ACTIO
Docência em Ciências

ACTIO: Docência em Ciências
ISSN: 2525-8923

http://periodicos.utfpr.edu.br/actio

A didactic proposal for teaching rational numbers using semiotic representation

ABSTRACT

O artigo apresenta os resultados de uma investigação que buscou identificar de que forma uma sequência didática organizada com base na Teoria da Aprendizagem Significativa e na Teoria dos Registros de Representação Semiótica pode se constituir em material potencialmente significativo para o ensino do conjunto dos números racionais. A pesquisa foi realizada com 21 estudantes de uma turma de 6º ano do ensino fundamental de uma escola da rede estadual do RS. Os dados foram coletados por meio do diário de bordo da professora-pesquisadora e dos materiais produzidos pelos participantes no decorrer da implementação da proposta. A análise demonstrou que atividades pensadas com base nas teorias favoreceram a predisposição dos estudantes em aprender significativamente, a identificação dos conceitos subsunçores da área, a promoção da diferenciação progressiva e da reconciliação integrativa dos temas abordados, assim como a transferência dos conceitos estudados para diversos contextos.

PALAVRAS-CHAVE: TAS; TRRS; Números racionais; Representação semiótica.

Fernanda Gheno
151037@upf.br
orcid.org/0000-0003-6264-0492
Secretaria de Estado da Educação do Rio Grande do Sul, Passo Fundo, Rio Grande do Sul, Brasil.

Luiz Marcelo Darroz
ldarroz@upf.br
orcid.org/0000-0003-0884-9554
Universidade de Passo Fundo (UPF), Passo Fundo, Rio Grande do Sul, Brasil.

Cleci Teresinha Werner da Rosa
cwerner@upf.br
orcid.org/0000-0001-9933-8834
Universidade de Passo Fundo (UPF), Passo Fundo, Rio Grande do Sul, Brasil.

Uma proposta didática para o ensino dos números racionais utilizando representação semiótica

RESUMO

O artigo apresenta os resultados de uma investigação que buscou identificar de que forma uma sequência didática organizada com base na Teoria da Aprendizagem Significativa e na Teoria dos Registros de Representação Semiótica pode se constituir em material potencialmente significativo para o ensino do conjunto dos números racionais. A pesquisa foi realizada com 21 estudantes de uma turma de 6º ano do ensino fundamental de uma escola da rede estadual do RS. Os dados foram coletados por meio do diário de bordo da professora-pesquisadora e dos materiais produzidos pelos participantes no decorrer da implementação da proposta. A análise demonstrou que atividades pensadas com base nas teorias favoreceram a predisposição dos estudantes em aprender significativamente, a identificação dos conceitos subsunçores da área, a promoção da diferenciação progressiva e da reconciliação integrativa dos temas abordados, assim como a transferência dos conceitos estudados para diversos contextos.

PALAVRAS-CHAVE: TAS; TRRS; Números racionais; Representação semiótica.

INTRODUCTION

Mathematics is a science that studies numerical symbols, formulas, and theorems, and it is considered the science of logical and abstract reasoning. It has essentially helped the evolution of humanity throughout the ages. Linhares (2016) states that mathematical knowledge enabled humans to define environment management strategies, creating and designing instruments to seek explanations about human existence and natural facts and phenomena.

In this sense, Walle (2009) explains that science presents a pattern, a regularity, and a logical order that must be discovered and explored to provide an everyday meaning to its concepts. In the author's words:

> The world is filled with patterns and order in nature, art, construction, and music. Patterns and order emerge in commerce, science, medicine, industries and factories, and sociology. Mathematics reveals this order, giving it meaning and using it in various fascinating ways, improving our lives and expanding our knowledge (Walle, 2009, p. 32).

Regarding Mathematics teaching, Kuhn and Bayer (2023, p. 151) emphasize that the content worked in the school environment must "be meaningful for students, that is, they need to feel that such knowledge is relevant for their lives or that it will be useful to understand the world in which they live." However, students still perceive the subject as inaccessible, distant from their daily lives, and with concepts difficult to understand.

Duval (2013) reports that one of the main challenges in Mathematics teaching is that mathematical objects do not have a physical existence and can only be accessed through a semiotic system. Thus, understanding the diversity of semiotic systems during the learning process is essential for acquiring and constructing new concepts (Duval, 2013).

This reality includes the different representations of elements in the set of rational numbers. Junqueira (2021) affirms that, during the teaching process in the school context, fractions are taught separately from topics related to the decimal form. Walle (2009, p. 322) explains that this approach prevents the understanding that decimal and fractional symbolisms represent the same ideas, leading to multiple challenges in calculating operations with fractions and decimals, interpreting percentages, using fractions in measures, and understanding the concepts of ratio and proportion.

In this sense, several studies have sought teaching methodologies to develop meaningful learning. Moreira (1999) explains that Ausubel's Meaningful Learning Theory (MLT) supports this learning approach, as it occurs when new ideas, concepts, and propositions are non-arbitrarily and substantively linked to other relevant, inclusive, and well-established knowledge already present in the individual's cognitive structure. This argument corroborates Duval (2013), who states that understanding semiotic representations of rational numbers is essential for promoting the meaningful learning of the set of rational numbers.

This context raises the following research question: How can a didactic sequence supported by MLT and the Theory of Semiotic Representation Registers (TSRR) represent potentially meaningful material for developing the learning of rational numbers in the sixth year of primary education?

This study aimed to solve this problem by presenting the findings of an investigation to identify how a didactic sequence organized from MLT and TSRR precepts may constitute potentially meaningful material for teaching the set of rational numbers.

Hence, this research is organized as follows: First, a brief description of TSRR and MLT; then, the presentation of the elaborated didactic proposal; next, a description of the research methodology and the exposure of findings; and lastly, final considerations.

THEORETICAL FRAMEWORK

THEORY OF SEMIOTIC REPRESENTATION REGISTERS

The TSRR, proposed by the French philosopher and psychologist Raymond Duval, is a cognitive approach that evaluates the difficulties encountered in the learning process and the peculiar cognitive development of Mathematics (Duval, 2013). The author considers the mode of access to objects within the domain, the variety of semiotic systems that provide their representation, and the necessary distinction between the mathematical object and its representation.

