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Bisection method and algorithmic thinking: an unplugged instructional sequence in high school

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ABSTRACT

This study aimed to analyze manifestations of algorithmic thinking and difficulties in formalization within an unplugged instructional sequence centered on the Bisection Method, conducted in the still underexplored context of an integrated technical high school program. This qualitative, applied study used a descriptive and exploratory design and involved 21 students and was conducted over three meetings, totaling five class periods. The data comprised field notes, 20 flowcharts, five natural-language algorithmic descriptions, and a questionnaire answered by five students. The analysis followed the principles of content analysis and considered the sequencing of actions, the explicit representation of decisions, the repetition of procedures, interval-endpoint updating, and stopping criteria. Of the 13 flowcharts produced for the "Guess the Number" game, three were classified as adequate, one as partially adequate, three as incomplete, three as using incorrect flowchart notation, and three as simplified or unclear. One flowchart with omissions was submitted for the "Slicing Intervals" game. Among the six flowcharts representing the Bisection Method, two were adequate and four presented inadequacies. None of the five algorithmic descriptions fully explained how to select and update the new interval. It is concluded that the Bisection Method provides a relevant context for articulating Mathematics and Computational Thinking, but the transition from playful activities to formalization requires additional instructional time and teacher support.

KEYWORDS: Mathematics education; High school; Educational games; Technology and education; Digital technology.

Método da bissecção e raciocínio algorítmico: uma sequência didática desplugada no ensino médio

RESUMO

O objetivo deste estudo foi analisar as manifestações do raciocínio algorítmico e as dificuldades de formalização em uma sequência didática desplugada centrada no Método da Bissecção, desenvolvida em um contexto ainda pouco explorado do Ensino Médio técnico integrado. A pesquisa, de abordagem qualitativa, natureza aplicada e caráter descritivo-exploratório, envolveu 21 estudantes e foi realizada em três encontros, totalizando cinco períodos de aula. Os dados compreenderam notas de campo, 20 fluxogramas, cinco registros algorítmicos elaborados em linguagem corrente e um questionário respondido por cinco estudantes. A análise seguiu os princípios da Análise de Conteúdo e considerou o sequenciamento das ações, a explicitação das decisões, a repetição do procedimento, a atualização do intervalo e o critério de parada. Dos 13 fluxogramas do jogo “Adivinhe o número”, três foram classificados como adequados, um como parcialmente adequado, três como incompletos, três apresentaram simbologia inadequada e três foram classificados como simplificados ou sem clareza. No jogo “Fatiando intervalos”, foi entregue um fluxograma com omissões. Entre os seis fluxogramas do Método da Bissecção, dois foram considerados adequados e quatro apresentaram inadequações. Nenhum dos cinco registros algorítmicos explicitou completamente a seleção e a atualização do novo intervalo. Conclui-se que o Método da Bissecção oferece um contexto pertinente para articular a Matemática e o Pensamento Computacional, mas a passagem das atividades lúdicas para a formalização requer mais tempo e mediação pedagógica.

PALAVRAS-CHAVE: Educação matemática; Ensino médio; Jogos didáticos; Tecnologia e educação; Tecnologia digital.

INTRODUCTION

The growing presence of digital technologies in contemporary society has increased the need for skills that go beyond the mere operation of electronic devices. Faced with increasingly complex and rapidly changing challenges, Computational Thinking (CT) has emerged as a relevant competence for twenty-first-century citizenship because it contributes to critical and systematic problem solving across different fields (Raabe et al., 2018; Vicari et al., 2018). Jeannette Wing, one of the main scholars responsible for popularizing the term, argues that CT should be regarded as an analytical skill as essential as reading, writing, and arithmetic and should therefore be part of everyone's basic set of competences (Wing, 2006). In an interview published in ACTIO, Almeida and Motta (2025) likewise identify problem solving as a central aim of CT in Basic Education.

In the context of Brazilian Basic Education, the Base Nacional Comum Curricular (BNCC; National Common Curricular Base) recognizes this importance by establishing an explicit connection between CT and Mathematics (Barichello, 2023; Brasil, 2018). The BNCC defines CT as the ability to "understand, analyze, define, model, solve, compare, and automate problems and their solutions in a methodical and systematic manner through the development of algorithms" (Brasil, 2018, p. 475, authors' translation). The document also states that mathematical processes such as problem solving and modeling are "potentially rich" for developing CT (Brasil, 2018, p. 267, authors' translation), which supports the search for activities that integrate both fields. In this study, algorithmic thinking, also referred to as algorithmic thinking, is understood as a facet of CT centered on designing and analyzing algorithms for problem solving.

Although the literature on integrating CT into Mathematics Education is expanding, gaps remain that justify the present study. A systematic mapping of Brazilian research indicates that most studies in Basic Education focus particularly on elementary and lower secondary education (Ferreira et al., 2021), leaving high school comparatively underexplored. In addition, a recent systematic review shows that high school studies tend to focus on technological tools, that is, plugged approaches (Azevedo, 2024). These findings point to the need for qualitative investigations into the articulation between specific mathematical concepts and the pillars of CT through unplugged activities in integrated technical high school programs, with attention to students' strategies, artifacts, and observed difficulties.

