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Training aspects in an environment for teacher education: the supervised internship with mathematical modelling

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ABSTRACT

Training teachers in Mathematical Modelling from the perspective of Mathematics Education has become a topic of collective interest. Investigating the aspects or contexts that give meaning to this training is also relevant within the scope of recent research on the subject. In light of this, we pose the following question: What training aspects do future Mathematics teachers express through their experience of conducting Mathematical Modelling during the Supervised Internship? To answer this question, we briefly present the context of the Internship practice, from which the reflections arising from the research emerge. This is followed by a dialogue with the interns regarding their experiences, which was transcribed and analyzed from a phenomenological perspective. The research revealed that the training aspects expressed by future teachers can be organized into three main axes: theoretical, didactic, and pedagogical. In general, these axes are not exclusive; rather, they complement each other, providing meaning to the training process. They seem to create a favorable environment for Teacher Education in Mathematical Modelling within the Supervised Internship, incorporating various training aspects such as understanding Mathematical Modelling and its practice, navigating professional vulnerability, and fostering self-identification through the teaching activities based on lived experiences.

KEYWORDS: Teacher education; Teaching Mathematics; Mathematics Education.

Aspectos formativos em um ambiente para a formação de professores: o estágio supervisionado com modelagem matemática

RESUMO

Formar professores em Modelagem Matemática na perspectiva da Educação Matemática tem se mostrado um interesse coletivo. Investigar aspectos ou contextos que atribuem sentido a esta formação também se mostra relevante ao escopo de pesquisas recentes sobre o tema. Com isso, estabelecemos a seguinte interrogação: **que aspectos formativos manifestam futuras professoras de Matemática, desde a regência com Modelagem Matemática no Estágio Supervisionado?** Para responder a indagação é apresentado, brevemente, um contexto da prática de Estágio de Regência, donde emergem as reflexões oportunizadas pela pesquisa, bem como um diálogo estabelecido com as estagiárias a respeito da experiência, o qual foi transcrito e analisado numa perspectiva fenomenológica. A pesquisa revelou que os aspectos formativos manifestados pelas futuras professoras se articulam em três **eixos norteadores**, a saber: **teórico**, **didático** e **pedagógico**. Em linhas gerais, esses **eixos** não sendo excludentes, dão sentido à formação e parecem conferir ao Estágio Supervisionado um ambiente favorável à Formação de Professores em Modelagem Matemática, desde os vários aspectos formativos como, compreensões sobre Modelagem Matemática e sua prática, situações de vulnerabilidade profissional e autoidentificação pela atividade docente dadas às experiências vividas.

PALAVRAS-CHAVE: Formação de Professores; Ensino de Matemática; Educação Matemática.

INITIAL CONSIDERATIONS

Training teachers in Mathematical Modelling within Mathematics Education, henceforth referred to as Mathematical Modelling, is a recent topic in research in this field. Different practices have been implemented, and various theoretical perspectives have supported reflections that highlight formative aspects regarding what Mathematical Modelling is, how it is done, and how it is taught (Almeida, Silva & Vertuan, 2013). This scenario invites us to establish and investigate different spaces that promote teacher training, while also discussing their possibilities and/or limitations, and, in a more epistemic sense, problematizing what this training actually entails (Oliveira, 2023).

Invariably, we know that both institutionalized and non-institutionalized spaces can enhance formative experiences (Larrosa, 2002). When it comes to Mathematical Modelling, curriculum components have stood out as spaces for training, as evidenced by Oliveira's (2016) research, which investigated the course *Mathematical Modelling* in state higher education institutions in Paraná. However, other components have also served as venues for facilitating such formative experiences, such as the Supervised Internship, henceforth referred to as the Supervised Internship.

This argument is supported by the results of several Brazilian studies, such as those by Gavanski (1995), Almeida (2009), Oliveira (2020), and Pereira (2023), among others, which suggest discussions regarding Mathematical Modelling in the Supervised Internship or the Supervised Internship with Mathematical Modelling. There is a consensus among these authors that the Supervised Internship provides the experience of being a teacher with Mathematical Modelling. However, there are differing opinions about its potentialities and/or challenges, especially when discussing the type of Mathematical Modelling that takes place during the Supervised Internship, which may reveal another area for further investigation.

