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Educational activities in mathematical modeling promoting experiences in basic education: practices and reflections

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ABSTRACT

This paper aimed to investigate the reflections prompted by a teacher training course in Mathematical Modeling. It covers the development of activities by the course participants and the outcomes of applying course content in Basic Education. The data, which guided a qualitative analysis, were collected from written records and audio recordings of the planning sessions and discussions regarding the activities proposed during the course. The reflections are presented through the details of three mathematical modeling activities, bringing to the debate the experiences of the students during the forty hours of the course, in the planning and implementation of mathematical modeling activities in the classroom. Based on the reflections, the results point to reflections that focus on the unpredictability of the mathematical contents that emerge in mathematical modeling activities, obstacles in the school organization, concerns about the limits in the development of activities due to insufficient knowledge about the emerging mathematical contents, their investigative nature and their duration in the classroom, lack of familiarity of teachers with mathematical modeling activities, as well as with mathematical contents.

KEYWORDS: Mathematics Education; Mathematical Modeling; Formative Reflections; Activity Planning.

Atividades formativas em modelagem matemática desencadeadoras de vivências na educação básica: práticas e reflexões

RESUMO

Este artigo teve como objetivo investigar as reflexões proporcionadas pelo curso de formação de professores em Modelagens Matemáticas. O artigo contempla o desenvolvimento de atividades pelos cursistas e o resultado da aplicação do conteúdo do curso na Educação Básica. Os dados, que balizaram uma análise qualitativa, foram coletados a partir de registros escritos e áudios dos planejamentos e discussões acerca das atividades propostas durante o curso. As reflexões são apresentadas por meio do detalhamento de três atividades de modelagem matemática, trazendo para o debate experiências dos cursistas durante as quarenta horas de curso, no planejamento e realização das atividades de modelagem matemática em sala de aula. A partir das reflexões, os resultados apontam para reflexões que incidem sobre a imprevisibilidade dos conteúdos matemáticos que emergem nessas atividades, obstáculos da organização escolar, preocupações com os limites no desenvolvimento das atividades devido à insuficiência de conhecimentos sobre os conteúdos matemáticos emergentes, à sua natureza investigativa e a duração delas em sala de aula, falta de familiaridade dos professores com atividades dessa natureza, bem como com os conteúdos matemáticos.

PALAVRAS-CHAVE: Educação Matemática; Modelagem Matemática; Reflexões formativas; Planejamento de atividades.

INTRODUCTION

Official documents emphasize the importance of connecting mathematics education to real and contextualized situations. According to the Curriculum Guidelines for Basic Education in Paraná (Paraná, 2008, p. 20), school education should contribute to students' formation by "aiming at transforming the social, economic, and political reality of their time." We see mathematical modeling as a way to foster this connection through "students' engagement with real-world social and cultural problems, which in turn contributes to their critical development" (Paraná, 2008, p. 65).

> Mathematical Modeling, as a research field within Mathematics Education, is gaining prominence, with a growing number of publications in academic journals and increasing participation of professionals interested in mathematics teaching and learning at specialized events. This rising interest can partly be attributed to the expansion of graduate programs nationally, including professional master's programs that emphasize applied research in school education (Vertuan; Silva & Borssoi, 2017, p. 2).

Research highlights the importance of incorporating mathematical modeling activities in teaching and learning, especially regarding teacher training and the implementation of these activities in the classroom (Bellei & Klüber, 2018; Nunes, Nascimento & Sousa, 2020; Barbosa, 2001; Mutti & Klüber, 2018; among others). However, evidence suggests that few teachers have encountered mathematical modeling during their initial training (Tambarussi & Klüber, 2014; Malheiros, Forner & Souza, 2020; among others), with limited awareness of it as an approach to teaching and learning mathematics (Malheiros, 2016).

Over two decades ago, Biembengut (1999) pointed out that the lack of exposure to methodological approaches in teacher training might lead educators to teach mathematics primarily through rote content, exercises, and techniques that lack meaning for students. This underscores the need to rethink teaching practices and provide teacher training that offers insights into mathematical modeling in Mathematics Education, encouraging educators to develop a critical and investigative approach with their students.

One of the ways to rethink teaching practices and enhance the teaching and learning processes in mathematics through modeling lies in teacher training. Martens and Klüber (2023), in their investigation into the practice of Mathematical Modeling in classrooms by teachers attending modeling courses, highlighted the value of these training opportunities but also noted their limited time due to ongoing research requirements. Lozorio and Carvalho (2024) examined the results of implementing mathematical modeling in a teacher training course centered on methodological innovation, while Pereira (2023) emphasized the importance of modeling as an active methodology in teacher education programs.

This context highlights the importance of teacher training courses in mathematical modeling that offer hands-on practice and provide continuity beyond the formal training period, extending its impact to future research and teaching contexts.

In this scenario, this article aims to investigate reflections generated from the development of modeling activities within a teacher training course on mathematical modeling. This initiative is contemporary in that the training program supported not only the development of modeling activities by the participants but also their planning and implementation in regular classrooms.

The text is organized as follows: theoretical foundations on mathematical modeling in Mathematics Education; aspects of training in mathematical modeling; the modeling activities developed and planned during the course; reflections on the practices among the participants; and discussion of results and final considerations.