Against this background, the present study uses the Bisection Method as a pedagogical tool for exploring aspects of CT. This choice is justified by the method's algorithmic and iterative structure, which makes it possible to demonstrate how algorithmic thinking can be systematized and formalized. In addition to reporting the implementation, the study examines how this approach unfolds in practice and which difficulties emerge as students formalize the method.

The research question guiding this study is: how does the implementation of an unplugged instructional sequence based on the Bisection Method reveal manifestations of algorithmic thinking and difficulties in its formalization among high school students? To answer this question, the general objective is to analyze manifestations of algorithmic thinking in the students' activities and work. The specific objectives are to: (a) investigate how the Bisection Method can provide a basis for designing and formalizing algorithmic procedures; (b) analyze the contributions and limitations of unplugged educational games for the empirical exploration of the method; (c) identify and analyze difficulties and affordances encountered during the instructional sequence; and (d) examine the evidence found in the students' work and the limitations of the intervention.

In addition to this introduction, the article is organized as follows: Section 2 presents the theoretical framework; Section 3 describes the study methodology; Section 4 reports and analyzes the data in detail; and Section 5 presents the final remarks.

THEORETICAL FRAMEWORK

This section presents the theoretical framework underpinning the study. It first discusses the origins, definitions, and four pillars of CT. It then examines how these pillars are manifested in the Bisection Method, the unplugged approach, and, finally, the relationship between CT and Pólya's (1995) problem-solving approach.

COMPUTATIONAL THINKING: DEFINITIONS AND ORIGINS

Although the term CT gained worldwide prominence following Wing's (2006) publication, it had previously been used by Papert (1980, as cited in Barichello, 2023), who investigated interactions among children, computers, and thought processes. Wing (2006) popularized the concept by defining it as "solving problems, designing systems, and understanding human behavior, by drawing on the concepts fundamental to computer science" (p. 33). She argued that CT should be part of every child's fundamental analytical abilities, alongside reading, writing, and arithmetic.

The concept has been reformulated over time. Wing (2010) describes CT as the thought processes involved in formulating problems and their solutions so that they can be effectively carried out by an information-processing agent, whether a person, a machine, or a combination of both. Other authors offer complementary perspectives. Liukas (2015) summarizes CT as a way of framing problems so that a computer can solve them, whereas Brackmann (2017) presents it as a human capacity to solve problems using principles from Computing.

Despite the diversity of definitions, CT is understood as a problem-solving capacity grounded in Computer Science concepts rather than as a standalone discipline (Vicari et al., 2018). Its structure is commonly organized into four pillars: decomposition, pattern recognition, abstraction, and algorithms (Brackmann, 2017; Raabe et al., 2018).

THE FOUR PILLARS OF CT AND THEIR MANIFESTATION IN THE BISECTION METHOD

The four pillars of CT provide a conceptual structure for addressing problems systematically. Their relationship with Mathematics becomes explicit when examining how the Bisection Method mobilizes each pillar.

Decomposition is the process of breaking a complex problem into smaller, more manageable parts (Brackmann, 2017; Liukas, 2015). This division is strategic rather than arbitrary because it focuses on functional elements that can be solved individually and subsequently recombined to reach a solution to the original problem (Raabe et al., 2018; Shute et al., 2017).

In the Bisection Method, the task of approximating a function root is decomposed into a sequence of simple, repetitive subproblems: (1) define an initial bracketing interval whose endpoint function values have opposite signs; (2) calculate the midpoint; (3) evaluate the sign of the function at that point; and (4) select the new subinterval. This reduction process characterizes algorithmic decomposition.

Pattern recognition is the ability to identify similarities and regularities that support the generalization of solutions (Brackmann, 2017; Liukas, 2015). Generalization is also associated with this pillar because it involves recognizing common features across problems and their solutions, thereby simplifying the process and enabling strategies to be applied in other contexts (Vicari et al., 2018).

In the Bisection Method, a fundamental pattern follows from the Intermediate Value Theorem: for a continuous function over an interval $[a, b]$, the condition $f(a) \cdot f(b) < 0$ guarantees at least one root in that interval. At each iteration, sign analysis guides selection of the subinterval in which the procedure continues. The games were designed to help students intuitively recognize this pattern of successive interval reduction before formalization.

Abstraction entails focusing on the essential details of a problem while disregarding information that is irrelevant to its solution (Brackmann, 2017). Wing (2017) assigns abstraction a central role in CT because it enables the creation of a simplified, focused representation of the problem.

The Bisection Method exemplifies abstraction because it does not depend on the function having a specific algebraic form, such as polynomial or exponential, but on the properties required for the method, particularly continuity and a sign change over the interval. To select the new subinterval, the procedure considers the signs of the function values at the endpoints and midpoint. This reduction of complexity to essential properties occurs in both mathematical and computational reasoning.

Finally, an algorithm is a finite and ordered set of clear, unambiguous instructions for solving a problem (Brackmann, 2017). Once formulated, it can be executed repeatedly by a person or a machine, provided that its steps, conditions, and termination are adequately specified.

In light of these definitions, algorithmic thinking is treated in this study as a manifestation of the practical articulation among the pillars of CT, especially abstraction and algorithms, when students design, represent, and analyze step-by-step procedures for solving problems.

