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Teacher education and authenticity in mathematical modeling activities

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ABSTRACT

This article aims to investigate how authenticity in modeling activities can be fostered in mathematics teacher education. The research is based on a structure for pre-service teacher education in mathematical modeling that has been proposed in the literature and includes three axes: learning about modeling, learning through modeling and teaching using modeling. Empirical research is developed with a group of students from a Mathematics Degree course considering their contact with mathematical modeling in two disciplines of this course: the Mathematical Modeling discipline and the Supervised Internship discipline. The analysis carried out indicates that articulating the learning of mathematical modeling with teaching using mathematical modeling, based on the assumption that the three axes configure training in mathematical modeling and mathematical modeling, is a perspective with the potential to excel in the incorporation of authenticity in classroom modeling activities.
KEYWORDS: Mathematical Modeling; Authenticity; Teacher Education.

Formação do professor e autenticidade em atividades de modelagem matemática

RESUMO

No artigo investiga-se como a autenticidade em atividades de modelagem pode ser fomentada na formação do professor de matemática. A investigação se fundamenta em uma estrutura para a formação inicial do professor em modelagem matemática que vem sendo proposta na literatura e inclui três eixos: aprender sobre modelagem, aprender por meio da modelagem e ensinar usando modelagem. Uma pesquisa empírica é desenvolvida com uma turma de estudantes de um curso de Licenciatura em Matemática considerando o seu contato com a modelagem matemática em duas disciplinas desse curso: a disciplina de Modelagem Matemática e a disciplina de Estágio Supervisionado. A análise realizada indica que articular a aprendizagem da modelagem com o ensino usando modelagem, a partir do pressuposto de que os três eixos configuram uma formação em modelagem matemática e para a modelagem matemática, se mostra uma perspectiva com potencial para primar pela incorporação da autenticidade nas atividades de modelagem na sala de aula.
PALAVRAS-CHAVE: Modelagem Matemática; Autenticidade; Formação de professores.

INTRODUCTION

In recent decades, mathematical modeling has been highlighted as a possibility for improving teaching and learning processes in the classroom. Its inclusion in curricula can be observed in different countries, including Brazil (Brasil, 2018; Ferri, 2018; Spooner, 2017).

The use of mathematical modeling is generally based on aspects that indicate that it can help students understand the real world, enhance their learning (motivation, concept formation, understanding, retention) and develop skills (Blum & Ferri, 2009; Castro & Almeida, 2023).

Mathematical modeling activities seek to explain or predict specificities regarding real-world situations under study (Geiger et al., 2022; Blum, 2015; Almeida, 2022; Trindade & Silva, 2024), thus requiring an interlocution between mathematics and reality in the classroom. In fact, the introduction of mathematical modeling in the classroom poses challenges that have not been faced before. One of these challenges, as suggested, for example, by Vos (2018) and Almeida and Omodei (2022), is the introduction of authentic activities instead of problems in which anything that does not relate to mathematics is not considered in its proposition in the classroom.

In fact, discussions about what *authenticity* is and how it can be incorporated into school activities, particularly mathematical modeling activities, can be identified in the literature (Galbraith, 2007, 2013, 2015; Palm, 2007, 2009; Kaiser & Schwarz, 2010, Vos, 2011, 2015, 2018; Carreira & Baioa, 2018).

In general terms, authenticity refers to situations that are characterized by originality rather than being characterized as a copy. In this context, Almeida (2022), when proposing a *design* for the inclusion of mathematical modeling activities in classes, includes authenticity as an important characteristic, pointing out that it requires that it be a real situation and not a simulated situation or one in which the data is simulated.

> This implies using situations in which the data are real and refer to a genuinely real situation. [...] unlike what can sometimes be observed in the literature, the situation does not necessarily need to be in the student's daily life, considering this as the physical space that surrounds them. However, one cannot consider problems such as those frequently found in textbooks that, even presenting situations that are likely to occur, the problems are completely structured and only require obtaining an answer to a question produced by others. (Almeida, 2022, p. 134, our translation).

Without distancing ourselves from the idea of bringing the originality of external situations into the classroom, in this environment, authenticity requires a relativization that, above all, needs to be considered by teachers and students. Therefore, thinking about teacher education for the use of mathematical modeling, including authentic activities, requires including in teacher education programs the specificities, challenges, and demands that modeling activities require. In this sense, Malheiros *et al*. (2020, p. 5, our translation) indicate that "one of the main concerns is teacher education, so that changes occur in Mathematics classes".