Duval (2012) defines the semiotic system as a set of signs organized according to their rules of formation and conventions, which present internal relationships and allow the identification of the represented objects. That is a system that performs the communication function, as it is capable of producing and transmitting information.

Duval (2012) chose the word “register” to represent the specific semiotic systems of Mathematics. The author's conception states that a representation register is a semiotic system that performs the cognitive functions of objectification, treatment, and conversion in addition to communication. Hence, Duval (2012) considers the existence of four representation registers: natural language, writing systems, Cartesian graphs, and geometric figures.

The author states that various semiotic systems allow diversifying representations of the same mathematical content, which increases individual capacities, as no representation is complete compared to the object it represents from a cognitive perspective. In this direction, Duval (2012) reports that the mobilization and coordination of various representation registers are essential to avoid confusing mathematical matters with their representations and, at the same time, recognize them in each representation.

Thus, Duval (2012) affirms that the transformation of mathematical knowledge into actual understanding requires students’ spontaneous mobilization of different semiotic representations of the same object. The author also emphasizes three cognitive activities that must be provided to avoid confusing a mathematical object with its representation registers: formation, treatment, and conversion.

Duval (2012) explains that formation involves representing a register through its statement in its natural language in an intelligible way. It is the description of a problem and/or activity, that is, the content's characteristics, with the "function of units and rules of formation of the cognitive register whose product is the representation" (Duval, 2012, p. 271). Treatment, in turn, is the transformation of the register without changing its representation, that is, an internal transformation. Thus, each register has its set of treatment and functioning rules, which are not necessarily valid for another register. Finally, conversion is the transformation of one representation into another register. That means a transformation between different registers. According to Duval (2012), the representation of an object in a given register is converted into a representation in another register, which retains the reference but not the meaning, not maintaining the same properties of the object. Hence, the conversion operation allows understanding different aspects of the same object, leading to knowledge.

The author affirms that teaching usually prioritizes the first two activities: formation and treatment. However, he also emphasizes that conversion must be included in teaching activities, as it is through changes in representational registers that conceptual formation and the transformation of information into knowledge become most effective (Duval, 2013).

In this sense, Duval (2013) mentions that there is no conceptual acquisition of an object without resorting to semiotic systems. The fundamental law of the cognitive thought function highlights that "there is no noesis (conceptual acquisition of an object) without semiosis (apprehension or production of a semiotic representation)," representing the essence of solving Mathematics learning problems (Duval, 2013, p. 31).

However, the author highlights that if students know how to solve a situation in a specific representation (semiosis), it does not guarantee they have understood the concept. The conceptual acquisition of an object (noesis) requires mobilizing several representations. That occurs because each representation register reveals a given content and characteristic, another sense of the object.

Formation, treatment, and conversion may allow individuals to construct meaningful learning. Thus, the Theory of Semiotic Representation Registers can be aligned with the Meaningful Learning Theory, which will be summarized next.

MEANINGFUL LEARNING THEORY

Proposed in 1963 by American psychologist, educator, and researcher David Paul Ausubel (1918 - 2008), MLT assumes that the more individuals know, the more they learn. In this sense, the theory anticipates learning as an expansion and reconfiguration of knowledge in the learner's mental structure (Darroz, Rosa, & Kuiava, 2023). That means that the new information must non-arbitrarily and substantively (non-literally) relate to some knowledge or body of knowledge called "subsuming concepts" in the individual's cognitive structure (Ausubel, 1973). These concepts grow and change during the meaningful learning process.

Contrary to this learning mode, Ausubel (1973) emphasizes that learning new information without interacting with existing concepts in the learner's cognitive structure promotes mechanical learning. However, the author explains that, in MLT, the two learning types do not oppose as both are continuous. That means that mechanical learning may evolve and become meaningful if the subject addresses new concepts and builds new relationships (Ausubel, Novak, & Hanesian, 1980).

Meaningful learning requires meeting three conditions (Moreira, 2012). The first regards the existence, in the student's cognitive structure, of adequate subsuming concepts specifically relevant to providing meaning to new knowledge. The second relates to the student's readiness for non-arbitrary and non-literal learning. The third refers to the instructional material, which must be potentially meaningful to establish the relationship between the learner's new and subsuming concepts. Mechanical learning occurs when one of the three conditions is not met (Moreira, 2012).

Ausubel, Novak, and Hanesian (1980) propose using advance organizers to assist in the cognitive interaction between new and subsuming concepts available in the cognitive structure. These resources are introductory and must be presented before the material for learning but at higher levels of abstraction, generality, and inclusiveness. They "serve as a bridge between current and required knowledge to learn the future task meaningfully" (Ausubel, Novak & Hanesian, 1980, p. 171).

Meaningful learning is possible by developing, elaborating, and differentiating concepts as the consequence of successive interactions that materialize in two processes: progressive differentiation and integrative reconciliation (Moreira & Masini, 2006). Progressive differentiation is assigning new meanings to a given subsuming concept. It results from successively using this subsuming concept to provide meaning to new knowledge. Conversely, integrative reconciliation analyzes the relationships between ideas, indicating and recombining the relevant similarities and differences of concepts with pre-existing elements in the cognitive structure.

After these processes, how do you know whether they have resulted in meaningful learning? Ausubel, Novak, and Hanesian (1980) admit this is a difficult question. According to the authors, short-term perceptions are signs of meaningful learning. The best way to achieve these perceptions is by formulating questions and problems related to the developed work but in a context unfamiliar to students, requiring them to transform the acquired knowledge (Ausubel, Novak, & Hanesian, 1980, p. 21).