Table 1

Mapping the instructional-sequence activities to the pillars of CT

Teaching activity	Pillars of CT mobilized	Rationale
"Guess the Number" game	Pattern recognition; algorithms	The game encourages students to compare guesses with "higher than" or "lower than" feedback, identify search strategies, and organize a sequence of steps to reach the chosen number.
"Slicing Intervals" game	Algorithms; abstraction	The game introduces the rule of choosing the midpoint of the remaining interval, making the sequence of steps more precise. It also requires students to focus on the interval bounds, which are essential information for the procedure.
"Finding Roots" activity	Pattern recognition; abstraction	Graphical analysis guides students to recognize the sign change, expressed by $f(a) \cdot f(b) < 0$, as a condition that guarantees at least one root in the interval for continuous functions.
Construction of flowcharts and algorithms	Algorithms; abstraction; decomposition	Constructing the flowchart and pseudocode requires students to decompose the method into logical parts, select essential information, and create an objective, executable sequence of instructions.

Source: Prepared by the authors (2025).

The Bisection Method is, by definition, an algorithm. Formalizing it through a flowchart and pseudocode requires students to make the logical structure of the procedure explicit. In the instructional sequence, the task of producing a pseudocode algorithm was intended to support the translation of the method's steps into a precise and objective sequence of instructions that could be followed by either a person or a machine. Creating a flowchart or pseudocode requires students to represent and formalize this structure, including variable initialization, a repetition loop, and an if-then-else conditional structure for deciding which half of the interval should be discarded, thereby articulating the remaining pillars in an executable solution (Wing, 2017).

After presenting the four pillars and their relationship with the Bisection Method, Table 1 summarizes how each activity in the instructional sequence was designed to mobilize them. This mapping expresses the conceptual relationships that informed the activities without assuming that completing them in isolation ensures the development of these pillars.

UNPLUGGED COMPUTATIONAL THINKING

The unplugged CT approach enables computational concepts to be explored without electronic devices (Bell et al., 1998). It can be advantageous in contexts with limited technological resources and can also direct attention to the reasoning processes involved in understanding these concepts (Brackmann, 2017).

In classes using this approach, "unplugged activities frequently involve kinesthetic learning . . . and students work with one another to learn Computing concepts" (Brackmann, 2017, p. 50, authors' translation). School materials and physical examples are commonly used to simulate machine behavior (Vicari et al., 2018). In Basic Education classrooms, early records date from the late 1990s, when Bell et al. (1998) introduced the prototype of a digital book entitled Computer Science Unplugged: Offline Activities and Games for All Ages. In Portuguese, a resource derived from the CS Unplugged Classic project is available under the title Computação Desplugada (Barichello et al., 2022).

CT AND PROBLEM SOLVING

In Mathematics Education, numerous studies seek to understand and improve teaching and learning processes. Among them are studies grounded in problem solving. Within this field, Grave (2021) highlights George Pólya's proposal in How to Solve It (Pólya, 1995), which "now engages in dialogue with computational thinking" (p. 29, authors' translation).

Like CT, Pólya's (1995) problem-solving approach is organized into four phases:

- Understanding the problem: the first phase requires identifying the data, unknowns, and conditions involved. It is closely related to abstraction, which entails selecting information relevant to the solution.
- Devising a plan: the second phase involves drawing on prior knowledge and experience and is related to pattern recognition and the decomposition of the problem into smaller parts.
- Carrying out the plan: the third phase consists of implementing the selected strategy by following the planned steps in an organized manner. When these steps are sufficiently precise and ordered, the procedure can become algorithmic.
- Looking back: the fourth phase involves analyzing the solution and the strategy used, supporting pattern identification, revision of the procedure, and its possible application to other problems.

This correspondence indicates that the algorithmic thinking associated with CT is not external to Mathematics; rather, it systematizes processes of understanding, planning, execution, and strategy revision that are already present in mathematical problem solving.

By integrating the Bisection Method with a playful, unplugged approach, the instructional sequence seeks to explore the pillars of CT and align with Pólya's (1995) problem-solving approach, particularly by organizing activities into stages of understanding, planning, execution, and review. This articulation provides a theoretical foundation for the proposal and guides the analysis of the students' work.

STUDY METHODOLOGY

This section presents the study design, research context and participants, data-generation and analysis procedures, and the instructional sequence designed to meet the research objectives.

As complementary classifications to the qualitative approach, the study is applied in nature because it "aims to generate knowledge for practical application, directed toward solving specific problems" (Gerhardt & Silveira, 2009, p. 37, authors' translation). Regarding its objectives, the study is descriptive-exploratory because it describes the implementation of the instructional sequence and explores manifestations of algorithmic thinking and difficulties in formalizing it in the students' work (Gil, 2002).

The central study design is qualitative because the investigation seeks to understand manifestations of algorithmic thinking and difficulties in its formalization based on the students' activities and work. The focus is not on statistically generalizing the results but on an in-depth, contextualized analysis of the data generated (Gerhardt & Silveira, 2009).

The results are therefore interpreted contextually. The number of participants, attendance variations across meetings, and the short duration of the intervention are study limitations considered in the analysis and discussion.

The investigation was conducted through a pedagogical intervention planned and led by the teacher-researcher, who implemented the instructional sequence and produced the field notes. His dual role as teacher and researcher was considered when interpreting the data, and the observational records were compared with the students' artifacts and questionnaire responses.