Different perspectives and interests regarding teacher education have been presented. Some research highlights the need to invest in developing teachers' skills to teach mathematics through modeling, as well as to teach mathematical modeling (Blum, 2015; Kaiser *et al*., 2010; Almeida, Ramos & Silva, 2021). Others, however, have addressed aspects of professional development required to deal with modeling in the classroom, as is the case, for example, with Barquero *et al*. (2018) and Braz and Kato (2017), Omodei and Almeida (2022), Sousa and Almeida (2021).

In general terms, what is indicated regarding teacher education in mathematical modeling is that education, in addition to qualifying teachers to include activities of this nature in the classroom, must enable them to use it safely and have the audacity to break current paradigms in teaching and learning processes, whether due to the strength of the school structure, often invading classrooms and subtracting teacher autonomy, or for personal education reasons, as pointed out by Ceolim and Caldeira (2017) and Mutti and Klüber (2018). In this article, in particular, the discussion that we intend to cover concerns the valuing of authenticity in modeling activities, proposing a direction for teacher education in which familiarization with modeling activities, emphasizing their authenticity, is provided during teacher education. Thus, the objective of this article is to investigate how authenticity in modeling activities can be addressed in teacher education for mathematical modeling.

The investigation, on the one hand, is based on a structure for pre-service teacher education in mathematical modeling that has been proposed in the literature (Almeida & Dias, 2007; Almeida & Silva, 2015; Almeida; Silva & Vertuan, 2016; Omodei & Almeida, 2022) and includes three axes: learning about modeling, learning through modeling, and teaching using modeling.

On the other hand, empirical research is developed with a class of students from a Bachelor's Degree in Mathematics, considering their contact with mathematical modeling in two disciplines of that course: the Mathematical Modeling discipline and the Supervised Internship discipline.

AUTHENTICITY IN MATHEMATICAL MODELING ACTIVITIES AND TEACHER EDUCATION

Different understandings permeate what can be said about modeling in the field of Mathematics Education. There seems to be, however, as suggested by Castro and Almeida (2023), a consensus that central elements should always include: identifying a problem situation, deciding what to keep and what to ignore in the formulation of a mathematical model, using mathematics in the idealized situation based on a real situation and then deciding whether the results are appropriate for the problem. In the classroom, its development is guided by some phases that in Almeida, Silva and Vertuan (2016) are identified as: (1) Integration (with the situation; (2) Mathematization; (3) Resolution, which includes the construction of the mathematical model; (4) Interpretation of results and validation (5) Preparation of a report and presentation of results.

Thus characterized, as suggested by Almeida (2022, p. 134, our translation), mathematical modeling in the classroom is “based on an extra-school reality, but must meet the desires of a school reality”. In this sense,

> modeling in the classroom has a teaching character, which is not independent of pedagogical practice, in addition to a social character, in which, on the one hand, there is an

> interest in studying the situation of reality with its specificities and, on the other, being attentive to the repercussion that the results obtained in the modeling can have on the interpretation of the situation and on the formation of the student (Almeida, 2022, p. 134, our translation).

Hence the relevance of thinking about authenticity in mathematical modeling activities. In fact, authors from the Mathematics Education community such as Kramarski *et al.* (2002) Galbraith (2007, 2013), Palm (2007, 2009), Vos (2011, 2015, 2018), Carreira and Baioa (2018), Almeida and Omodei (2022) have addressed authenticity in this type of activity.

In general, authenticity can be considered a characteristic of something that is true, real. According to Michaelis, Brazilian Dictionary of the Portuguese Language, the term authenticity consists of the nature, property or condition of what is authentic and is derived from authentic, from the Latin *authenticus* (authoritative; reliable; valid; true; faithful; genuine) and the Greek *authentikos* (which really comes from the one to whom we attribute it; genuine).

The recommendations in the works of Pauline Vos (Vos (2011, 2015, 2018)) indicate that the authenticity used in Mathematics Education must be aligned with this meaning of not being a copy and the situation to be studied in the classroom must, in fact, incorporate the reality outside of the classroom related to this situation. For Kramarski, *et al*. (2002), however,

> [...] being authentic is not a property of the situation under investigation itself, but of the relationship between the solver and the situation. The problem of evaluating the price of a pizza, for example, [...] may be authentic for students who are accustomed to going to restaurants and comparing prices, but it may not be authentic for those who live in communities where this practice is not common (Kramarski et al., 2002, p. 226, our translation).