Starting with the Supervised Internship or other formative components, there is a growing understanding that experiences with Mathematical Modelling in teacher training can become a pathway for actions involving Mathematical Modelling to resonate in classrooms in Basic Education. This could occur either through early experiences, as revisited in the research by Kaczmarek & Burak (2018), or by expanding democratic opportunities and participation within the school community (Bellei & Klüber, 2018), which justifies advocating for its presence in teacher training curricula.

As a practice, in this text, we understand Mathematical Modelling as a pedagogical alternative (Almeida, Silva & Vertuan, 2013). According to this understanding, Mathematical Modelling begins with an initial situation, goes through several procedures, and ends with a final situation, involving reflections and the construction of representations, called mathematical models. In other words, between these two situations (initial and final), there are four steps: interaction, mathematization, solution, and interpretation and validation of results.

This understanding of Mathematical Modelling is framed within the context of the Supervised Internship, which we conceive as a field for the production of knowledge and research. That is, it is a formative curricular component that moves

away from approaches that reduce practice to an instrumental activity and instead aligns with a problematizing and reflective perspective on and in practice. The Supervised Internship, conceived in this way, articulates "[...] essential aspects for the construction of the professional teaching identity, with regard to the development of knowledge, skills, and necessary attitudes" (Lima & Pimenta, 2004, p. 61).

Driven by these understandings, in this text, we ask: **What formative aspects are manifested by future Mathematics teachers through their experience with Mathematical Modelling during the Supervised Internship?** In a reflective horizon, we felt the need to present, in the next section, a brief contextualization of the Mathematical Modelling practice carried out as part of the Teaching Internship. Following that, we outline the methodological aspects that justify the paths taken in the research. It is important to clarify that this qualitative research, based on a phenomenological approach, does not assume a theoretical framework a priori; thus, the theoretical connections made with some authors from the Mathematics Education literature emerge within the analytical process we undertake. In this sense, we present the reflections supported by the research question, followed by the final considerations.

ON THE EXPERIENCE WITH MATHEMATICAL MODELLING IN THE TEACHING INTERNSHIP

The Mathematical Modelling practice was preceded by lesson planning and the development of simulated lessons in the Supervised Internship I course. These actions were taken so that the practice could be regarded as a reflective phenomenon, fulfilling the role assigned to the Internship, whether through reflections on components of the activity that would be proposed, or by providing better didactic and pedagogical conditions for the interns and for the practice itself, which would later take place in Basic Education.

In Basic Education, the experience took place in a 9th-grade class of Elementary School and involved 23 students, organized into groups. In general, during the co-participation phase in the classroom, it was observed that the students had few difficulties in solving the proposed activities, worked well in groups, displayed some impatience, but despite this, maintained a good relationship with the lead teacher. The class also included students with special needs, such as a wheelchair-bound student and an autistic student.

For the teaching practice, the mathematical content addressed was **Linear Functions**, as outlined in the Online Class Register (RCO). The practice was guided by the theoretical assumptions of Mathematical Modelling and, in that context, given the pedagogical intent, it was understood as a pedagogical alternative (Almeida, Silva & Vertuan, 2013). The lessons aimed to promote, through a problem situation created by the future teachers, a Mathematical Modelling practice involving the concept of **Linear Functions**, its graphical representation, and the function rule. Didactically, the actions were carried out over the course of six lessons.

The problem situation proposed to the students involved selecting a digital influencer with an Instagram profile[1], followed by searching for information on that profile regarding the *influencer's* earnings from sponsored posts by different brands. Based on this information, and guided by the questions in the activity, the students were required to estimate the *influencer's* earnings in the following months, assuming they maintained the same frequency of sponsored posts.

In general, we approached the students, asking them to organize into groups, followed by a slide presentation featuring photos of some Brazilian influencers. As the slides were shown, some guiding questions were posed, encouraging them to reflect on and investigate the topic *Earnings from Sponsored Posts on Instagram*. The printed activity was provided, followed by both individual and collective reading, as well as clarification of any doubts regarding the activity, such as the choice of any influencer and how to start — specifically, what data to search for.