Moreira (2010, p. 19) also argues that "immediately after meaningful learning, a second stage begins: obliterating assimilation." That means that if students already know the A' definition in their subsuming concepts and assimilate the new a' concept to this knowledge, they will integrate the "a" (A' + a') definition for some time. However, the combined knowledge of both concepts may fall into oblivion if it ceases to be part of their daily lives. In the author's words:

> [...] the interaction product A'a', for some time, is dissociable in A' and a', thus favoring the retention of 'a'. However, although the assimilation process favors retention, the acquired knowledge remains subject to the erosive influence of a reductionist tendency in cognitive organization: It is easier and more cost-effective to retain only the most general and stable ideas, concepts, and propositions than the newly assimilated knowledge. Immediately after meaningful learning, a second assimilation stage begins: obliterating assimilation. The new information becomes spontaneously and progressively less dissociable from its anchor ideas (subsuming concepts) until they are no longer

> available, i.e., irreproducible as individual entities. Then, dissociation becomes impossible beyond this point, and A'a' reduces to A' (Moreira, 2010, p. 19).

In this sense, considering that there is no teaching without learning, that teaching is the means and learning is the end, and aiming to achieve the recommended learning with positive rational numbers, we sought to approximate the precepts of the two theories: TSRR and MLT. The next item presents this approach.

DIDACTIC PROPOSAL

This section describes the didactic proposal elaborated for the present study. It is fully available at http://educapes.capes.gov.br/handle/capes/746124.

The organization of all steps of the didactic proposal values the active participation of students. Thus, they were divided into three stages. The first focused on understanding the fraction as part of a whole and its representation in decimal form. It addressed concepts related to positive rational numbers, such as fraction types, comparisons between numbers, ordering on the number line, and the four basic mathematical operations. The second stage discussed the relationship between fractions and percentages, transforming one representation into another. Finally, the third stage drew attention to transition activities involving various representations of rational numbers and applying the studied concepts in different contexts from those seen in the classroom.

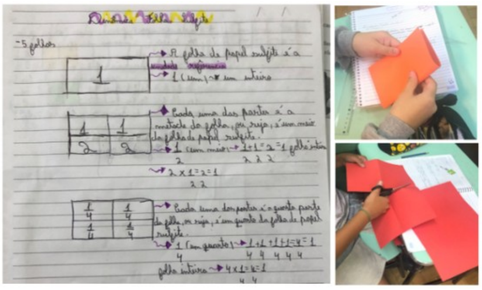
The first stage identified the subsuming concepts related to the subject in the cognitive structure of participants. Hence, a questionnaire with open-ended questions was applied to gather the participants' knowledge of the definition of fractions. Then, attempting to connect new to subsuming concepts and highlight others, the history of rational numbers was discussed with the students through a text.

After reading the text, an activity using clay represented an advance organizer to establish links between subsuming concepts and semiotic representations. In this activity, students made a clay cake and divided it into equal parts. Its implementation emphasized various representations of quantity to provide students with opportunities to understand that the same amount can be expressed in different forms.

Subsequently, the progressive differentiation process began with a folding and cutting activity with A4 paper sheets, in which students built flat shapes corresponding to parts of a whole (Figure 1). These materials allowed participants to compare fractions and locate the parts between the natural numbers expressed on a line.

Figure 1

Paper sheet activity

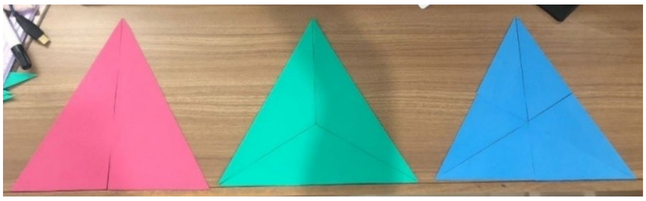


Source: Research data (2023).

Progressive differentiation was promoted with an activity in which students had to compose and decompose triangles. Hence, groups of approximately four components received a set with three equilateral triangles made in E.V.A. material. Each triangle had an assigned color and was cut in different ways but with equal parts (Figure 2).

Figure 2

Set of equilateral triangles



Source: Research data (2023).

The task was organized in steps: I) Assemble each of the whole triangles; II) Assemble whole triangles with different colors; III) Assemble half of a whole triangle using two colors; IV) Assemble one-third of the whole triangle. At each step, the students should draw in their notebooks the different representations and the fraction that each part represented from the whole. They should also write, in the additive form, the fractions equivalent to the whole triangle.

Next, an activity was developed using Cuisenaire Rods (Figure 3) to promote the learning of equivalent fractions.

Figure 3

Cuisenaire Rods

Source: Research data (2023).

Cuisenaire Rods comprise a wooden box containing little wooden rods that compose five families. The colors relate to the length of the rods and help students identify the standard number, contributing to the fixation of the number and its relationships with other numbers. This material motivated students to perceive the number as a continuous amount and establish new rational possibilities to identify rational numbers.

Regarding the integrative reconciliation of the concepts addressed so far, the group was invited to prepare a cake in the school kitchen, following the ingredient amounts and the instructions indicated in the recipe. This process addressed the operations of addition, subtraction, multiplication, and division with rational numbers in fractional and decimal forms.

The second stage promoted a dialogue with the participants to identify the subsuming concepts related to percentages in the cognitive structure of each one. Then, newspaper reports of economic indicators and store inserts advertising cash discount sales conditions involving interest served as a cognitive bridge between current and new knowledge.

Subsequently, in an attempt to acquire new knowledge anchored in the determined subsuming concept, the students were explained that percentages correspond to a fraction of the denominator 100, demonstrating percentages as irreducible fractions and presenting the corresponding drawing of the fraction. The activity also included drawings of a whole divided into equal parts, potentially favoring the understanding of diverse representations of fractions, decimal numbers, and percentages.

The situations of discounts, interest, and economic indicators presented earlier were then analyzed to promote progressive differentiation and integrative reconciliation, allowing students to understand the value corresponding to a given percentage of a whole. The activity demonstrated that 100% is equal to one whole; thus, percentages are fractions of this whole. Furthermore, the corresponding value of a percentage stems from the product between the value equivalent to a whole and the fractional or decimal representation of the percentage. Likewise, the determination of an amount plus interest was explained.

The third stage focused on activities that would prompt students to work with the four representations (fractions, percentages, decimals, and drawings) of rational numbers and the transition between them.