MATHEMATICAL MODELING IN MATHEMATICS EDUCATION

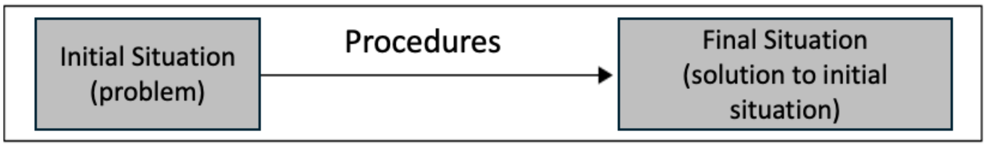
Mathematical modeling in Mathematics Education is understood, as described by Bassanezi (2002), as the investigation of non-mathematical situations through mathematics. Different researchers present a range of interpretations. Some emphasize the construction of mathematical models, while others focus on the teaching and learning processes. Tambarussi and Klüber (2014) suggest that the establishment of this research field brings about a diversity of understandings, making it difficult to standardize theories and epistemological and ontological conceptions of modeling.

Our perspective aligns with the conception of mathematical modeling in Mathematics Education presented by Almeida, Silva and Vertuan (2012), who see it as a pedagogical alternative that allows a problem-based approach to a situation that is not inherently mathematical through the use of mathematics. This conception is significant because it supports the formation provided, based on a strand of modeling aligned with these authors' view.

According to these authors, mathematical modeling aims to investigate real-world problem situations and is generally considered an investigative activity. They also state that a modeling activity moves from an initial situation (problematic) to a desired final situation (representing a solution to the initial problem) (Figure 1), through a series of procedures connecting the initial and final situations.

Figure 1

Initial and Final Situations in Mathematical Modeling

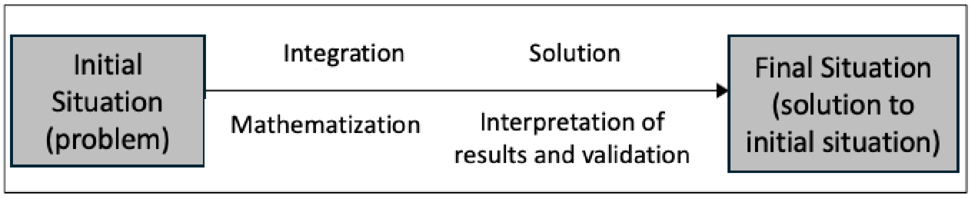


Source: Adapted from Almeida, Silva and Vertuan (2012).

The initial situation originates from a real-world problem, and the desired final situation is associated with the solution to that problem. A mathematical model is used in developing the activity to describe or explain the situation studied using mathematical language or structure. In the application of mathematical models, various representations may be used depending on the options available for solving the problem. This type of mathematical activity involves a series of phases defined by Almeida, Silva and Vertuan (2012) as: interaction, mathematization, resolution, and finally, interpretation of results and validation (Figure 2).

Figure 2

Phases of a Mathematical Modeling Activity



Source: Adapted from Almeida, Silva e Vertuan (2012).

Interaction involves engaging with information about the problem situation, collecting data, formulating the problem, and defining strategies for its solution. Mathematization includes transitioning from natural language to mathematical language, formulating hypotheses, selecting variables, and making simplifications. The resolution phase involves creating a mathematical model to represent the initial situation. Finally, the interpretation of results and validation of the findings provide an answer to the problem. However, although these phases are described in this order, they do not necessarily occur linearly during the activity's development.

Such activities, however, are not common in classroom practice. To provide teachers in-service with theoretical and practical access to mathematical modeling, continuing education courses are seen as an option to address gaps present in initial training and highlight the importance of implementing mathematical modeling, particularly in Basic Education:

> [...] The interpretations and discussions conducted throughout the investigation reveal an urgent need for the Modeling community to pursue studies focused on teacher training, particularly in the area of continuing education. Such studies should move beyond exploratory research, offering in-depth reflections and tangible contributions to the professional development of teachers in Mathematical Modeling (Tambarussi & Klüber, 2014, p. 17).

According to Tambarussi and Klüber (2014), these studies would aim to "[…] broadly address or remedy a gap in the training of Basic Education teachers" (Tambarussi & Klüber, 2014, p. 19). In this context, we discuss some elements from previously published works that indicate a possible direction for training in mathematical modeling.

TEACHER TRAINING IN MATHEMATICAL MODELING

The publications we bring to the debate in this article address initial and continuing teacher training and are associated with the dissemination of texts on mathematical modeling (Luna & Barbosa, 2016, Silva & Oliveira, 2014) regarding the evaluative process in modeling activities (Oliveira & Kato, 2017). Other studies explore student motivation, teacher actions in such activities, and the use of digital technologies in teaching (Rosa & Kato, 2014; Souza et al., 2013; Domingues & Borba, 2017; Silva & Oliveira, 2014; Vertuan, Silva & Borssoi, 2017; Burak & Martins, 2015), as well as teachers' difficulties and insecurities regarding the use of mathematical modeling in the classroom (Malheiros, 2016; Ceolim & Caldeira, 2015; Pereira, Schlünzen Júnior & Palharini, 2015; Rosa, Zampieri & Malheiros, 2015).

Particularly concerning teacher training in mathematical modeling, recent studies indicate: influences of the school community on the development of modeling activities (Bellei & Klüber, 2018); the knowledge needed by teachers to use it in the classroom (Sousa & Almeida, 2021; Omodei & Almeida, 2022); its use as an active methodology in training courses (Pereira, 2023); the need for training in mathematical modeling for early childhood education teachers (Belo & Zimer, 2023); and the intertwining of initial teacher training and the mentors who receive interns (Oliveira & Kato, 2023).

Luna and Barbosa (2016) identify three domains of emerging texts in teacher training in mathematical modeling: the teacher's own experience with mathematical modeling, the experience with other teachers' modeling, and the experience with modeling in the school context of ongoing teacher training. Silva and Oliveira (2014) argue that certain factors shape how teachers transform the text while developing mathematical modeling activities to meet pedagogical objectives. Generally, the reason for text transformation by the teacher is tied to the teacher's commitment to the pre-planned scheme.