Peter Galbraith's work (Galbraith, 2007, 2013, 2015) proposes that authenticity can refer to some aspects of modeling activities. An essential characteristic is that problem solving in the modeling activity is guided by the demands of the problem and "pedagogical (and other) educational choices should be decided by the needs of the problem-solving process and not the other way around" (Galbraith, 2015, p. 344). In this sense, modeling "[...] as a solution to real-world problems cannot be fully authentic if the system values, above all, the prescribed curricular mathematical content" (Galbraith, 2013, p. 34).

Carreira and Baioa (2018) suggest that authenticity can be permeated by simulations since they provide "a reflection of what is happening in the real world, in addition to being an adaptation of reality under controlled conditions, a search for similarity with reality and a way of verifying how the practice works in reality" (Carreira & Baioa, 2018, p. 203, our translation).

Almeida and Omodei (2022) highlight the importance of investing in the development of modeling activities that meet certain specificities of authenticity and, to characterize this, the authors propose six attributes observed in the activities: (1) the mathematics used in the modeling activity must emerge from the needs of the mathematical approach to the situation; (2) students must experiment with different strategies and tools (mathematical tools, technological tools and empirical tools); (3) simplifications that lead to an idealized situation cannot distort the reality of the situation; (4) pedagogical choices cannot overcome the needs of the mathematical approach to the situation; (5) there must be a balance between teacher guidance and student autonomy so that the latter prevails; and (6) the results obtained must reach interests and generate discussions that go beyond the classroom. For the authors, modeling activities may, however, meet some of these attributes and not others.

The introduction of mathematical modeling activities that meet characteristics or attributes of authenticity, however, is not dissociated from teacher training. Blum (2015, p. 89) considers that, when introducing mathematical modeling activities in classes, “the teacher is what matters most!” Silva and Almeida (2019) in this line of thought, state that “the implementation of modeling activities in Mathematics classes presupposes that teachers are prepared to play an active role in the organization, implementation and evaluation of these activities” (Silva & Almeida, 2019, p. 3, our translation). Forner and Malheiros (2020) also suggest that Mathematics teachers “have experience with Modeling in the classroom, so that they can understand its possibilities as a pedagogical approach, in addition to discussing it, considering its practice in the classroom” (Forner & Malheiros, 2020, p. 508, our translation). Among the actions to promote these experiences and this education is the introduction of mathematical modeling in Mathematics undergraduate courses, whether in specific disciplines or in topics from other disciplines (Silva & Almeida, 2019; Oliveira, 2020; Trzaskacz & Veronez, 2019).

This introduction also resonates with research by Klüber and Tambarussi (2017), Ferri (2018), Oliveira (2020), for example, who argue that the knowledge necessary for teachers to develop modeling activities in the classroom should be provided to them while they are still at university, during their pre-service education, with specific and mandatory teaching experiences. Oliveira (2020), in particular, argues that offering experiences with mathematical modeling in pre-service education is “an opportunity for experiences to be articulated and lived during internship practices and viewed in a reflective way” (Oliveira, 2020, p. 75, our translation).

In this article, the approach to introducing modeling in mathematics teacher education and in undergraduate courses in Mathematics, in particular, follows assumptions presented and discussed over the last decades (Almeida & Dias, 2007; Almeida & Silva, 2015; Almeida; Silva & Vertuan, 2016; Omodei & Almeida, 2022). According to these authors, teacher education in modeling should include three axes: learning about mathematical modeling; learning through mathematical modeling; and teaching using mathematical modeling.

In the axis of learning about mathematical modeling, future teachers are given the opportunity to learn about what mathematical modeling is, how activities of this type are characterized, and how the organization of a mathematical modeling activity can be systematized (Almeida & Dias, 2007). Therefore, the scope of this axis includes elements that relate to what mathematical modeling is, highlighting relevant characteristics of these activities in the classroom.

The axis related to learning through modeling includes experiences with mathematical modeling in which learning is mediated by the modeling activity itself, i.e., learning about the problem situation, learning mathematics or how mathematics can be taught that emerges from the development of the modeling activity (Almeida & Silva, 2015). In this axis, future teachers are expected to identify, in specific situations, the characteristics of mathematical modeling that are already known and to develop modeling activities as students, i.e., it is they who seeks to solve the problem. Thus, they must go through all the phases of a modeling activity (Almeida & Dias, 2007).