The first activity of this stage, which also provided the integrative reconciliation of the addressed subjects, consisted of reading a part of the book entitled *Aritmética da Emília* (Emilia's Arithmetic) by Monteiro Lobato. After reading the book, the students should locate the excerpts describing parts of a whole and write them as fractions. Then, they should represent these amounts in a drawing, writing them in the form of decimals and percentages.

The second part of this stage involved solving a set of mathematical problems based on situations similar to students' daily lives, aiming to support the assimilation process outlined by MLT and to encourage the spontaneous mobilization of different semiotic representations of positive rational numbers. This set was available in an open questionnaire on Google Forms, and participants had 100 minutes to answer it.

Finally, an activity was performed with candy-coated chocolate. Each student received 100 colored candy-coated chocolates, a ruler, and three A4 paper sheets. The task was to separate the chocolate candies by color and count the number corresponding to each color (Figure 4).

Figure 4

Chocolate candy activity

Source: Research data (2023).

Then, the students should register the results in a spreadsheet and organize the data in fractions, decimals, percentages, and graphs. Finally, a one-minute paper exercise was performed, motivating students to write about their perceptions during the activity.

THE RESEARCH

The developed qualitative research analyzed how a didactic sequence based on MLT and TSRR implemented with a group of 21 sixth-grade students from a primary state school in a municipality in upstate Rio Grande do Sul may represent potentially significant material to teach the set of rational numbers.

The research is qualitative because it corroborates the concepts by Minayo (2002), who highlights that qualitative research aims to study aspects of reality that cannot be quantified, seeking to explain the dynamics of social relationships. Furthermore, this investigation is consistent with Gil (2017) and Creswell (2007) in characterizing qualitative research as interpretative, applying data collection and analysis strategies based on texts and images, with researchers often involved with the participants.

The data highlighted elements to answer the central question of this study. Hence, the selected collection instruments were the logbook maintained by the teacher-researcher and materials produced by the participants during the implementation of the proposal. Zabalza (2004, p. 10) mentions that the logbook helps reflect teaching practices through records throughout the activity and promotes a reflective distance from the activity, allowing study evaluation from another perspective. The choice to examine the materials produced by the students was because these materials required participants to apply the studied concepts to new contexts, which, according to Moreira (2016), may show evidence of meaningful learning.

The collected data were analyzed according to five categories from MLT assumptions, defined *a priori*: readiness to learn, subsuming concepts, advance organizers, progressive differentiation and integrative reconciliation, and application in new contexts.

The material analysis was based on the described methodologies, providing the findings below.

RESULTS

The findings of the implementation of the didactic proposal are presented according to the established analysis categories.

READINESS TO LEARN

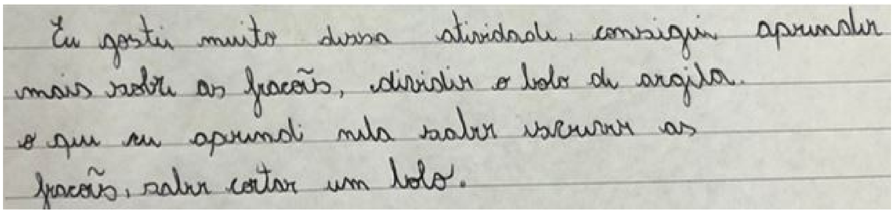
Meaningful learning effectiveness requires potentially meaningful material and the readiness of individuals for meaningful learning. Moreira (2016, p. 62) states that “to learn meaningfully, students have to be willing to non-arbitrarily and non-literally (substantively) relate the captured meanings of their curriculum's potentially meaningful educational materials to their cognitive structure.” Therefore, this category aimed to highlight students’ readiness to learn meaningfully during the implementation of the proposal.

The data demonstrates that participants were interested and willing to learn meaningfully about the subject matter at various times during the implementation of the proposal.

One of the first evidence of motivation and interest appeared in the students' reports after the clay activity. The image (the excerpts and figures in this article exemplify the entire research data, noting that analyses and conclusions were based on all data) in Figure 5 shows that the activity drew the students' attention to the addressed subjects, arousing their curiosity and desire to learn.

Figure 5

One-minute paper exercise for the clay activity


Eu gostei muito dessa atividade, consegui aprender
mais sobre as frações, dividir o bolo de argila.
o que eu aprendi nela saber escrever as
frações, saber cortar um bolo.

(*"I liked this activity a lot, I learned more about fractions and divided the clay cake. I learned to write fractions and cut a cake."*)

Source: Research data (2023).

Another evidence of readiness to learn meaningfully emerged in the cake preparation activity. After addressing the addition and subtraction of rational numbers, the participants were asked to solve addition and subtraction operations from a list of exercises. According to the researcher's logbook record transcribed below, the students sought the teacher's constant help and were happy to realize their ability to perform such operations.

> After explaining the addition and subtraction of fractions with different denominators, the students started solving some exercises. Many questions arose, but they maintained the willingness to resolve them. The students consistently tried, asked for the teacher's help, commented among themselves, and continued wanting to solve the proposed exercises (Logbook record from August 8, 2023).

It is worth noting that, after studying the fraction division operation, one of the students did not remember exactly what was proposed in the classroom when performing the task at home. The student sought another way to develop the activity. This fact, recorded in the logbook, indicates that the proposal promoted readiness for the meaningful learning of the addressed subjects.

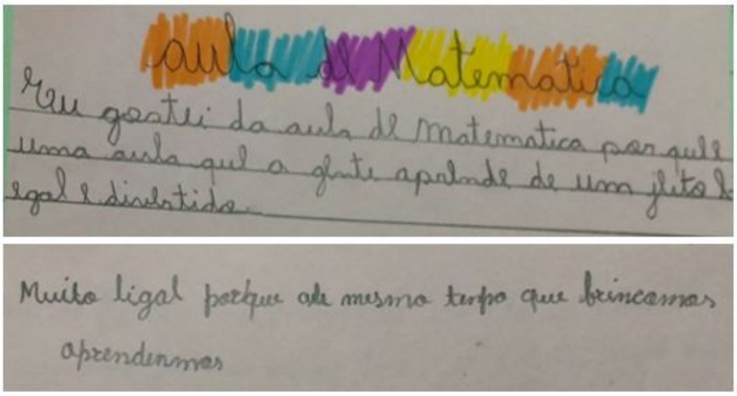
The activities performed with the A4 paper sheet, the triangles, and the Cuisenaire Rods also provided elements that demonstrate students' readiness to learn meaningfully. The researcher's logbook (excerpts transcribed below) reveals that students were motivated, interested, and curious about the activities throughout the development of the proposal.