DESCRIPTION OF THE INSTRUCTIONAL SEQUENCE

The study was conducted in a fourth-year class of the Technical Program in Administration integrated with high school at the Federal Institute of Rio Grande do Sul (IFRS), Canoas Campus. Twenty-one adult students participated and signed an informed consent form before the activities began. The instructional sequence was implemented over three school days, totaling five 50-minute class periods. Twenty-one students attended the first meeting, while 11 attended each of the second and third meetings.

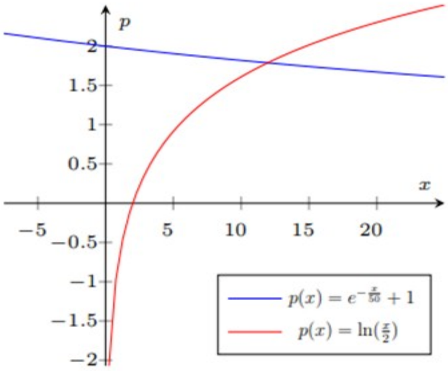
The instructional sequence was organized into five stages comprising activities designed to explore CT through its pillars. The proposed learning objectives were to: (a) recognize an iterative process; (b) construct flowcharts for the games; (c) define function roots or zeros; (d) describe the Bisection Method for approximating function roots; and (e) produce a natural-language algorithmic description of the Bisection Method. This design was intended to reveal manifestations of algorithmic thinking in the students' activities and work.

Stage 1: Motivation

The first stage introduced a market-equilibrium problem formulated using demand and supply functions. Because these functions were exponential and logarithmic, solving their equality analytically was not straightforward. The problem was therefore rewritten as the search for a root of the function defined by the difference between the demand and supply functions, motivating the use of a numerical method and the subsequent introduction of the Bisection Method. Figure 1 presents the graphs used to provide visual context for the problem.

Figure 1

Demand and supply functions in the motivating problem



Source: Prepared by the authors (2025).

Stages 2 and 3: Games for exploring the method

The second and third stages were devoted to a playful exploration of the Bisection Method through two games designed by the researchers: "Guess the Number" and "Slicing Intervals." Played in pairs, both games involved discovering a natural number between 1 and 100.

The "Guess the Number" game explored iterative thinking through unrestricted guesses guided by "higher than" or "lower than" feedback. The activity was intended to help students intuitively recognize the need for a search strategy.

The "Slicing Intervals" game systematized this strategy by introducing the selection of the midpoint of the search interval. It thus brought students closer to the iterative structure of the Bisection Method in a playful context.

After the games, the strategies used, procedural logic, and decisions involved were discussed. Students were then asked to construct flowcharts representing the steps of each game.

Stage 4: Formalization of the Bisection Method

This stage aimed to formalize the procedures explored empirically in the games. Using graphical examples, the "Finding Roots" activity was intended to help students recognize that, for a continuous function over an interval $[a,b]$, the condition $f(a) \cdot f(b) < 0$ guarantees at least one root in that interval. The Bisection Method was then formally presented, including midpoint calculation, selection of the new subinterval, and repetition of the procedure. Finally, students were asked to represent the method through a flowchart and a natural-language algorithmic description.

Stage 5: Application and connection

The final stage applied the Bisection Method to the initial market-equilibrium problem. Students were asked to determine, to one decimal place, an approximation of the root of the function defined by the difference between the supply and demand functions. This stage sought to articulate the playful exploration undertaken in the games, the formalization of the method, and its application to the initial problem.

DATA GENERATION AND ANALYSIS

To strengthen the consistency of the analysis, data-source triangulation was used (Marcondes & Brisola, 2014), involving field notes, student-produced artifacts, and a post-intervention questionnaire.

The following data sources were used:

Field notes: records produced by the teacher-researcher concerning the activities, student participation, strategies articulated during class, and observed difficulties.

Student-produced artifacts: notes made during the games, 20 flowcharts (13 for “Guess the Number,” one for “Slicing Intervals,” and six for the Bisection Method), and five natural-language algorithmic descriptions of the method.

Post-intervention questionnaire: an electronic instrument comprising closed Likert-scale items and open-ended questions designed to record students’ perceptions of the activities. Five participants completed the questionnaire.

The data were analyzed according to the principles of Content Analysis (Bardin, 2002). The corpus was initially organized by data source and instructional-sequence activity. Each student artifact was treated as a unit of analysis, and the artifacts were examined in terms of action sequencing, explicit decisions, representation of repetition, updating of information or the interval endpoints, and definition of the stopping criterion.

Each “Guess the Number” flowchart was assigned to one of the following categories: adequate, when it presented the steps, decisions, repetition, and termination; partially adequate, when the overall logic could be reconstructed despite an omitted element; incomplete, when insufficient information was provided to reconstruct the procedure; inappropriate notation, when process and decision symbols were used incorrectly; and simplified or unclear, when actions and conditions were not sufficiently explicit.

The “Slicing Intervals” flowchart was also assessed for midpoint calculation and updating of the search interval. The Bisection Method flowcharts and algorithmic descriptions were analyzed in terms of the initial interval, midpoint calculation, sign evaluation, selection and updating of the new interval, repetition, and stopping criterion.