In the third axis, teaching using mathematical modeling, the future teacher develops modeling activities in teaching practices, experiencing the teaching of mathematics, teaching how to propose and solve a problem, teaching the interpretation of a situation in reality through Mathematics (Almeida, Silva & Vertuan, 2016).

This is, therefore, a proposal that sheds light on the movement inherent in facing the need to associate a theoretical discussion with mathematical modeling practices in the classroom. There is, however, no prescription that these axes should follow the order in which they are presented, since teaching and learning may not be isolated phases in teacher education and theory and practice are interdependent in the sense that one is in constant dialogue with the other. The proposed structure is in line with Oliveira's (2020) recommendations that:

> If there is an intention for mathematical modeling to be incorporated into practices, it needs to be instilled, debated, experienced and explored, in the field of pre-service and in-service teacher education, as a condition for these experiences to be promoted. (Oliveira, 2020, p. 83, our translation).

Considering this assertion, in this article the focus on modeling experiences promoted in pre-service education focuses on the inclusion of authenticity attributes in the activities developed.

METHODOLOGICAL APPROACH

The investigation of how the authenticity of modeling activities can be fostered in the education of mathematics teachers for modeling is supported by an empirical study conducted with a class of nine students in a Mathematics undergraduate course.

Specificities of learning and teaching mathematical modeling are addressed according to three axes: learning about modeling, learning through mathematical modeling, and teaching using mathematical modeling. The first two are included in the Mathematical Modeling discipline; the third, teaching using modeling, is explored in the Supervised Internship discipline in which future teachers taught classes to high school students. One of the authors of this article was the teacher who taught these two disciplines.

In the Mathematical Modeling discipline, students were given the opportunity to learn what characterizes mathematical modeling and how to do modeling. In this characterization of modeling, valuing authenticity in these activities was discussed considering the attributes that qualify it (Almeida & Omodei, 2022) and the possibility of adding these specificities to the development and modeling activities in the classroom.

The axis of learning through mathematical modeling was incorporated from the first contact with the class, with mathematical modeling activities inserted throughout the classes of the subject.

The Supervised Internship in High School was the context of the research in the part related to the axis of teaching using mathematical modeling, including: the guidelines for the internship, the discussions about the lesson plans, the implementation of the mathematical modeling classes, which took place during the second part of the supervised internship in High School, the conversation circles about the internship carried out and the writing of the internship report. The classes during the internship were held in workshops that took place over two days of five classes each day for 2nd grade classes of High School of a public school in Northern Paraná.

The collection of information during the activities in the three axes was done through audio and video recording, the delivery of reports by student-teachers (students of the Mathematics Degree course when completing their supervised teaching internship), questionnaires answered by the student-teachers, and the researcher's field diary. The methodological path follows guidelines from the qualitative and interpretative approach (Bogdan & Biklen, 1994).

EDUCATION STRUCTURE AND AUTHENTICITY IN MATHEMATICAL MODELING ACTIVITIES

The empirical research guided by the structure related to the axes of learning about modeling and learning through modeling took place in the Mathematical Modeling discipline over the course of 72 class hours.

Among the fundamental components for education related to the axis of learning about mathematical modeling is the future teacher's knowledge of what mathematical modeling is with regard to understanding the elements that characterize it. To learn about mathematical modeling and, particularly, to characterize authenticity in these activities, studies and seminars were carried out considering references such as Almeida, Silva and Vertuan, (2016). In addition, modeling activities with themes proposed by the teacher are also included in this axis.

An example of these activities had the theme The Big Cap, or Bonezão in Portuguese, a statue built at the entrance to the city of Apucarana to draw attention to a relevant activity in the city: cap factories. The theme emerged from the observation that family members and friends are professionals who work in cap factories in the city (Figure 1), serving as inspiration for detailing the activity and defining different problems in each group.

Figure 1

Activity introduction details indicating for authenticity

Source: Research data (2023).

In general terms, it is possible to infer authenticity in this axis from the students' understanding that a mathematical modeling activity must start from genuine real problems, mathematics must emerge from the demands of studying these problems, and its development must include phases in which mathematics and reality are articulated.