The development of the activity was monitored by the trainee teachers, observing the consensus among group members regarding which influencer to consider and which information was relevant, such as how much the influencer earns per sponsored post on their Instagram feed. It was necessary to guide the groups collectively in filling out a table, suggesting that, to complete it, they should "stalk" the influencer's Instagram profile, noting the date of each sponsored post, the total number of sponsored posts up to that date, and the estimated total earnings up to that point.

After the guidance, the group members had no difficulties conducting the research and recording the results in the activity. However, a significant challenge was the graphical representation of the calculated values, as well as developing a mathematical relationship that expressed earnings as a function of the number of posts counted. Regarding the graphical representation of the situation, several groups chose to create a histogram.

In the groups, they were questioned and encouraged to think about possible representations, so that they could realize that the graph could help them write, in mathematical language, what they had already written in a descriptive way, that is, in natural language. It was observed that all groups had completed the proposed activity, and that the students' choices, although some did not turn out as expected, were useful for later discussions.

Continuing with the practice, in subsequent lessons, the groups were given a white poster board, a photo of the Instagram profile of the influencer they had chosen, as well as the records of the activity that had been collected. They were asked to represent, on the poster board, the solutions to the questions from the printed activity, similar to the layout of the chosen influencer's Instagram page.

With the completed work, the posters were displayed on the board, according to the established didactic objectives. The mathematical representations, ranging from less sophisticated to more sophisticated, allowed for discussions about Linear Functions. This organization is shown in Figure 1:

Figure 1

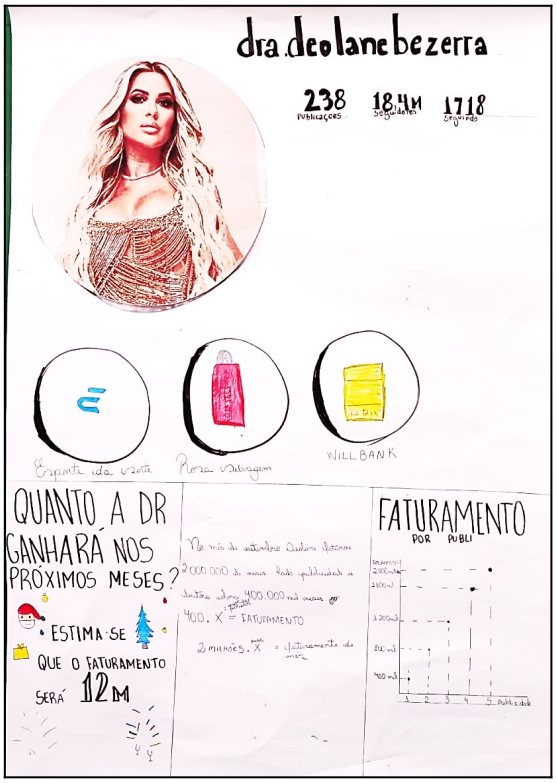
Socialization in the practice of Mathematical Modelling.

Source: The authors (2024).

Regarding the more sophisticated solution, from the perspective of the discussion on Linear Functions, this was handled by the group that chose the influencer Deolane Bezerra. The members of this group represented the situation, in Figure 2, using a mathematical relationship and also managed to create a graphical representation on the Cartesian plane.

Figure 2

Representation drawn up by a group of students.



Source: The authors (2024).

Based on the information presented by this group, the graphical representation was enlarged on the chalkboard. Next, the representation f(x)=400·xf(x) = 400 \cdot xf(x)=400·x, where f(x)f(x)f(x) is the earnings as a function of xxx (the number of posts), with x∈Zx \in \mathbb{Z}x∈Z, was connected to the general formula of the Linear Function, f(x)=a·x+bf(x) = a \cdot x + bf(x)=a·x+b. It was then concluded that the algebraic model expressed by the group was a particular case of the Linear Function, as it did not have the linear coefficient, the constant "b."

Finally, the algebraic representation of the Linear Function was explored with the students, based on the graphical representation generated through the exploration of the topic. At the end of the teaching lessons, the posters and answer sheets were collected, and acknowledgments were made to the students and the lead teacher for their participation, as they had contributed to the professional growth.