> [...] While cutting the paper sheet, the students commented on how enjoyable it is to study and, at the same time, cut and play [...] (Logbook record from August 8, 2023).
>
> [...] During the activity with Cuisenaire Rods, a student stated the following: I want to try doing other things, can I, teacher? (Logbook record from August 15, 2023).
>
> [...] The students were questioning today. They consistently wanted to know if they could do the same activity (triangle composition) with other geometric shapes [...] (Logbook record from August 15, 2023).

Finally, the activities related to percentages, especially involving chocolate candy, allowed students to demonstrate their readiness to study the subject. The participants' answers in the one-minute paper exercise (Figure 6) confirm that the group had fun performing the activities.

Figure 6

One-minute paper exercise reports



(*"I liked the Mathematics class because we learn in a cool and fun way." / "It was very nice because we played and learned at the same time."*)

Source: Research data (2023).

That indicates that the participants were interested and motivated to learn meaningfully the topics covered in the activities of the didactic proposal.

SUBSUMING CONCEPTS

This category verified whether the proposal provided situations capable of evidencing the existence of subsuming concepts in the participants' cognitive structure.

Students' answers to the first questions of the questionnaire at the beginning of proposal implementation indicate their understanding that fractions correspond to the parts of a whole. However, these same answers reveal their difficulties in representing these fractions in equal parts. These findings also appear in the excerpt of the teacher's logbook transcribed below:

> When answering the initial questionnaire, students demonstrated that fractions represent parts of a whole, but they do not realize that these parts are the same. That means that, when representing the fraction of two-thirds in drawings, they did not divide the whole into three parts of the same size [...] (Logbook record from August 12, 2023).

The initial responses to the questionnaire also indicate that participants recognize the numerator and denominator of a fraction. Additionally, the logbook record transcribed below evidences the participants' understanding of the correspondence of each of these terms to fractional representations:

> [...] During the meeting, I realized that students know that the denominator of a fraction corresponds to the number of parts of the divided whole and that the numerator corresponds to the number of parts of the considered whole [...] (Logbook record from August 12, 2023).

Regarding the decimal form, the questionnaire data demonstrates that students can readily address situations related to addition and subtraction. However, when operations involve multiplication and division, they present difficulties in calculations. As in the logbook section transcribed below, most participants do not understand that decimal and fractional representations correspond to the same ideas.

> [...] During the initial activities, it was possible to realize that students understand what a fraction is and work well with the addition and subtraction of decimal numbers. However, it was evident that they did not perceive fractions and decimal numbers as the same idea as the whole. In a comment made today, one of the students asked why the teacher said that half was equal to 0.5 [...] (Logbook record from August 22, 2023).

In this sense, as in the logbook excerpt transcribed below, students' speeches during the second stage demonstrate that they neither understand percentage calculations nor identify them as representing rational numbers. The logbook section also reveals that students relate percentages to discount rates offered in purchases and involving interest.

> [...] During the class, the group was asked about the definition of sales, and the answers were "something that pays less," "there is a sale, like pay one and get two." Then, the students were inquired about what usually appears along with the sales information, and the comments included "the percent" and "a symbol." They know what it is but cannot calculate it. For instance, they could not tell the amount paid for a product costing 100 Brazilian Reais after a 10% discount. They also could not identify that a percentage corresponds to a fraction with the denominator 100 [...] (Logbook record from August 22, 2023).

The analyses indicate that most students presented subsuming concepts in their cognitive structure. They understand the different representations of a rational number and that such knowledge might anchor the development of the learning anticipated by the proposal. However, this knowledge was poorly structured and required development to encompass new meanings, becoming more differentiated and unalterable.

Therefore, structuring the activities for the proposal enabled the identification of the subsuming concepts capable of anchoring new knowledge.

ADVANCE ORGANIZERS

Moreira (1999) understands that an advance organizer is an instructional resource presented at a higher level of abstraction, generality, and inclusiveness related to the learning material. In this sense, this category sought to demonstrate whether the activities proposed as advance organizers aided the connection between students' previous and new knowledge.

The proposal organized two activities for this purpose, constituting comparative advance organizers. Loreian, Darroz, and Rosa (2020) report that a comparative advance organizer is used when the new material is relatively familiar to learners, and it may help integrate new knowledge into their cognitive structure. At the same time, it may differ from existing knowledge in this structure, which may be mistaken for the recent concepts, although they are essentially different.

Thus, the first exercise used as an advance organizer was the activity using clay, which sought to relate the knowledge in the cognitive structure of students to the subjects linked to the fractional form of rational numbers.

According to the logbook excerpt transcribed below, the activity provided participants with the opportunity to recognize which fraction represents a part of the whole. The section also shows that participants identified the numerator and denominator of a fraction.

> [...] During the activity, it was possible to notice that, as the students were making the cakes with clay and dividing them into parts, they talked and commented on the meaning of a part. That means they commented that it was a part of six, for example, and that it was the one-sixth fraction. One of the students commented to their classmates: It is a part of three, and number one is the numerator [...] (Logbook record from August 8, 2023).

Also, another logbook record shows that the activity enabled students to compare fractions.

> [...] One student made a rectangle-shaped cake and divided it into four parts, and his groupmate made a similar one and divided it into eight parts. I noticed that they commented to each other that one-fourth is greater than one-eighth and that they needed two-eighths to equal one-fourth (Logbook record from August 15, 2023).

The second advance organizer activity consisted of newspaper reports with economic indicators and inserts of local store sales with situations involving interest. This proposal aimed to connect the subsuming concepts to the idea of percentages. According to the teacher’s logbook excerpts transcribed below, the students demonstrated an understanding that percentages may represent a part of the whole.