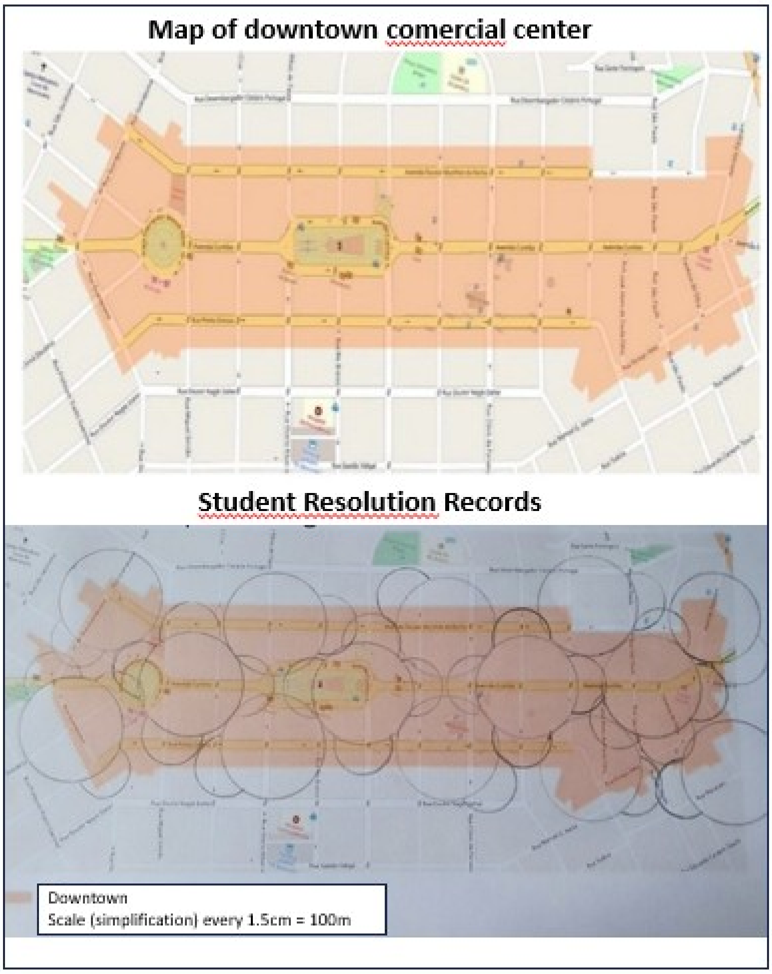
In the axis of learning through mathematical modeling, the main elements are: understanding how a modeling activity is developed; identifying the characteristics of mathematical modeling; developing modeling activities in which the situation or problem is suggested by the teacher; choosing a problem situation and approaching it using all phases, from thinking about the problem situation to solving it, validating the answer, presenting and discussing it with all members of the discipline, and writing the report. In this axis, the teacher's attention was focused on identifying attributes of authenticity in the activities as well as incorporating these attributes in activities in which the themes were chosen by the students themselves.

Regarding this axis, the three groups developed and discussed modeling activities with their peers. Examples of these activities include themes suggested by the students themselves: Free Wi-Fi in the downtown region of the city (group 1); Feet dirty with mud? Never again! (group 2); Making and selling cookies (group 3).

In the Free Wi-Fi in the downtown region of the city activity, a map was obtained from the city hall to help define the location of modems and signal repeaters downtown. The location of these devices was simulated using different techniques, with Geogebra being the most commonly used to define how they would be allocated in space (Figure 2), considering that the range of these devices is circular.

Figure 2

Information of the activity Free Wi-Fi in the downtown region of the city

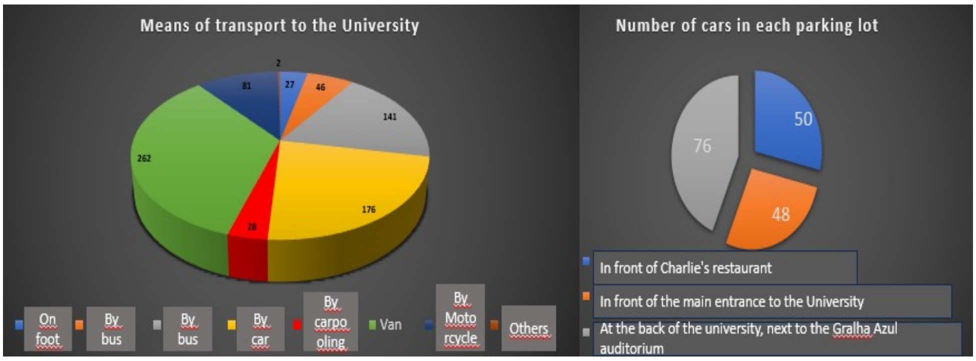


Source: Research data (2023).