During interpretation, the artifact-analysis results were compared with the field notes and questionnaire responses. Closed items were presented as frequencies without inferential statistical treatment, while open-ended responses were used as complementary records of participants’ perceptions.

RESULTS AND DISCUSSION

This section presents the results from the perspective of algorithmic thinking as a facet of CT by integrating the three data sources: field notes, student-produced artifacts, and the perception questionnaire. The analysis connects the activities and observed difficulties with evidence from the students' work and questionnaire responses to understand manifestations of algorithmic thinking and the challenges involved in formalizing it at the intersection of Mathematics and CT.

REPORT OF THE INTERVENTION: CLASSROOM DYNAMICS AND CHALLENGES

The instructional sequence was implemented over three school days, totaling five 50-minute class periods. The "Guess the Number" game was introduced during the first meeting, attended by 21 students. According to the field notes, the activity encouraged participation and was well received by the class, which is consistent with the interactive possibilities associated with unplugged activities (Bell et al., 1998). During the game, one student spontaneously adopted a successive interval-reduction strategy similar to bisection, providing an indication of an intuitive understanding of the procedure before formalization.

Time management was one of the challenges observed in the first meeting. The period allocated to formalization, particularly constructing the game flowchart, proved insufficient. Although students reported being familiar with this type of representation, only 13 submitted a flowchart, and some artifacts contained omissions, lacked clarity, or used notation inappropriately. The difference between the students' stated knowledge and the artifacts produced indicates difficulties in formally representing the strategy, without supporting the claim that all students fully understood the game logic.

The second meeting, attended by 11 students, featured the "Slicing Intervals" game, which introduced midpoint selection and updating of the search interval. The field notes recorded student participation, although some pairs still had difficulty understanding the rules. Students were then asked to construct a flowchart of the procedure, but only one artifact was submitted.

During the third and final meeting, also attended by 11 students, the Bisection Method and its application to the initial market-equilibrium problem were formally presented. Students were then invited to represent the method through a flowchart and a natural-language algorithmic description. Six flowcharts and five algorithmic descriptions were collected. The limited time devoted to the preceding activities hindered the articulation between the games and formalization of the method. The field notes and artifacts indicated difficulties particularly in making decisions explicit, updating the interval endpoints, and organizing instructions, all of which require additional time and instructional support.

ANALYSIS OF STUDENT ARTIFACTS: EVIDENCE OF REASONING AND FORMALIZATION DIFFICULTIES

Analysis of the student-produced artifacts—flowcharts and natural-language algorithmic descriptions—complements the field notes and enables an examination of how the procedures were represented and formalized. The artifacts were analyzed according to the dimensions, categories, and criteria presented in the methodology: action sequencing, explicit decisions, repetition, updating of information or the interval endpoints, and the stopping criterion. Omissions, ambiguities, and inappropriate notation were therefore interpreted as indications of formalization difficulties without directly attributing unobservable cognitive processes. Content Analysis made it possible to organize and interpret these recurring features in light of the theoretical framework (Bardin, 2002).

Table 2 systematizes the results by presenting the distribution of artifacts according to the categories and criteria defined in the methodology and relating the observed characteristics to the identified formalization difficulties.

Table 2

Classification of student-produced artifacts

Artifact	Classification of results	Observed features
"Guess the Number" flowcharts (13 artifacts)	Three adequate; one partially adequate; three incomplete; three using inappropriate notation; and three simplified or unclear. Examples are shown in Figures 2–4.	Adequate artifacts specified the steps, decisions, repetition, and termination. The remaining artifacts contained omissions, lacked clarity, or showed difficulties in using flowchart notation.
"Slicing Intervals" flowchart (1 artifact)	The artifact included midpoint calculation but did not fully specify the decisions or how the interval endpoints should be updated.	The artifact attempted to represent successive interval reduction but remained incomplete regarding the conditions and sequence of the procedure.
Bisection Method flowcharts (6 artifacts)	Two were classified as adequate, as shown in Figures 5 and 6. The other four contained inadequacies: one had an incorrect starting point, one was unclear, and two did not adequately specify how the interval endpoints should be updated, creating a risk of repetition without procedural progress.	The main difficulties concerned defining the initial stages, updating the interval endpoints endpoints, and articulating repetition with the stopping criterion.
Natural-language algorithmic descriptions of the Bisection Method (5 artifacts)	All five descriptions were written in natural language and implicitly contained decision structures. None fully specified how to select the new interval and update its endpoints. Figure 7 provides an example.	Difficulties were observed in formulating precise, unambiguous instructions, particularly for updating the interval endpoints endpoints and continuing the procedure.