> [...] When analyzing the economic indicators in the newspaper reports, the students talked to each other and demonstrated knowing that a high percentage represented a greater amount [...] in a conversation between the students in a group, I realized that they commented that, in the case of a discount, the buyer would not pay the full amount (Logbook record from August 29, 2023).

Another logbook section found that the students did not understand the meaning of percentage and that the activity prompted them to seek answers to the discounts offered in the store inserts.

> When analyzing the offers, students commented that a discount consisted of paying less. However, they could not say the amount of the discount. They were limited to saying that, for example, a 20% discount is higher than a 15% discount. That led to a debate among the students where one tried to explain to the other. As they could not give such explanations, they continuously asked for the teacher's help [...] (Logbook record from August 8, 2023).

The analysis of these data indicates that these activities represented advance organizers, establishing cognitive connections between advance and new knowledge. That provided a favorable environment for constructing meaningful learning and a deeper integration of concepts.

PROGRESSIVE DIFFERENTIATION AND INTEGRATIVE RECONCILIATION

Moreira (2012) reports that two processes must occur for the learner's cognitive structure to learn in organized and hierarchical ways: progressive differentiation and integrative reconciliation. The author states that “when we learn in a meaningful way, we have to progressively differentiate meanings from newly acquired knowledge to perceive differences between them, but integrative reconciliation must also proceed” (Moreira, 2012, p. 7). Establishing these two processes in the learner's cognitive structure requires them to differentiate concepts and their meanings, subsequently integrating this information and excluding conceptual differences.

In this sense, this category aimed to analyze whether participants could progressively differentiate the concepts of fractions, percentages, and decimals and identify potential integrative reconciliation, that is, whether students understood that, although different, the concepts may represent the same amount. The geometric representation was also presented when exploring the involved semiotic representations.

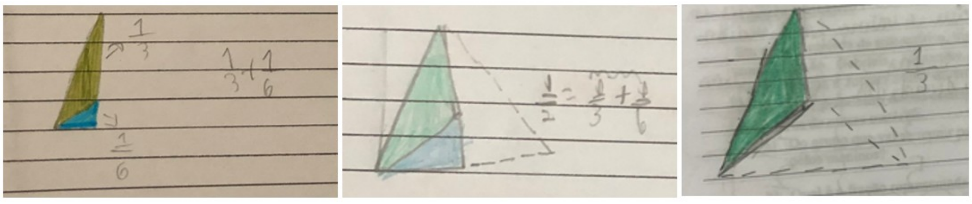
The first indication that the participants could progressively differentiate the addressed subjects appears in the teacher's logbook when reporting the activity on a paper sheet. The excerpt transcribed below demonstrates that the students differentiated and reconciled representations of rational numbers in the form of fractions, decimals, and drawings.

> [...] When they cut the rectangle into two parts, one of them commented that it was a whole sheet divided into two. At the same time, another student said: Each is a half corresponding to half the sheet. A third student showed the drawing of a half in the notebook and explained that all forms represented the same thing [...] (Logbook record from August 15, 2023).

In the triangle composition activity, the drawings made in the students' notebooks (Figure 7) showed elements indicating the development of progressive differentiation and integrative reconciliation of fraction concepts. The participants demonstrated through these data that they understood that smaller triangles correspond to parts of the whole. The drawing data also shows that students understood that the junction of smaller triangles represents the addition of fractions.

Figure 7

Representations of the triangle activities



Source: Authors' collection (2023).

Two other logbook segments transcribed below show that, based on the proposed activities, students recognized the different forms of rational numbers, understood that they correspond to the same part of a whole, and defined the amount of one whole represented as percentages.

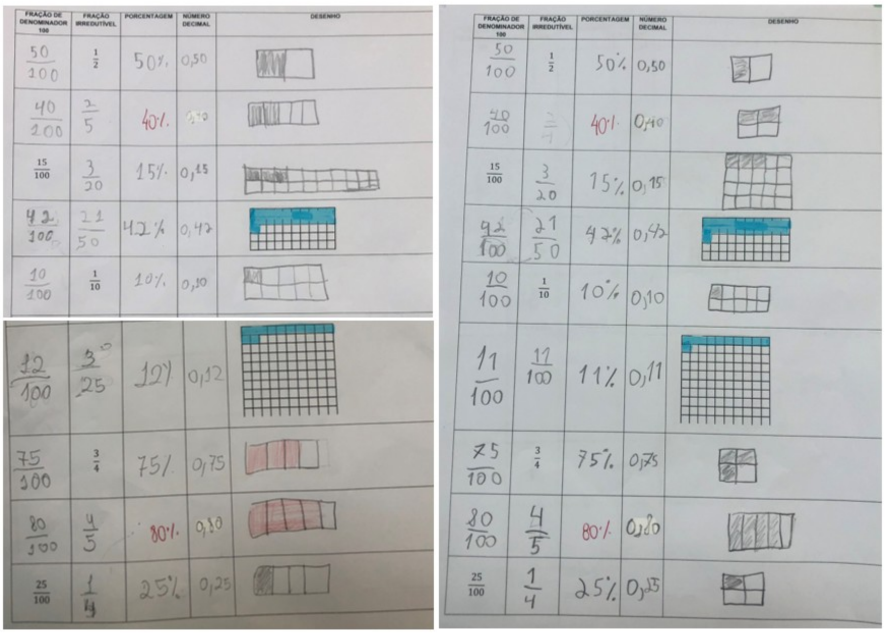
> [...] During the Cuisenaire Rods activity, students assembled geometric shapes and then explained to each other how much an amount represented from the whole. For instance, a group formed a rectangle and used a given number of parts. During the activity, they commented that half of the parts represented half of the whole [...] (Logbook record from August 22, 2023).

> [...] The activity with chocolate candy taught students that the amount of a percentage corresponded to the whole. Throughout the activity, I noticed that the students divided an amount into equal parts and stated that 20%, for example, corresponded to a given value, 25% was the fourth part corresponding to another value, and so on (Logbook record from August 29, 2023).

