but also fundamental in mitigating the common resistance in the study of geometry and preparing the ground for the practical activities.

Figure 13

Number of students satisfied with the lecture

Você ficou satisfeito com a palestra?

14 respostas



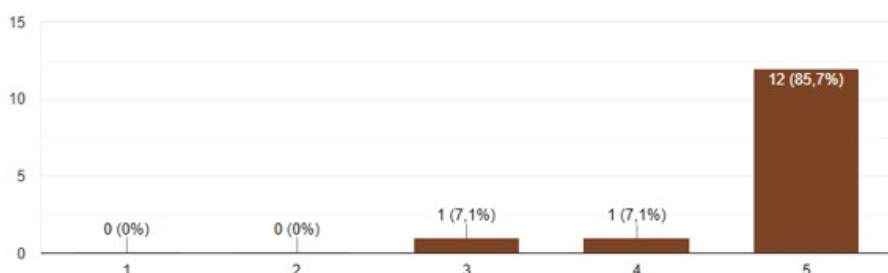
Source: Elaborated by the author (2025).

Figure 14

Number of students who found the lecture useful

A palestra foi relevante e útil?

14 respostas



Source: Elaborated by the author (2025).

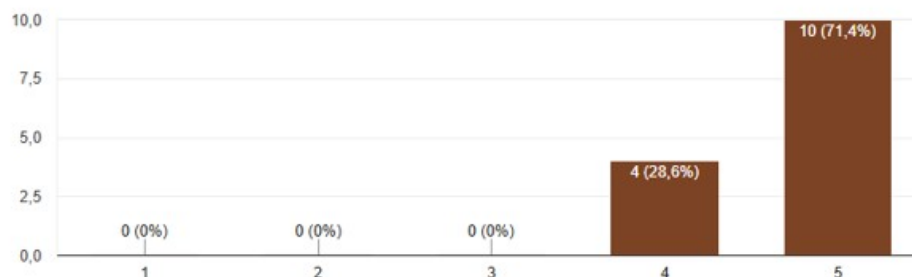
The preference for practical activities mediated by technology was also evaluated by adopting the same methodological structure, as illustrated in Figures 15 and 16. The horizontal axis of these graphs maintains the parametric scale from 1 to 5 (in which 1 indicates total displeasure and 5, total approval), while the vertical axis denotes the number of students and their respective percentages. The post-activity evaluation demonstrated high acceptance for both online games. The *Tangram* (Figure 15) obtained massive approval, with 71.4% of the students assigning the maximum level (5). The *Stomachion* (Figure 16), due to presenting a cognitive challenge of greater complexity and possessing more irregular pieces, obtained a slightly broader distribution, but still concentrated the evaluations positively, with 57.1% of the students marking level 5 and 35.7% marking level 4. These metrics attest that the students manifested significant satisfaction with the experiences provided by the digital tools, validating the playful approach.

Figure 15

Percentage of what students thought of the Tangram

O que você achou do Tangram?

14 respostas



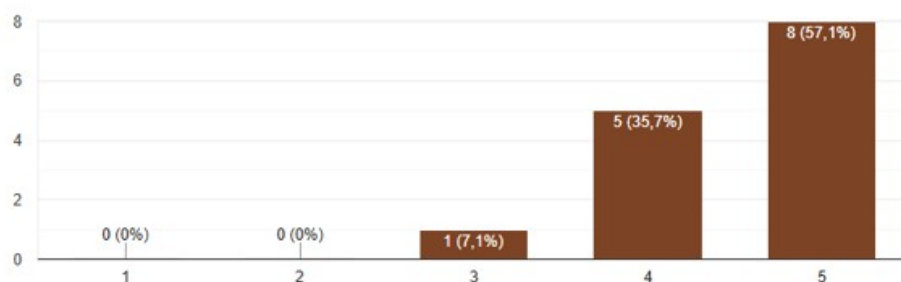
Source: Elaborated by the author (2025).

Figure 16

Percentage of what students thought of the Stomachion

O que achou do Stomachion?

14 respostas



Source: Elaborated by the author (2025).

The questionnaire also included open-ended questions, which allowed the students to express their perceptions qualitatively. The responses to questions such as "What did you like most about the lecture?" and "What suggestions/tips/compliments can you give?", exemplified in Figure 17, frequently highlighted the interactive nature of the activities, as well as the clarity of the explanation of the equidecomposition concept. Many students praised the applied didactics and the opportunity to learn in an innovative way, reinforcing the value of the use of technologies in the mathematics learning process.

Figure 17

Responses to the question: What did you like most about the lecture?

O que você mais gostou na palestra?

14 respostas

A parte do dinâmica de montar as peças
Gostei como foi dinâmico e interativo. formas divertidas de aproximar alunos pra a matemática
Saber que um forma geométrica pode se transformar em outra, e as diversas formas de fazer isso.
Da interatividade dos jogos
Do tangram
Me interessei pela por uma área da matemática que não possuía conhecimento sobre.
Montar formas
Da forma divertida de aprender sobre as formas geométricas
Gostei da parte dos jogos, e de descobrir esses jogos.

Source: Elaborated by the author (2025).

Overall, the analysis of the collected data, both quantitative via evaluation scales and qualitative via open-ended responses, suggests that the educational product had a largely positive impact among the students. The active participation and engagement observed during the application, added to the favorable responses from the questionnaire, indicate that the methodology of teaching areas through equidecomposition, mediated by digital games, was successful in awakening interest and facilitating the comprehension of the proposed content.

FINAL CONSIDERATIONS

The present research sought to investigate and evidence the didactic potential of the equidecomposition concept as a powerful tool for teaching the areas of planar figures in Basic Education. Throughout this study, it was demonstrated that the idea of transforming figures without altering their measure—formalized by the Wallace-Bolyai-Gerwien Theorem—is fundamental not only for the understanding of geometric principles but also for the development of more flexible and creative mathematical thinking. The articulation between the theoretical robustness of equidecomposition and pedagogical practice culminated in the creation and application of an innovative and effective educational product.

The development and implementation of the didactic proposal, composed of interactive online games of Tangram and *Stomachion*, confirmed the central hypothesis that playfulness and manipulation mediated by technology are essential catalysts for the meaningful learning of geometric concepts. Although the case study was conducted with a limited number of participants [14 third-year High School students from Caucaia/CE], direct observation and the qualitative *feedback* obtained through post-intervention questionnaires revealed remarkable

engagement and contagious enthusiasm. The active participation and cooperative interaction during the activities were clear evidence of the positive impact of the approach, validating the importance of teaching methods that transcend the mere transmission of content and the memorization of formulas.

More than the specific mastery of area formulas, the success of the educational product lies in its capacity to foster a set of essential competencies and skills, aligned with the National Common Curricular Base (BNCC). The students demonstrated the development of scientific, critical, and creative thinking by seeking innovative solutions to the challenges proposed by the games. Additionally, the insertion of online games promoted an educational digital culture, enabling the youth to utilize technology as a tool for meaningful learning, stimulating autonomy, proactivity, and persistence in solving complex problems.

In short, this article reiterates that mathematics can be explored in a thought-provoking and relevant way, going far beyond traditional textbooks. By offering a didactic approach that integrates equidecomposition with active and playful methodologies mediated by technology, it was possible not only to facilitate the understanding of important geometric concepts but also to cultivate a set of skills and competencies that are indispensable for the integral development of youth in the 21st century. It is believed that this work and its educational product represent a valuable contribution to mathematics teaching, inspiring future pedagogical practices that unite scientific rigor with joy and discovery.

NOTES

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