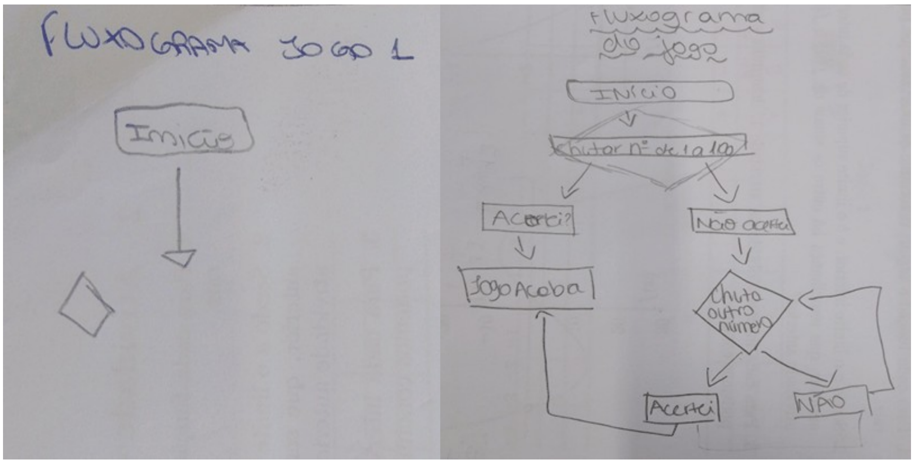
Source: Research data (2025).

Figures 2–4 provide examples of the categories used to classify the "Guess the Number" flowcharts and illustrate the characteristics underpinning the analysis.

Figure 2 presents two examples of difficulties in formally representing the "Guess the Number" game. In Figure 2a, the artifact was classified as incomplete because it did not contain enough information to reconstruct the sequence, decisions, and termination of the procedure. In Figure 2b, although some actions and decisions can be identified, the symbols for processes and decision points were used inappropriately. The examples indicate distinct difficulties: omission of steps required to understand the procedure in the first case and inappropriate graphic representation of the game's conditional logic in the second.

Figure 2

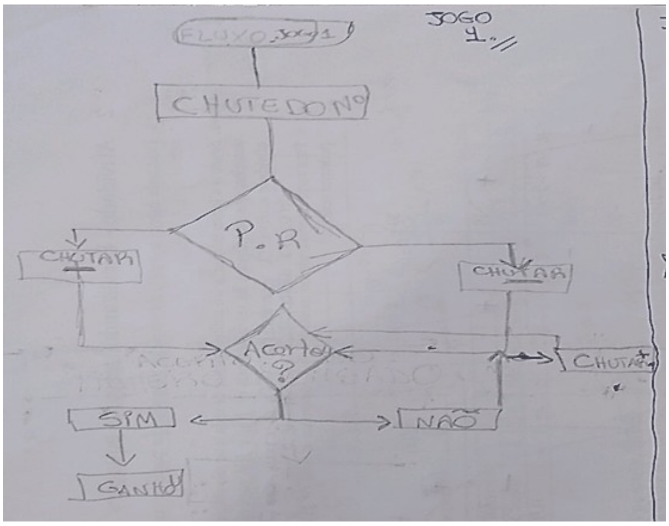
"Guess the Number" flowcharts: (a) incomplete artifact; (b) artifact using inappropriate notation



Source: Research data (2025).

Figure 3

"Guess the Number" flowchart classified as simplified or unclear



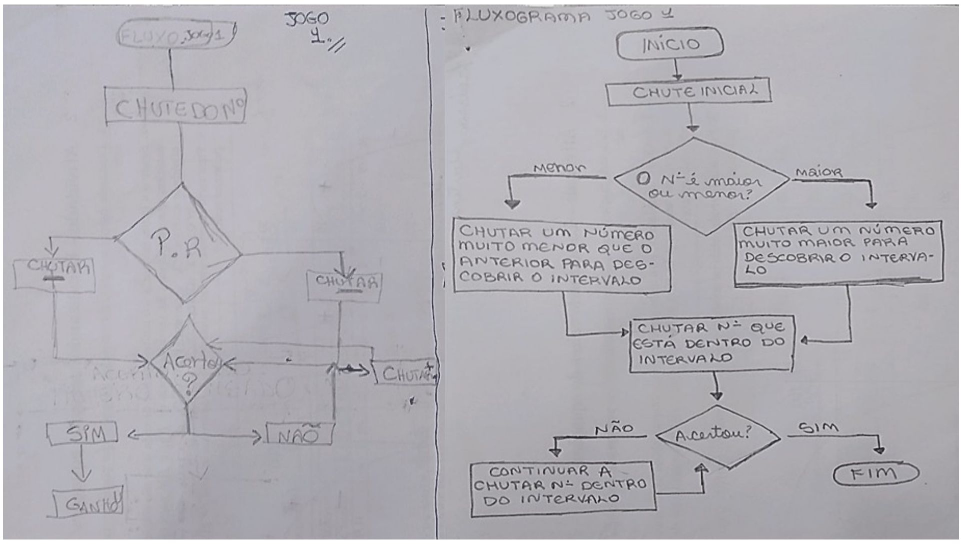
Source: Research data (2025).

Figure 3 presents an artifact classified as simplified or unclear. Although it records the action of choosing a number, the flowchart does not specify the conditions governing continuation of the game, changes to the guesses, or termination of the procedure. Without these relationships, the sequence is ambiguous and difficult for another person to execute.

Figure 4 presents two artifacts classified, respectively, as adequate and partially adequate. In Figure 4a, the flowchart specifies the sequence of actions, decision points, repetition, and termination, allowing the game logic to be reconstructed. In Figure 4b, repetition is represented, but the initial verification of a correct guess is incomplete. The artifact was therefore classified as partially adequate. Comparing the examples shows that the termination condition is a specific and necessary component of formalization.