The activity *No more dirty feet!* is themed around the renovation of the university campus parking lot. The students chose this option due to the difficulties in parking vehicles on the university premises. Based on a survey of night-shift students on campus, as shown in Figure 3, the group was able to determine how many cars, motorcycles, vans and buses the parking lot should accommodate (approximately) and the improvements suggested by the students. The questionnaires were answered by 762 students (Figure 3).

Figure 3

Data collected by students



Source: Research data (2023).

In the activity related to the manufacturing and selling of cookies, the initial idea was to investigate what was more advantageous financially: producing cookies at home or buying them at the university. However, the students saw in the activity an opportunity for entrepreneurship and decided to investigate what the production of cookies should be so that they could leave their paid jobs and start making and selling cookies.

Figura 4

The activity Making and selling cookies

Before starting any calculations, we tested the recipe to check its yield. It yielded a total of 22 cookies. Since the oven could hold two baking tins that could hold 12 cookies each, only one baking load was necessary. After testing, we looked at the hourly rate to produce each cookie. After some research and guidance from the teacher, we decided that the average month of work would be considered 4.5 weeks. So, we multiplied the total number of hours worked in a week by 4.5 and divided it by the corresponding salary for each.

Name	Time per month (H)	Monthly salary (R$)	Value of the hour worked (R$)
Jane	39,74	400,00	10,07
Isabela	109,74	800,00	7,29
Ana	179,24	1000,00	5,58

To calculate the value of each member's hourly wage, the simple arithmetic mean was used, obtaining the value of R$7.65 per hour.

$$\text{Hourly rate} = \frac{10{,}07 + 7{,}29 + 5{,}58}{3} = 7{,}65$$

The preparation time for a recipe was also calculated: 15 minutes to prepare the dough; 30 minutes to let the dough; 8 minutes to roll the cookies; 15 minutes to bake the cookies; 3 minutes to package the cookies. This makes a total of 1 hour and 11 minutes or 1.18 hours of work to prepare a recipe.

Source: Research data (2023).

Regarding the teaching using modeling axis, the main element consists of developing modeling activities in teaching practice. In this case, the three groups developed with the high school students during the Supervised Internship classes the activity that each group developed in the Mathematical Modeling discipline. In other words, in high school, the activities were developed with the themes Free Wi-Fi in the downtown region of the city, Feet dirty with mud? Never again! and Making and selling cookies.

It is important to consider that in this axis and in this circumstance of using mathematical modeling, the student-teachers proposed to the high school students the activity that they had carried out in the Mathematical Modeling discipline. In this case, therefore, it is necessary to consider that, at the same time that the high school students were not familiar with mathematical modeling, the student-teachers were having their first experience as teachers using mathematical modeling in the classroom.

What can be observed, above all, is that the student-teachers reproduced, to some extent, actions and procedures that they, as students of the Mathematical Modeling discipline, had experienced. For example, they began activities with high school students by forming groups and providing information about the situation to the groups.

To some extent, the authentic aspects of the activities developed in the Mathematical Modeling discipline were preserved when the activity was introduced in the Supervised Internship classes. In fact, one of the students in group 3 (Roberto), when referring to the introduction of the Wi-Fi activity for high school students, draws attention to one of these aspects:

> Roberto: Since this is a topic that is generally of interest to young people, which is access to the internet, we thought about developing this activity. Another point [...] is that it is present in the reality of the students, as we tried to develop it in a region of the city that everyone knows (Transcript of recording of the students' presentation, our translation).

In the activity *Making and selling cookies*, the group presented data and information that they themselves, in the Mathematical Modeling discipline, obtained for this real-life situation. The groups of students, in fact, became surprisingly familiar with the topic and became fully involved in the activity. Although, from the point of view of the use of mathematics, no generalization was made, the situation investigated with the data obtained was very well explored, with the groups of students even creating commercial brands to market their products (Figure 5).