Finally, the data from the last activity performed in the second stage demonstrate that students were able to write the rational numbers in different representations (Figure 8).

Figure 8

Activity performed at the end of the second stage



Source: Research data (2023).

The analysis conducted in this category indicates that the participants understood each concept separately, progressively differentiating them. Then, from the transition between semiotic representations, they performed integrative reconciliation. These notes corroborate Moreira (2016), highlighting that progressive differentiation and integrative reconciliation are dynamic processes that occur during the acquisition of meaning and help promote meaningful learning.

Thus, the activities developed during the implementation of the didactic proposal were organized hierarchically, progressively strengthening the cognitive structure of students. This arrangement enhanced the subsuming concepts, enabling them to anchor new information.

APPLICATION IN NEW CONTEXTS

Students must understand ideas to construct new concepts and apply knowledge to different contexts from those addressed in the classroom. Moreira (2016, p. 17) affirms that student learning analysis requires

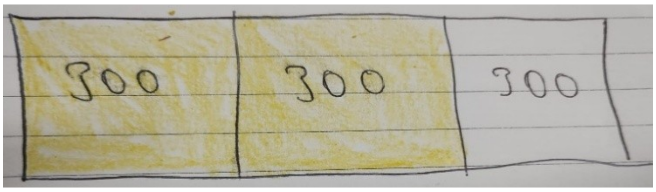
> comprehension tests, which must at least be written differently and presented in another context from the original instructional material. Problem-solving is undoubtedly a valid and practical method to seek meaningful learning evidence. Ausubel reports that it may be the only way to assess whether students had a meaningful understanding of the ideas they verbalized.

Given this perspective, this category aimed to understand whether participants transposed the developed concepts to contexts different from those addressed during the implementation of the proposal. In addition to the researcher's records in the logbook, students' answers to the activities in the third stage of the proposal were analyzed.

The first material of this stage, which seeks evidence of the transposition of the studied concepts into new contexts, was the proposal of simulating purchases from store inserts. This activity challenged the students to simulate the purchase of three products whose sums could not exceed two-thirds of 900 Brazilian Reais. Figure 9 shows that, during the activity, the students used the concept that a fraction corresponds to a part of the whole.

Figure 9

Development of the first activity in the third stage



Source: Research data (2023).

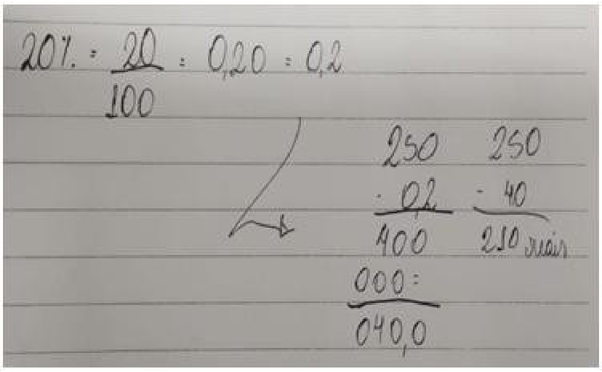
This fact demonstrates students' understanding of the transition of rational numbers between their semiotic representations. The researcher's logbook reports the same evidence in an excerpt stating that "all students immediately drew a rectangle, divided it into three equal parts, and calculated the value corresponding to two-thirds of 900" (Logbook record from August 22, 2023).

These data also demonstrate that students applied the operations of addition, subtraction, multiplication, and division of rational numbers in decimal form. That is evidence that students inserted the subjects in a different situation than that initially studied, understanding and capturing the meanings of these topics.

The students were also challenged to determine the value of a product from another store insert after the offered 20% discount for cash payment. Figure 10 exhibits the exercise solution by one of the groups. The participants understood that a rational number presents several semiotic representations and knew how to solve problems involving percentages.

Figure 10

Development of the first activity in the third stage



Source: Research data (2023).

In this sense, two logbook excerpts (transcribed below) demonstrate that motivating participants to comment on the data in the economic indicator reports prompted them to use explanations different from those addressed during the proposal implementation, including analogies to communicate their understanding.

> [...] One student commented that inflation is high because product prices in the supermarket have risen. When he completed the speech, another student mentioned that inflation is almost at 10% and concluded that product prices will increase by 10 Brazilian Reais per 100 Brazilian Reais. The class listened and agreed [...] (Logbook record from August 29, 2023).

> [...] To explain the savings return index, which they called interest, presented in the newspaper report, one student made the following comment: Saving is like keeping chocolate to eat later, and you will still have more chocolate to eat [...] (Logbook record from August 29, 2023).

The analyzed data allows inferring that, after developing the proposal, the students transposed the concepts into contexts that required transforming the studied subjects. This transposition reveals evidence of meaningful learning by the participants. Moreira (2016, p. 16) states that when individuals perceive regularities in events or objects, they begin to represent them by a given symbol and no longer depend on a concrete reference of the event or object to give meaning to this symbol, promoting conceptual meaningful learning.

FINAL CONSIDERATIONS

Walle (2009) affirms that the prevailing traditional teaching method begins with an explanation from the textbook, followed by exercises indicated to students. Sometimes, these exercises have no meaning and are answered only through mechanical procedures without understanding.

The challenge of students to understand the representations of the set of rational numbers indicates that the means for handling this topic (addressing fractions, followed by decimals and then percentages, each topic with different concepts) does not favor an association among representations. Consequently, this strategy isolates the representations, often preventing students from making connections between them.

Given this scenario, a didactic proposal was developed to assist students in understanding the elements of this numerical set based on the assumptions of the Theory of Semiotic Representation Registers and the Meaningful Learning Theory.

During the implementation of the proposal, the participants understood that a value may be expressed in different ways and still represent the same thing based on semiotic representations. Moreover, the planning and execution of tasks and practical explorations of this pedagogical resource helped visualize mathematical concepts, highlighting the nuances of each representation: fractions, decimals, and percentages. This approach allowed assimilating the particularities of each representation, including the geometric representation, revealing and evidencing the relationships between them.