Discussing the results of a continuing education course for teachers, Rehfeldt, Quartieri and Giongo (2017) point to the motivation in using mathematical modeling activities, emphasizing the development of creativity and critical thinking.

The teacher's role was investigated by Rosa and Kato (2014), who see the teacher as the creator of structures, dynamics, and stimuli that can foster learning, considering the development of competencies, abilities, and attitudes. These authors also examine the motivational factor, indicating the importance of working with themes of interest to students to engage them in acquiring knowledge related to the contents used in the activity, valuing diverse opinions and prior knowledge.

The choice of topic in modeling activities also influences students' performance, as highlighted by Silva and Oliveira (2014). They analyzed the decisions, reasons, interests, and rules involved in choosing a topic for modeling activities when the choice is made by the teacher, based on data from a Continuing Education Course for Basic Education teachers working in Elementary and Youth and Adult Education. In this context, it is essential for students to access knowledge related to a given reality, whether it be global, state, regional, or local.

The importance of continuing education courses is also reported by course participants (Goulart, Neumann & Quartieri, 2016) and in teachers' mediation using digital technologies (Pereira, Schlünzen Júnior & Palharini, 2015). Furthermore, the use of mathematical modeling in initial teacher training is investigated from different perspectives. For example, Domingues and Borba (2017) researched the use of videos in mathematical modeling work within an initial training course for Biological Sciences.

Among the arguments researchers use to analyze training in mathematical modeling are teachers' difficulties and insecurities about using it in the classroom, insufficient training, challenges in engaging students, and the need to overcome mathematics teaching that lacks connections within itself and with other subjects (Malheiros, 2016; Ceolim & Caldeira, 2015; Rosa, Zampieri & Malheiros, 2015).

Among these arguments, Malheiros, Forner, and Souza (2020, p. 5) suggest that teacher training in mathematical modeling should influence classroom practices, permeating the school environment.

For Ceolim and Caldeira (2015), three categories are mobilized in the investigation of a training course with in-service teachers and graduates of Mathematics Education programs: insufficient training in mathematical modeling as well as the content to be taught, difficulties in applying mathematical modeling due to the conservative posture of the school system, and difficulties in engaging students in an environment with modeling activities

Regarding insufficient training in mathematical modeling, Malheiros (2016) also raises this argument when reporting that, in proposing modeling activities during supervised internships, sometimes supervising teachers argue against the activities due to time constraints, the use of government-provided teaching materials, and a lack of knowledge about mathematical modeling.

In the experience of Rosa, Zampieri and Malheiros (2015) in a continuing education course, insecurities relate to the school schedule, the traditional stance of the school system, and expectations for working with modeling activities in the classroom after completing the courses. However, there are challenges in implementing these activities following the course.

According to Bellei and Klüber (2018, p. 330), “Mathematical Modeling lacks research that not only develops activities to be carried out but also examines its use in the classroom and how it affects the school as a whole.”

Lozorio and Carvalho (2024, p. 1596) highlight the importance of practicing mathematical modeling. For the authors, “When we do not know the object, it is exotic, distant from reality. As we approach it, it becomes familiar.” The search for this familiarity can be fostered through training moments, such as those presented in this article.

In this context, we systematized the findings and structured a mathematical modeling training course offered specifically to Basic Education teachers, aiming to explore theoretical issues, provide dynamic modeling activities, and present opportunities for teachers to work with mathematical modeling in their classrooms. Next, we present the methodological aspects that guided the research, the material analysis, and the results.

METHODOLOGICAL ASPECTS

This research is qualitative, aiming to investigate reflections fostered during the development of modeling activities as part of a training course on mathematical modeling. According to Godoy (1995, p. 62), “qualitative research has the natural environment as the direct source of data and the researcher as the fundamental instrument.” To guide our discussions, data were collected during a training course on mathematical modeling conducted at a public university in Northern Paraná, with the participation of seven basic education teachers, here referred to as “course attendees.” Data collection instruments included audio recordings, field journal notes, attendees' activity records, and responses to pre-structured questionnaires, coded as shown in Table 1.

Table 1

Coding of Attendees and Data Collection Instruments

Descriptions	Indicators
Activity	A1, A2, A3, A4
Course Attendee	C1, C2, C3, C4, C5, C6 e C7
Group of attendees	G1, G2 e G3
Questionaries	Q1, Q2, Q3 e Q4
Dialogues	D1, D2 e D3

Source: the authors (2024).

Despite having seventeen registered participants, only seven attended all sessions, even though the course was free and advertised to regional education centers. This aligns with the literature indicating the low enrollment rate of basic education teachers (Tambarussi & Klüber, 2014).

To contextualize the reflections of the study participants in the training course on mathematical modeling, certain characteristics were identified, such as the type of continuing education, teaching experience, and education level. All attendees had prior knowledge of modeling activities, gained through initial or continued training. Of the participants, only attendee C1 was still in initial training in mathematics; attendees C2, C3, C4, C5, and C6 had been teaching for less than five years, while attendee C7 had more than ten years of experience. Furthermore, attendees C2 and C7 taught in the early years of elementary school, while the others taught in the later years and high school.

In developing course activities, especially those involving the attendees' modeling practice, we assumed that a shift in attitude towards mathematical activity is necessary, as investigative activities are challenging and uncommon. Thus, we based our approach on Almeida and Dias (2004) and Almeida, Silva, and Vertuan (2012), who suggest that an invitation be extended for developing modeling activities, which solidifies through participants' experiences. We followed these authors' guidance, gradually introducing modeling activities in teacher training courses, with the premise that learning mathematical modeling involves learning "about" modeling, learning "through" modeling activities, and "teaching using" modeling.