Figure 5

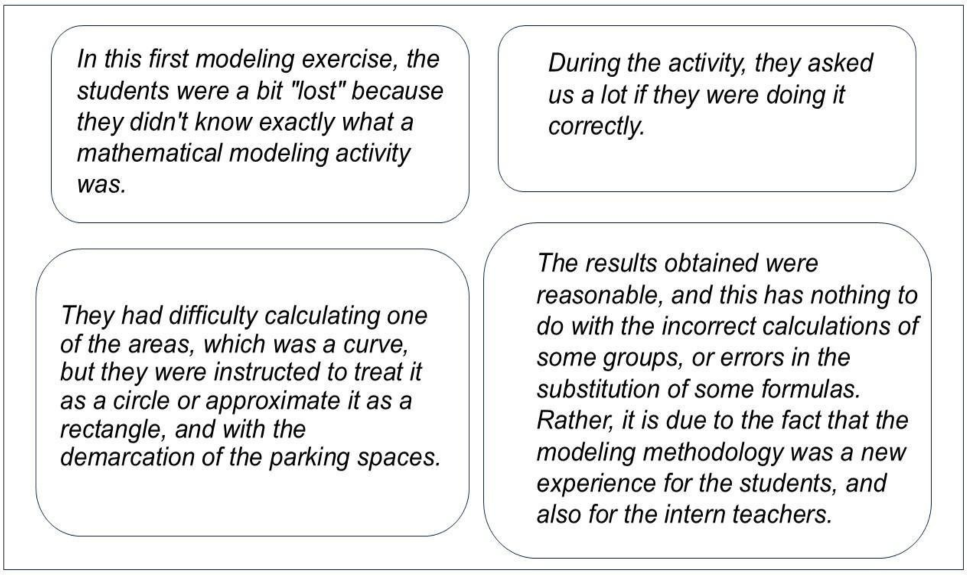
High School Student Results

Source: Research data (2023).

Regarding the theme *Feet dirty with mud? Never again!*, in their actions in the classroom, the student-teachers showed interest in familiarizing their students with the topic, presenting the data from the research carried out as well as pointing out possible approaches to construct a response on the possibility of building a parking lot on the university campus. On the other hand, however, they also pointed out in their report challenging aspects and in which they do not consider themselves successful, as indicated in parts of the internship report submitted by the students (Figure 6).

Figure 6

Student observations on the development of mathematical modeling activities at school



Source: Research data (2023).

RESULTS

The empirical research in which mathematical modeling activities were developed points to the identification of attributes that characterize authenticity in modeling activities and how these attributes were incorporated into the activities based on teacher education mediated by the axes: learning about, learning through, and teaching using mathematical modeling.

The analytical process in relation to the research data allows us to identify the understanding and consolidation of authentic aspects manifested during the students' involvement with mathematical modeling. A summary of these aspects is presented in Table 1.

Table 1

Indications of authenticity throughout the course of the education axes

Discipline	Axis	Examples of indications of authenticity	Authenticity attributes identified
Mathematical Modeling Discipline	Learn about Mathematical Modeling	**Study of texts and seminars:** -mathematical modeling must always start from genuine real problems. -indicate the relevance and intensity of the simplifications used; -authenticity derives from the entire modeling process and the relationship between the modeler and the situation; -pedagogical choices cannot exceed the needs of the mathematical approach to the situation;	(1) the mathematics used in the modeling activity must emerge from the needs of the mathematical approach to the situation; (2) students must experiment with different strategies and tools (mathematical tools, technological tools and empirical tools); (3) simplifications that lead to an idealized situation cannot distort the real situation; (4) Mathematical models are always idealizations, but by seeking similarity with reality, they indicate how reality works. (5) The results obtained must reach interests and generate discussions that go beyond the classroom. (6) Pedagogical choices cannot exceed the needs of the mathematical approach to the situation; (7) There must be a balance between teacher guidance and student autonomy so that the latter prevails
	Learning through Mathematical Modeling	**Activity The Big Cap** - The theme emerged from the observation that family members and friends of students are professionals who work in cap factories in the city; **Activity Free Wi-Fi** -(Excerpt from a dialogue): Roberto: Since the objective of this activity was to find the best distribution of internet equipment that would provide the greatest coverage at the lowest cost, this work can be compared to that carried out by companies that participate in a bidding process, in this case, to implement Wi-Fi internet in the downtown region of the city. -(Excerpt from the report submitted): With the concept of area, it was possible to relate the range of a router. [...] This is a satisfactory model, but we also made this allocation using the Geogebra software.	