Figure 4

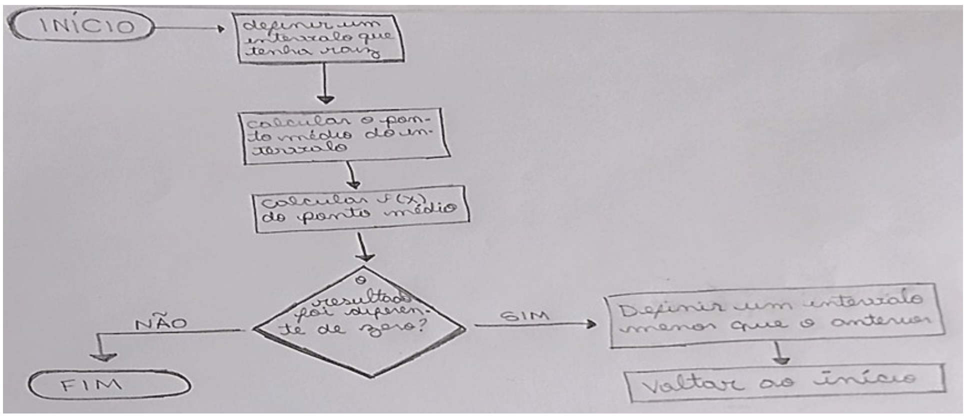
"Guess the Number" flowcharts: (a) adequate artifact; (b) partially adequate artifact



Source: Research data (2025).

Figure 5

Bisection Method flowchart classified as adequate—artifact 1



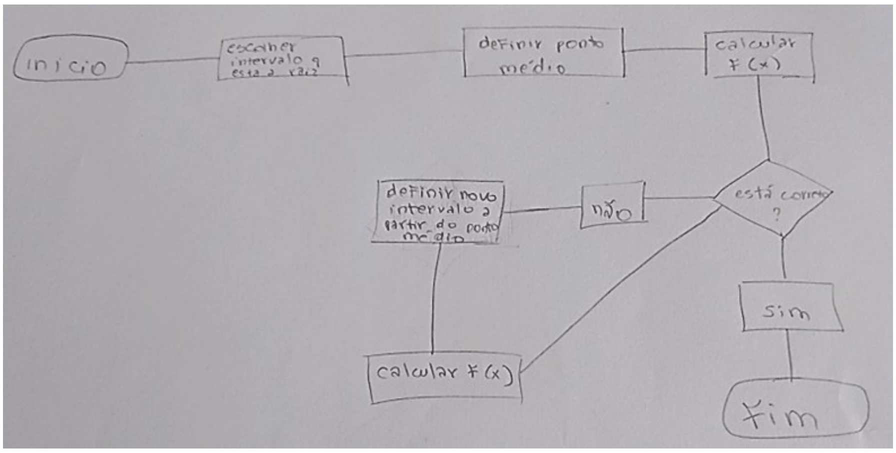
Source: Research data (2025).

Formalizing the Bisection Method required the flowcharts to represent not only sequencing and decisions but also midpoint calculation, interval-endpoint updating, repetition, and the stopping criterion. Two of the six artifacts were classified as adequate and are shown in Figures 5 and 6. In these artifacts, the elements are organized so that the procedure can be reconstructed from the students’ representations.

The other four artifacts exhibited different inadequacies: an incorrect starting point, an unclear sequence of operations, and insufficiently specified updates to the interval bounds. In the two artifacts with insufficiently specified updating, the representation could result in repeated execution without an effective change to the interval and, consequently, without progress toward approximating the root. These artifacts indicate difficulties in articulating the decision about the function sign, updating variables, and continuing the process. Although related, interval-endpoint updating and the stopping criterion are distinct components and must both be explicit if the algorithm is to be executed unambiguously.

Figure 6

Bisection Method flowchart classified as adequate—artifact 2



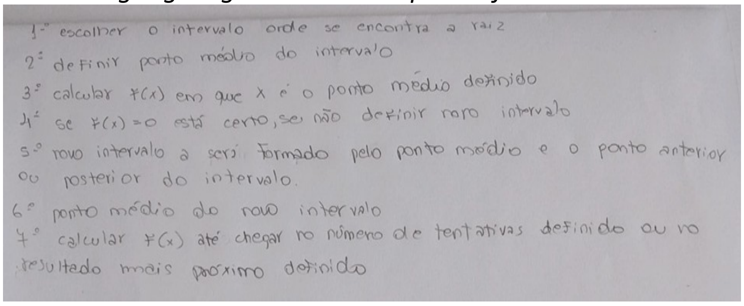
Source: Research data (2025).

The transition from visual representation to a natural-language algorithmic description also presented difficulties. The five descriptions included sequences of actions and implicitly formulated if-then-else decision structures. However, none fully explained how to select and update the new interval after evaluating the midpoint. Figure 7 shows one such artifact, which mentions the need to define a new interval but does not specify the conditions determining its new bounds. This omission compromises the precision of the instructions and prevents unambiguous execution.

In summary, besides corroborating the intervention report, artifact analysis identified recurring difficulties in formalizing algorithmic thinking. The results suggest difficulties in three dimensions: (1) use of a representation language; (2) articulation of algorithmic logic; and (3) textual formalization. A major pedagogical challenge therefore lies in moving from playful exploration to systematic formalization in each dimension.