To achieve this, literature on mathematical modeling was presented through discursive practices aimed at collaborative reflection. Modeling activities were developed in three familiarization phases: initially, the researcher-teacher proposed a topic for the activities, providing necessary data for resolution; in the second phase, the researcher-teacher suggested a problem or topic, and attendees, in groups, formulated a problem and developed the stages of the modeling activity; finally, attendees led an activity, from choosing a topic to analyzing the solution. This third-phase activity was shared with the class, discussed, and planned for application in the attendees' classrooms, thereby initiating a reflective process.

The reflective process initiated by the planned activities was underpinned by connections with literature on teacher training and mathematical modeling, highlighting its challenges and potential through attendees' classroom practices. At this stage, emphasis is placed on the implications of what Bellei and Klüber (2018) and Lozorio and Carvalho (2024) emphasized about the importance of mathematical modeling practice extending beyond research and reaching the school environment.

In the following section, data analysis is presented, including the modeling activities developed and planned by the attendees, elements of attendee familiarization, and analysis of reflections that emerged from the course based on teaching practice in mathematical modeling. This includes dialogues from the attendees to provide a clearer understanding of the study subject.

The analyses were based on data segmentation and the emergence of categories, relating these categories to theoretical references through Discursive Textual Analysis. Data were drawn from the attendees' records during the second familiarization phase, in which all participated, and two activities proposed by groups for the third familiarization phase, implemented by the attendees in their basic education classrooms.

DEVELOPMENT OF MATHEMATICAL MODELING ACTIVITIES

In this text, we present three activities developed during the course as examples to guide our analysis. One activity from the second familiarization stage, titled "Oil Tank Task," and two activities from the third familiarization stage, titled "Emotions" and "Dengue Epidemic in Londrina," which were conducted independently by the participants and implemented in the classes they were teaching.

EXEMPLARY GUIDED PRACTICES: CONSTITUTING FAMILIARIZATION WITH MATHEMATICAL MODELING

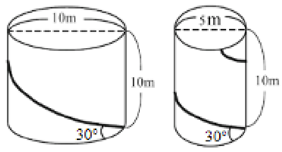
The problem situation "oil tank task" was developed by two groups: G1 (C6, C1) and G2 (C2, C3, C5). Proposed by the researcher-teacher, the topic was suggested for study along with the following materials: string, scissors, glue, adhesive tape, a toilet paper tube, and a protractor (Figure 3).

Figure 3

Problem Situation "Oil Tank Task"

Oil Tank Task

There are several types of oil tanks. They have the same heights, but different diameters. Are the lengths of the spiral ladders in these oil tanks the same or not? The angle of the ladder relative to the ground is 30º.



Toilet Paper Tube Task

Since it is impossible to "unfold" the actual ladder of the oil tank, as it is in a spiral shape, we can use a toilet paper tube as a similar shape to that of an oil tank, because the toilet paper tube can be opened along its seam to show its 2D shape. In other words, consider the shape of an opened toilet paper tube.

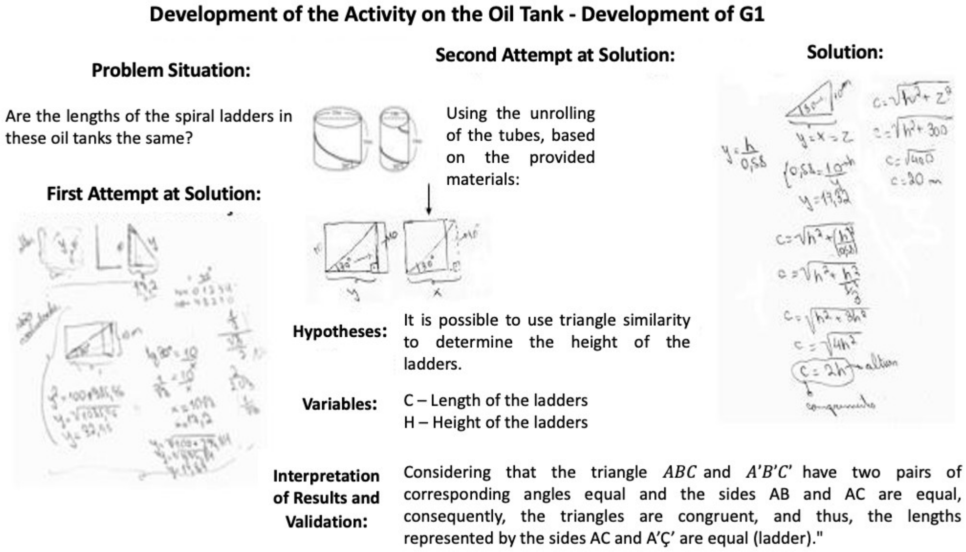
Source: Adapted from Kawakami, Saeki e Matsuzaki (2013).

Both groups began their investigation by drawing the unfolding of the oil tank on the sheet provided for notes. The participants from group G1 used the mathematical concepts of the Pythagorean Theorem and trigonometric relationships in right triangles; however, they were unable to solve the given problem situation. In a second attempt to find a solution, the members of the group unfolded toilet paper tubes in order to investigate the unfolding of oil tanks on a smaller scale. Using mathematical procedures related to the similarity of triangles, they were able to find a solution to the problem presented (Figure 3).