The data also indicated the effective involvement of participants in most meetings of the didactic sequence, promoting enthusiasm in the execution of the proposed activities. This engagement facilitated the understanding of the addressed topics and promoted questioning and dialogue. Besides the activities, the developed dynamics supported the readiness of participants, highlighting their expressed perceptions and turning the classroom into an ideal environment for learning and socialization.

Regarding implementation, the dynamics and activities of the didactic proposal provided students with sensory and visual experiences, favoring learning and understanding of the topic and helping to create a playful and cooperative environment with active participants.

The analysis of the data from the categories listed in the study showed that most participants had information in their cognitive structure capable of anchoring new information and concepts, that advance organizers were essential to establish the link between subsuming and recent concepts, that progressive differentiation and integrative reconciliation were crucial for understanding the representation and particularities of each definition, and that participants successfully applied the constructed knowledge to different contexts from those discussed in the classroom.

The research concluded that a didactic proposal supported by semiotic representation characterizes potentially meaningful material capable of aiding the development of meaningful learning of rational numbers in the sixth year of primary school. That means that a didactic proposal organized hierarchically, seeking understanding and transition among representations and following MLT assumptions, becomes valuable educational material for teachers and student learning progress.

REFERENCES

Ausubel, D. P. (1973). *Algunos aspectos psicológicos de la estructura del conocimiento*. Buenos Aires: El Ateneo.

Ausubel, D. P., Novak, J. D. & Hanesian, H. (1980). *Psicologia educativa*. 2. ed. Rio de Janeiro: Editora Interamericana.

Creswel, J. W. (2007). *Projeto de pesquisa: método qualitativo, quantitativo e misto*. 2. ed. Porto Alegre: Artmed.

Darroz, L.M., Rosa, C. T. W. da. & Kuiava, H. L. (2023). Instrumentos utilizados para a identificação de indícios de aprendizagem significativa no ensino de física: uma revisão de literatura. A*ctio: Docência em Ciências*, 8(2) p. 1-20. https://periodicos.utfpr.edu.br/actio/article/view/15709/9802

Duval, R. (2012). Registros de representação semiótica e funcionamento cognitivo do pensamento. *Revemat*, 7(2), 266-297. https://periodicos.ufsc.br/index.php/revemat/article/view/1981-1322.2012v7n2p266

Duval, R. (2013). Registros de representações semióticas e funcionamento cognitivo da compreensão em Matemática. In S. D. A. Machado. *Aprendizagem em Matemática: registros de representação semiótica* (pp. 17-32). Campinas, SP: Papirus.

Gil, A. C. (2017). Como elaborar projetos de pesquisa. 6. ed. São Paulo: Atlas.

Junqueira, J. R. (2021). *Aplicação de metodologias ativas no ensino e aprendizagem de números racionais com questões da OBMEP*. (Dissertação de Mestrado em Ensino de Ciências Exatas), Universidade Federal de São Carlos, São Carlos. https://repositorio.ufscar.br/handle/ufscar/15209

Kuhn, M. C. & Bayer, A. (2023). A presença das práticas socioculturais na Matemática abordada nas escolas paroquiais luteranas do Rio Grande do Sul. *Revista Matemática, Ensino e Cultura*, Belém, 18(45). e2023004. https://doi.org/10.37084/REMATEC.1980-3141.2023.n45.pe2023004.id543

Linhares, I. R. (2016). *História na educação matemática: uma proposta para o ensino de medidas no Ensino Fundamental*. (Dissertação de Mestrado Profissional Educação e Docência), Universidade Federal de Minas Gerais, Belo Horizonte. https://repositorio.ufmg.br/bitstream/1843/BUBD-AWFH64/1/disserta__o_promestre_ilma.pdf

Loreian, I., Darroz, L. M. & Rosa, C. T. W. da. (2020). Organizadores prévios no processo de ensino de Física: o que dizem os periódicos da área. *Amazônia: Revista de Educação em Ciências e Matemática*, 16(37), 210-223. https://periodicos.ufpa.br/index.php/revistaamazonia/article/view/7690

Minayo, M. C. S. (2004). *O desafio do conhecimento*. São Paulo: Hucitec.

Moreira, M. A. (2012). *O que é afinal aprendizagem significativa?* Porto Alegre: Instituto de Física da UFRGS. http://www.if.ufrgs.br/~moreira/

Moreira, M. A. (1999). *Teorias de aprendizagem*. São Paulo: EPU.

Moreira, M. A. (2010). *Mapas conceituais e aprendizagem significativa*. São Paulo: Centauro Editora.

Moreira, M. A. (2016). *A teoria da aprendizagem significativa*. 2. ed. Porto Alegre, Artmed.

Moreira, M. A. & Masini, E. A. F. S. (2006). *Aprendizagem significativa: a teoria de David Ausubel*. 2. ed. São Paulo: Centauro

Walle, J. A. V. (2009). *Matemática no ensino fundamental: formação de professores e aplicação em sala de aula*. Porto Alegre: Artmed.

Zabalza, M. (2004). *Diários de aula: um instrumento de pesquisa e desenvolvimento profissional*. Porto Alegre: Artmed.

Recebido: 25 fev. 2025
Aprovado: 30 maio 2025
DOI: https://doi.org/10.3895/actio.v10n2.19983

Como citar:
Gheno, F.; Darroz, L. M.; & Rosa, C. T. W. da. (2025). Uma proposta didática para o ensino dos números racionais utilizando representação semiótica. ***ACTIO***, 10(2), 1-23. https://doi.org/10.3895/actio.v10n2.19983

Correspondência:
Fernanda Gheno
Rua Emílio Spadotto, 35, apto 302, Vila Zucchetti, Nova Araçá/RS.

Received: Fev. 25, 2025
Approved: May 30, 2025
DOI: https://doi.org/10.3895/actio.v10n2.19983

How to cite:
Gheno, F.; Darroz, L. M.; & Rosa, C. T. W. da (2025). A didactic proposal for teaching rational numbers using semiotic representation. ***ACTIO***, 10(2), 1-23. https://doi.org/10.3895/actio.v10n2.19983

Address:
Fernanda Gheno
Rua Emílio Spadotto, 35, apto 302, Vila Zucchetti, Nova Araçá/RS.