Figure 7

Natural-language algorithmic description of the Bisection Method


1° escolher o intervalo onde se encontra a raiz
2° definir ponto médio do intervalo
3° calcular f(x) em que x é o ponto médio definido
4° se f(x) = 0 está certo, se não definir novo intervalo
5° novo intervalo a será formado pelo ponto médio e o ponto anterior ou posterior do intervalo.
6° ponto médio do novo intervalo
7° calcular f(x) até chegar no número de tentativas definido ou no resultado mais próximo definido

Source: Research data (2025).

STUDENTS' PERCEPTIONS: TRIANGULATING THE RESEARCH FINDINGS

Although the small number of questionnaire respondents—five students—limits generalization, the data provide information about participants' perceptions in accordance with the qualitative perspective adopted (Gerhardt & Silveira, 2009). The questionnaire thus complements observations of the intervention by presenting the students' own perspectives.

The results presented in Table 3 complement the preceding discussion. All five respondents agreed that CT can be applied in areas of the Technical Program in Administration, indicating that they recognized its applicability beyond the mathematical context, an aspect related to one of the intervention objectives.

At the same time, the data reinforce the observed difficulties. Some students' perception that the allocated time was insufficient converges with the intervention report (subsection 4.1). Likewise, responses concerning algorithm construction are consistent with artifact analysis (subsection 4.2), which identified formalization as one of the main difficulties in the intervention.

Table 3

Distribution of responses to selected items in the perception questionnaire (n = 5)

Questionnaire item	Strongly agree	Partially agree	Partially or strongly disagree	Not applicable
Computational thinking can be applied in areas of my technical program	5 (100%)	0 (0%)	0 (0%)	0 (0%)
The time allocated to each activity was sufficient	2 (40%)	0 (0%)	3 (60%)	0 (0%)
I was able to write the Bisection Method algorithm	1 (20%)	1 (20%)	2 (40%)	1 (20%)
The workshop contributed to my learning not only in Mathematics but also in other areas	2 (40%)	2 (40%)	1 (20%)	0 (0%)

Source: Research data (2025).

The open-ended questionnaire responses complement the quantitative data by presenting students' perceptions of the experience. One response suggests that the participant connected the activity with technical-course content and everyday situations: "I was able to practice constructing flowcharts, which we use in the technical Administration subjects, and, most importantly, to realize that we use algorithms in our daily lives, even if indirectly" (authors' translation).

Another student, in turn, described a difficulty related to the final stage of the sequence: "I was confused about the final topic" (authors' translation). Together, the responses summarize different perceptions of the experience: on the one hand, the identification of relationships among flowcharts, algorithms, and other situations; on the other, difficulty formalizing the Bisection Method. The latter response converges with the difficulties observed in the artifacts and reinforces the need for additional time and instructional support when moving from playful activities to formalization of the method.

FINAL REMARKS

The general objective of analyzing manifestations of algorithmic thinking in the students' activities and work was achieved. The evidence suggests that the Bisection Method is a promising alternative for exploring aspects of CT through Mathematics, although it does not support the claim that this capacity was developed. According to the field notes, the game-based approach encouraged student participation. Moreover, all five questionnaire respondents agreed that CT can be applied in areas of the Technical Program in Administration, indicating recognition of its applicability in that context.

A major contribution of the study lies in interpreting the observed difficulties as pedagogical findings. The conclusion that insufficient time was devoted to formalization is a relevant methodological result. The data indicate that the playful activities supported intuitive exploration of the concepts, whereas the transition to formalization through flowcharts and algorithmic descriptions concentrated the main obstacles. Inadequacies in the artifacts, such as the risk of indefinite repetition, reveal problems in making decisions explicit, updating the interval endpoints, and defining the stopping criterion, thereby reinforcing the need for additional time and instructional support.

The study reinforces the relationship between Mathematics and CT by indicating that mathematical concepts can provide contexts for exploring algorithmic thinking. The difficulties observed in producing algorithmic descriptions, together with some respondents' perception that they had not previously recognized how mathematical thinking can be structured and systematized, demonstrate the relevance of pedagogical interventions such as the one proposed, which seek to make the logical and procedural nature of mathematical thinking explicit.

The implications for teaching practice are clear. The unplugged sequence supported the initial formulation of steps and conditions and the systematization of procedures; however, planning should allocate more time for students to analyze and refine their work, particularly regarding stopping criteria, interval-endpoint updating, and justifications of convergence.

The study was short and involved a limited number of participants; its conclusions are therefore contextual and cannot be generalized. Triangulating artifacts, field notes, and the questionnaire increases interpretive consistency but does not eliminate the study's biases and limitations.

Future research should consider: (i) designing new instructional sequences for other topics, such as matrices and Gaussian elimination; (ii) developing instruments that make stopping criteria and convergence explicit; and (iii) adopting assessment strategies that combine artifact analysis with performance measures, such as pre- and post-intervention assessments or transfer tasks, to estimate possible learning gains.

In conclusion, this qualitative study offers support for integrating CT into high school Mathematics Education and identifies aspects of formalizing algorithmic thinking that require additional time and instructional support.

NOTES

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