Figures 4 and 5 showcase the problem situation, the solution, the hypotheses raised by the participants, the variables, the interpretation, and the validation carried out by both groups. The authors structured the cycle to enhance the presentation and extracted the data from the information collected by the participants during the development of the activities they carried out.

Figure 4

Records of the Oil Tank Activity by the G1 Attendees



Source: G1 written records of the oil tank activity (2024).

In Figure 4, it is possible to discern the back-and-forth movement in the modeling activity, highlighting aspects of the dynamic nature of mathematical modeling, as previously emphasized in the literature by Almeida, Silva, & Vertuan (2012). According to the solution obtained by the participants of Group G1, we can infer that if the ladder angle and the height of both oil tanks are the same, the ladders will have the same length; therefore, these are the two variables that influence the railing length, not the oil tank diameter. These findings were considered as hypotheses by the G2 attendees (Figure 5).

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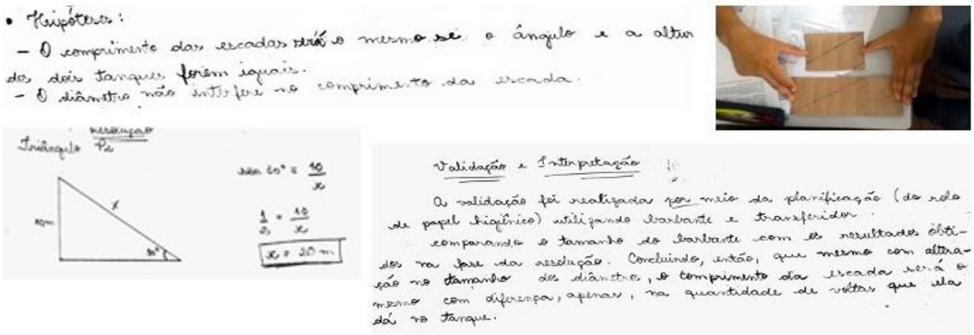
Figure 5

Records of the Oil Tank Activity by the G2 Attendees

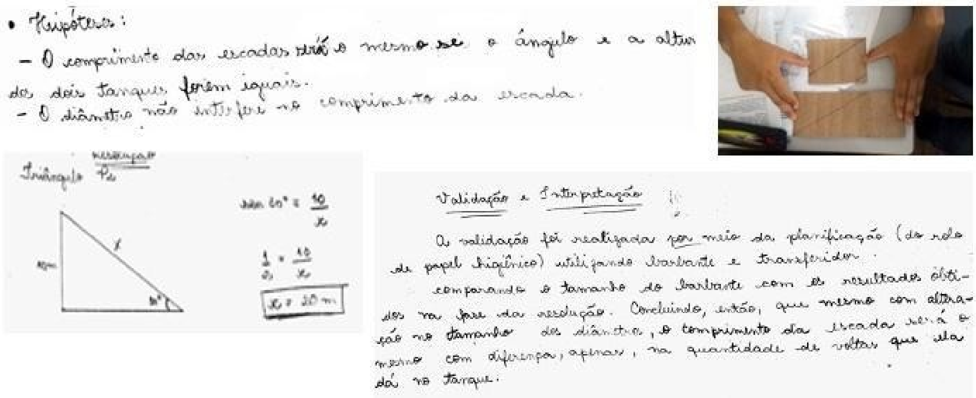
Development of the Activity on the Oil Tank - Development of G2



Hypotheses:
- The length of the ladders will be the same if the angle and the height of the two tanks are the same.
- The diameter does not interfere with the length of the ladder.

Validation and Interpretation: The validation was carried out through the unrolling (of the toilet paper roll) using string and a protractor, comparing the length of the string with the results obtained during the resolution phase. It was concluded that even with changes in the diameter sizes, the length of the ladder will remain the same, with only the number of turns it makes around the tank differing.

Desenvolvimento da Atividade sobre o Tanque de Óleo - Desenvolvimento de G2



Source: G2 written records (2024).

For the investigation, G2 attendees began by unfolding the toilet paper tube with two different diameters, as shown in Figure 5. The markings made with pen ink represent the oil tank railing, and after the unfolding process, G2 attendees moved on to the mathematization phase. In this process, the diameters of the tanks were selected as variables, as the text proposed two different measurements for the diameter of both the oil tank and the toilet paper tube. Through trigonometric relations in the right triangle, the problem was solved, and, during the data interpretation and validation phase, the participants observed that the length of the string used in the unfolded toilet paper tube remained constant, regardless of the tube's height. This reasoning was then applied to determine the length, using the same logic, in the case of the oil tanks.

Both groups realized that the railing length is the same and does not depend on the oil tank diameter. This solution was systematized by the participant teacher at the end of the sharing session, taking into account the hypotheses formulated by both groups. During the development, the attendees used a toilet paper tube to obtain a graphical representation of the problem situation, which focused their attention on a mathematical model associated with the geometric representation. In this context, the mathematical model interpretation gained prominence in the discussions, with the participant teacher taking a mediator role and revisiting the literature on mathematical models and their different forms of representation, emphasizing that we can define it as "[...] a simplified representation of reality from the perspective of those investigating it. Its formulation is not an end in itself, but aims to foster the solution of a problem" (Almeida, Silva & Vertuan, 2012, p. 13).

The discussion during the communication of the activity carried out by the participants also focused on integrating content from the Basic Education curriculum, such as the Pythagorean Theorem, trigonometric relations in right triangles, and triangle similarity. It also addressed the use of manipulable resources to investigate real-life situations, providing opportunities for the students to engage with aspects of the investigated situation through simplifications. For example, the toilet paper tube, which resembles the oil tank shape, allows for a more detailed investigation in the classroom. In the third moment of familiarization, two modeling activities, developed during the course, were implemented in Basic Education classrooms by the attendees.