Mathematical Modeling Discipline	Learn through Mathematical Modeling	**Activity No more dirty feet** -The theme was formed from data collection with the university community; -Excerpt from the report: We divided the area where the parking lot would be built into three parts; two of them we approximated to regular figures and in the other we used a definite integral to calculate the area to be paved. **Activity Making cookies** -Transcription of recording: We saw in the activity a possibility of entrepreneurship and investigated what the production of cookies should be so that we could stop working and start making and selling cookies; -The group made different simulations (of revenues and costs) to estimate profits. **In all activities:** -Students did not define mathematical content *a priori*; - Students were guided as they requested help or when mistakes were identified by the teacher.	(1) the mathematics used in the modeling activity must emerge from the needs of the mathematical approach to the situation; (2) students must experiment with different strategies and tools (mathematical tools, technological tools and empirical tools); (3) simplifications that lead to an idealized situation cannot distort the real situation; (4) Mathematical models are always idealizations, but by seeking similarity with reality, they indicate how reality works. (5) The results obtained must reach interests and generate discussions that go beyond the classroom. (6) Pedagogical choices cannot exceed the needs of the mathematical approach to the situation; (7) There must be a balance between teacher guidance and student autonomy so that the latter prevails

Supervised Internship High School	Teach using Mathematical Modeling	**Activities developed with students during the Supervised Internship Transcription of recording**. (Intern addressing high school students): - You received the downtown map of Apucarana and it is marked where there should be internet. So, the problem is how do we distribute modems and signal repeaters so that this entire region has internet access? Can you think of a way to do this? Report data: The group did the calculations correctly. But they simplified things and did not construct the circles correctly. But we showed them this when they handed it in. **A dialogue on the activity Making Cookies:** Maria: You can make whatever decisions you think are necessary, such as how much each person will earn and how much you will produce. You can also choose the brand of ingredients you prefer. A1: Is this recipe real? Can you make it at home? Maria: Yes. A2: Can you get a flyer from each supermarket? Maria: You can get as many as you need. A1: Look at the butter. See if you can find it cheaper. [A1 said to the other students in their group.]	(8) Situations of interest to students promote their participation. (9) There must be a balance between teacher guidance and student autonomy. (10) Modeling cannot be fully authentic if the use of prescribed curricular content predominates.

Source: Research data (2023).

What can be concluded is that indicators of authenticity permeate teacher education in mathematical modeling in the structure for education indicated in the article. In fact, permeated by education in the three axes, authenticity becomes part of the curriculum, lesson planning, the objective of modeling, as well as classroom practice. In this sense, Kramarski *et al*.'s (2002) assertion that being authentic is not a property of the situation investigated in itself, but of the relationship between the solver and the situation. In other words, the students' actions are what point to the presence of authenticity in mathematical modeling activities.

While in the learning about modeling axis, students became familiar with the characteristics and specificities of modeling and that add authentic attributes to them, the learning through modeling axis provided them with the opportunity to deal with activities and add these attributes to them. In the teaching using modeling axis, in turn, in the case of the empirical research carried out here and in which contact with modeling as a teacher occurred only in internship classes, the authenticity that can be observed seems to be a direct result of the students' previous experiences in the Mathematical Modeling discipline. Information from the collected material, however, points to the identification, by future teachers, of aspects that indicate authenticity, as is the case of items (7), (8) and (9) in the last column of Table 1.

FINAL CONSIDERATIONS

Articulating the learning of mathematical modeling with teaching using mathematical modeling based on the assumption that the three axes (learning about, learning through, and teaching using) indicated in Almeida and Dias (2007) configure teacher education in mathematical modeling and for mathematical modeling, this research shows a perspective with the potential to prioritize the incorporation of authenticity in modeling activities in the classroom. The approach indicated in this research is also supported by results such as that of Barbosa (2001, p. 14, our translation), who suggests that teacher education should go beyond mathematical experiences with mathematical modeling, "[...] it is necessary to involve them with knowledge associated with curricular, didactic, and cognitive issues of modeling in the classroom, which only make sense in practice itself." In this sense, teacher education for mathematical modeling in the Mathematics Undergraduate course can be strengthened through the use of modeling in different disciplines and instances of the course. In fact, as Pollak and Garfunkel (2013, p. 12) point out,

Modeling problems should be real-world problems, and we believe that we can teach these problems. There are good problems all around us: many people, including us, put a lot of effort into finding these problems. But we believe that teaching a real modeling problem takes time (Pollak & Garfunkel, 2013, p. 12).

The time we refer to in this article is not chronological, but rather a time that includes different facets of teacher education, which are characterized here in the three axes: learning about, through, and teaching using mathematical modeling. Incorporating authentic attributes into modeling activities is a process that, as indicated by the results presented here, can be fostered in teacher education for modeling structured in this way.

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