EXEMPLARY GUIDED PRACTICES: THE DEVELOPMENT IN BASIC EDUCATION

Groups G1 (C1, C4, C6 and C7) and G2 (C2, C3 and C5) were responsible for developing activities during the course, over a 5-hour class period; later, in Basic Education, the participants took on their roles as teachers, with 2 class hours allocated for the implementation of these activities.

The groups developed their activities around two specific themes. G1 focused on "Emotions", based on a subject that one of the attendees had already started discussing on a conversation circle of a second-grade class in the early years of Basic Education (later mentioned as early elementary); while G2 focused on "Dengue Epidemic" due to the contemporary issue being addressed on a seventh year of elementary school by one of the attendees of the group.

In early elementary, G1 gathered data from the students during the interaction phase, while in elementary school, the attendee teacher delivered the data related to Dengue cases. This marked a return to the interaction phase, where quantitative and qualitative data collection takes place, along with a problem formulation (Almeida, Silva & Vertuan, 2012).

Figure 6 presents information on the activity planned and developed by G1 attendees for early elementary students. During the interaction and data collection phase, the attendee teacher in charge of the class assisted the students in writing down the actions that made them happy. Eight actions emerged from this data collection: studying, playing tag, painting, hugging, resolving math exercises, P.E. classes, hanging out with friends, and having friends.

Figure 6

Problem Situation "Emotions"

According to our discussion circle, the actions that make us happy are: studying, playing tag, painting, hugging, resolving math exercises, P.E. classes, hanging out with friends, and having friends. In this way, we can organize our friends' responses in a table:

What makes me happy:	Amount of friends:
Studying;	
Playing tag;	
Painting;	
Hugging;	
Resolving math activities;	
P.E. classes;	
Hanging out with friends;	
Having friends.	

According to the table, how can we represent our friends' answers?

Source: G1 activities (2024).

Note that the main challenge lay in systematically associating the quantity (the number of students corresponding to each response) with the emotions that surfaced from the data collected during the discussion. In other words, Group G1 aimed for the primary school students to employ pictorial representations organized into sets, following the completion of a table that recorded both the actions and the corresponding student count. Thus, the group's approach focused on graphical representations, with the objective to "relate quantities; organize information using drawings or attempts to create a graph, with support from the Montessori Golden Beads material" (Group G1Q3 Records).

As an outcome, the attendees emphasized the students' creativity in their diverse approaches to representing data, using both graphical and representation of mathematical sets. They also highlighted the importance of group interaction, noting how collaborative work was essential to carrying out the activity effectively.

G2 engaged students with the topic by presenting texts, news reports related to the topic and information about the Aedes aegypti mosquito, the vector for the dengue virus (Figure 7). For classroom implementation, they planned an investigation aimed at estimating potential infection cases for the following month.

Figure 7

Problem Situation "Dengue"

Dengue Epidemic in Londrina

Dengue is a disease transmitted through the bite of the Aedes Aegypti mosquito, which is infected with the dengue virus. The Aedes Aegypti is considered an urban mosquito due to its proximity to humans. Around 80% of mosquito breeding sites are found inside homes, especially in backyards. Therefore, it is crucial for the population to remain vigilant regarding any place or object that can accumulate stagnant water, as it can become a breeding ground for mosquito larvae. The Aedes Aegypti mosquito measures less than 1 cm, is black, and has white stripes and spots on its body and legs. It can bite throughout the day but prefers morning hours, as it avoids strong sunlight (AGUIAR, 2013).

According to the Health Department of the state of Paraná, in the municipality of Londrina, during the months of September 2018 to May 2019, the incidence of the disease increased significantly. In September, three cases were diagnosed, and in the following months, the number of cases rose to 8, 13, 18, 37, 150, 701, 1151, and finally, in May, 1378 cases of dengue were reported.

	September	October	November	December	January	February	March	April	May
Cases	3	8	13	18	37	150	701	1151	1378

NOTE: Data collected in: <http://www.dengue.pr.gov.br>.

What is the estimated number of dengue cases for the month of June 2019 in the city of Londrina?

Source: G2 activity records (2024).

Since there is a significant number of confirmed cases between January and May, G2 selected data from 2018 and proposed a possible solution by fitting the data to a quadratic function, resulting in the mathematical model:

$f(x) = -81x2 + 818.4x - 593.5$

Where $f(x)$ represents the number of confirmed cases and x indicates the reference month. Using this model, G2 projected approximately 1,374 confirmed cases for June of that year. The participants discussed with their students the fitting process using a quadratic function and examined the trend of decreasing cases. Additionally, they explored other possible models to predict case numbers, such as an exponential function.

In examining the detailed communication and reflections of the participants, we focus on the contributions of this research to their roles as modelers and educators employing mathematical modeling. In their communication, G2 participants highlighted that their teaching practice using mathematical modeling was implemented in elementary school by one of their members, engaging 22 students divided into groups. The group noted their classroom objective as: "to raise student awareness of important environmental and social issues, such as the dengue epidemic that has affected the Londrina-PR region, while integrating mathematical approaches to estimate a possible increase in dengue cases" (Group G2 records via dialogue D4).

In proposing the development of these activities in Basic Education classrooms, we aimed to offer the type of training recommended by Almeida, Silva, & Vertuan (2012, p. 24), where "it is essential to structure teacher training in mathematical modeling based on the triad 'learning about,' 'learning through,' and 'teaching using' mathematical modeling." To introduce the problem scenario, G2 initiated a dialogue between the instructor and the students, discussing the dengue epidemic in Londrina, Paraná. Following this initial engagement with the topic, the students received a handout containing collected data and synthesized information on dengue.

Regarding the results achieved by G2, the proposed activity was carried out with students divided into groups, which, according to the instructor who guided the activity, encouraged participation and engagement from all students, making them active participants in the learning process. When sharing the activity results, opportunities for discussions were provided, enriching the mathematical content studied that emerged from the modeling activity. As for teacher training, the discussion focused on the need to systematize content with students, emphasizing that the teacher, as a guide, should not accept subpar work, but instead offer solutions and point out pathways for students to follow.

During the systematization of the activities proposed and developed by the participants in Basic Education, questions arose about the students' lack of awareness regarding the use of mathematical modeling. It was the responsibility of the researcher-teacher to highlight that the students in Basic Education would not have knowledge or familiarity with the activity stages, as the goal is the execution of mathematical content rather than mathematical modeling itself. In this regard, the lack of awareness of the activity stages was more closely linked to the students' prior knowledge of mathematical content.

Based on the modeling activities report, as modelers or teachers, a reflective moment was provided, which will be addressed in the next section.

EMERGING REFLECTIONS ABOUT TEACHING PRACTICE WITH MATHEMATICAL MODELING

During the course, the attendees reflected on teaching practices using mathematical modeling activities, along with their perceptions while observing Basic Education students during the execution of the proposed activities. Both topics were related to the implementation of the activities within the curriculum, the associated themes, students' interest and engagement in the context and mathematical concepts, and the positioning of the school structure to accommodate these activities.

Regarding student participation in the activities during the lessons and in comparison to regular classes, the attendees indicated that Basic Education students engaged more intensely in the activities, with their interest in performing the proposed tasks becoming evident. According to participant C5: “When we tried to solve the more traditional textbook activities and so on, they didn’t discuss the activity but rather other things. In the case of the modeling activity, the discussion centered around the activity” (Dialogue records D4).

Participant C7 highlighted that during the modeling activity by G1, a student diagnosed with Autism Spectrum Disorder (ASD) was able to relate well with their group mates. Furthermore, they emphasized the impact of this activity on their teaching practice:

> C7: "In this activity, he sat facing two others, and he participated. The only thing he wanted was a sheet of paper just for himself, because he thought he should have one. I found it interesting because, with his own sheet, he did it the way he thought was right. He socialized better because, while the students were talking and manipulating the Golden Beads Material, he joined the conversation, which he doesn’t usually do." [...] "Up until then, I hadn’t worked in a group with them because of this student’s limitations. Additionally, I noticed that one didn’t copy from the other; instead, they discussed the

activity. Then, he showed how he was doing it, and the other two continued doing it their way, but without aggression." (Records from Dialogue D4).

As noted in Dialogue D4 above, we can also observe the investigative process to which the students were guided, shifting their approach to the problem situation that was proposed. Thus, the search for a solution to the problem transcended the pursuit of a single answer.

In the integration of modeling activities in Basic Education classes, C2 and C5 emphasized the importance of selecting a topic that interests the students, highlighting the motivation this choice can generate. According to C5, mathematical modeling made their lesson more engaging, and working with topics related to the students' reality contributed to a more participatory class, allowing for the exploration of students' creativity, as well as indicating potential future actions with mathematical modeling.

Beyond regular classrooms, C4 presented a reflection on the implementation of modeling activities in extracurricular lessons with their students, highlighting the intention to work with mathematical modeling in spaces such as the support room, even if on an individual basis.

Another possible development was regarding the expectations related to working with mathematical modeling aimed at the early elementary school, both for the attendees and for the researcher who taught the course, as neither had previously experienced activities at this level of education.

> Researcher (R): I had never developed a mathematical modeling activity with the early years before, and when I saw that teachers from the early years had enrolled in the course, I thought it would be a challenge because the early years have a different characteristic compared to other levels. Whenever I've worked with mathematical modeling activities, it was aimed at basic education, and I always thought about high school.
>
> C7: Because then come the more traditional contents that appear in the activities, like exponential equations, quadratic functions, second-degree equations, this type of content —a progression. We immediately think of that, and it's hard to associate this type of representation.
>
> R: It seems like creativity is more evident in the early years, doesn't it? Look at how many different and creative representations emerged.
>
> C7: It's because they are still in the phase of literacy, familiarizing themselves with these mathematical objects. They are at the beginning of their mathematical literacy, and for them, everything is drawing. That's why I imagined that at first they would draw, and then they would discover, "Ah! This can be done another way?" or "Does this work too?" And then when that student made that little staircase with the golden material, I thought: "Wow! I'm happy!" It was exactly what I said—wouldn't anyone make my eyes shine in this activity? When I saw that student, I thought: "Wow! She nailed it!" A pearl for the activity. So, for me, it was very interesting. Personally, it was very enriching, and I believe it was for everyone in the group as well because the contact we had with modeling was very different from what we had been shown. It really added to our knowledge, and it was very interesting. I'm speaking for myself, but I believe it was the same for everyone in the group

The course attendees' perspectives reflected that the practices with mathematical modeling, despite limitations and difficulties, were considered successful, even with reports outside the classroom, where C7 received praise from the school principal and supervisor. In this context, regarding the reflection on teaching practice with mathematical modeling, Rosa and Kato (2014) define as teacher-strategists those who, when reflecting on their practice, seek teaching alternatives through reading, training, and experiences with other teachers, turning their classrooms into laboratories where they develop methodologies from literature or even created by themselves.

> C7: [...] The supervisor said, "Keep up with this practice”, it was also a comment from the principal, "Look, we are facing difficulty with the students in the following grades, in the subsequent years; my class is in an early elementary school. So, we have many difficulties with the students in the following years, especially in fourth and fifth grade. This is because the teachers, in addition to not having this affinity with mathematics, do not work in a differentiated way. It’s just what's in the book, and they limit themselves to using the book because the teacher's book already has the answers printed in blue." (Records from dialogue D4).

The emerging reflections on teaching practice with mathematical modeling address the teachers' attitudes toward creating a predictable classroom environment, concern for student learning, insecurity regarding the proposed activities, and the characteristics of a teacher who continually seeks to improve their practice.

RESULT DISCUSSION AND FINAL CONSIDERATIONS

To present the insights gained from a teacher training course—particularly regarding the teaching practices fostered by the course—we introduce modeling activities developed by participants, which served as the basis for reflections throughout the course. These modeling activities were offered as an invitation to participants who were not yet acquainted with such approaches, in line with the guidelines of Almeida and Dias (2004) and Almeida, Silva and Vertuan (2012).

Throughout the attendees' activities development, we observed the organization and structuring of modeling activity components, such as hypothesis formulation, variable definition, and validation of mathematical models, as demonstrated in the oil tank task. This activity not only highlighted the procedures within the modeling phases but also revealed iterative movements throughout each process. Hypothesis formulation does not occur at a fixed point in the modeling activity; rather, it can emerge progressively as investigations deepen. Nonetheless, hypothesis formulation serves as a guiding element in the investigation. According to Almeida, Sousa and Tortola (2015, p. 4), “the formulation of hypotheses in mathematical modeling activities, while requiring some knowledge about the phenomenon, also functions as a guiding line for the reading or description of that phenomenon.”

The reflections that emerged were closely aligned with theoretical discussions found in the literature on both initial and ongoing teacher education. The discussions that arose pertained to the unpredictability of activities developed by students, the need for a shift in teachers' attitudes to break from habitual practices, the relationship between the mathematical content addressed in the school curriculum and the content emerging from activities, and the uncertainties related to classroom application.

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In general, discussions about the existence of an established teaching practice for educators may be linked to their experience and mastery of mathematical content in teaching. This suggests that as teachers gain experience, they acquire a greater command of the mathematical content they will teach, making their lessons more predictable in terms of potential student questions. In this regard, Almeida, Silva and Vertuan (2012, p. 24) note that research findings "reveal that many teachers remain within a 'comfort zone,' preferring situations where nearly everything is known or predictable, with little room for 'unpredictability'."

The modeling activities incorporation suggests that teachers step out of their "Comfort Zone" into a "Risk Zone," where student activities may often become unpredictable. In this regard, Rosa and Kato (2014) describe characteristics of reflective teachers who reconsider their teaching practices through mathematical modeling.

According to Rosa and Kato (2014), a reflective teacher is a professional willing to learn while teaching, someone who plans lessons, revisits and reviews content and is open to listening, reflecting during action, and assessing their practice after each lesson. The authors further emphasize that "to train a reflective professional is to cultivate someone willing to learn, building new or deeper competencies and knowledge based on their prior acquisitions and experiences" (Rosa & Kato, 2014, p. 591).

Regarding teachers' insecurities with mathematical modeling, it was argued that discomfort arises from a lack of mastery in conducting activities of this nature or from the mathematical content itself. Such arguments align with the findings presented in the research by Ceolim and Caldeira (2015), which highlighted the insufficient training in mathematical modeling, as well as in the content to be taught.

In this regard, the reflections of the course participants, along with the research already published and presented in the theoretical framework, suggest that these obstacles can be overcome through continuing education courses. However, Tambarussi and Klüber (2014) highlighted the low demand from Basic Education teachers for continuing education courses, and one of the causes may be the discomfort caused by the implementation of investigative activities in environments where the school culture is linked to bureaucracy or traditional models of teaching and learning, either through teaching methods or by the provision of structured materials, such as workbooks, as well as the lack of familiarity of students and teachers with such activities.

Almeida, Silva and Vertuan (2012) argue that within the continuing education in mathematical modeling scope, reflections on the teacher's role in activities are formed when this training is framed by the triad "learning about," "learning through," and "teaching using." They assert that "only in this way is it possible to surpass the strictly empiricist and pragmatist view of the teacher's practice in relation to Modeling, shifting to a space where the 'how to do' is imbued with theory and practice" (Almeida, Silva & Vertuan, 2012, p. 24).

Thus, the teacher's change of attitude, stepping out of their "Comfort Zone," must be accompanied by a reflection on teaching practice, in order to avoid the concern raised by Bicudo and Klüber (2011) regarding the mere reproduction of modeling activities in Basic Education classrooms without reflecting on teaching practices.

The reflections on the teacher's stance in mathematical modeling were made by the course participants, who positioned themselves as facilitators, guiding students toward solutions to the problems by suggesting or recommending mathematical content, questioning, and encouraging their students. In this context, Rosa and Kato (2014) highlight that one of the characteristics of a reflective teacher is recognizing oneself as a facilitator of learning, valuing the student's participation in the learning process and their prior knowledge, whether related to mathematics or not.

The attendees' anxiety also focused on lesson planning with mathematical modeling, concerning the duration of classroom activities. Regarding the planning of a modeling activity, Almeida, Silva and Vertuan (2012) emphasized that there is no predefined definition regarding the duration of the activity, as there is a variety of approaches and developments that the activity carried out by students may take.

In this context, the research highlights the need for teacher training and spaces where reflection on teaching practice is fostered, in which participants act not only as modelers but also as teachers who use mathematical modeling in their classes. It is through this usage that obstacles and preconceived notions are demystified.

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