METHODOLOGICAL ASPECTS

Inquiring about the formative aspects manifested by future Mathematics teachers through their experience with Mathematical Modelling during the Supervised Internship requires an approach consistent with qualitative research. According to Severino (2017, p. 118), “the multiplicity of aspects through which reality manifests itself also opens up a multiplicity of methods for configuring phenomenological data, as well as a multiplicity of epistemological methods” to explain and produce scientific knowledge.

In this regard, since the phenomenon suggests formative aspects from the experience of teaching with Mathematical Modelling during the Supervised Internship, descriptive elements become necessary. These are required not only to present, albeit briefly, the lived experiences, but also to help us understand these aspects from the perspective of the future teachers, advancing toward an interpretive horizon. Thus, the interns were invited to reflect on their experience with Mathematical Modelling, which involved lesson planning and classroom practice in Basic Education. This reflection was guided by some questions that led to the production of data for this qualitative research, following a phenomenological approach.

> “... in qualitative approaches, the term **research** takes on a new meaning, being conceived as a circular journey around what is intended to be understood, not solely and/or a priori concerned with principles, laws, and generalizations, but focusing on quality, on the elements that are meaningful to the observer-researcher” (Garnica, 1997, p. 111, emphasis in the original).

Adopting this approach, the data were collected through semi-structured interviews with the interns, recorded in audio, after they had experienced the classroom teaching practice. The interviews were transcribed and, from the establishment of the research question, phenomenologically, they encompassed meanings that, in our understanding, need to be discussed within the community researching Mathematical Modelling. This is not only because they reveal aspects that can strengthen the presence of Mathematical Modelling in the Internship, but

also because they resonate with the thematic issue, "Creating environments for teacher education in mathematical modelling," to which this text is presented.

In light of the guidelines suggested by Bicudo (2011), we took the transcriptions of the interviews, highlighted and coded units of meaning (U.), which were then grouped and subjected to movements of convergence and/or divergence, leading to what we organized and labeled as **guiding axes of teacher education**, when we inquired about the formative aspects manifested by the future teachers, expressing another phenomenological movement.

When we asked, ***What formative aspects are manifested by future Mathematics teachers through their experience with Mathematical Modelling during the Supervised Internship?***, we understood that these aspects made sense within the guiding axes of teacher education, as they expressed meanings that permeate the field of Teacher Education (in Mathematical Modelling). Considering them as emerging within the analytical process we undertook, we chose to present them in Table 1, which gathers the codes of the units of meaning (U.), the emerging **axes** in/of the analysis process, and a brief description of each.

Table 1 - Emerging **training guidelines**.

Units of meaning	Formative aspects	Axes/descriptions	
U.4 – First, we did the interaction, we presented some influencers, had a conversation with them, then we presented the sheet with the activity, which led what we did in the interaction to the mathematization — we asked them to search for data about the influencers and then they worked on the solutions.	Understanding of the stages of Modelling manifested in relation to practice.	Theoretical axis - Mathematical Modelling	This axis brought together theoretical understandings of Mathematical Modelling, based on its characteristics as presented in the literature.
U.13 – I liked [the experience] because it allowed us to delve deeper into Modelling.	A theoretical deepening of mathematical modelling.		

Units of meaning	Formative aspects	Axes/descriptions	
U.2 – I think the part of doing... of prompting them to take the data they collected into the mathematics part, according to the solutions we wanted to achieve. Overall, I think that was the most challenging part.	Guide the interpretation of data to construct a mathematical representation.	Didactic axis - Mathematical Modelling Practice	This axis brought together didactic understandings of the practice of Mathematical Modelling, as well as the teaching and learning attitudes guided by both theory and practice.
U.3 – We see that we have to ask some questions to guide them in making the solutions. Because the other parts, they developed on their own, it was easier. To ask these questions, you know, to look at the solutions and think, *What can I say so they understand, without giving them a specific way to do it, but closer to what we want, to our goal.*	Mediate and suggest ways forward without offering ready-made answers, valuing the ongoing research carried out by the groups.		
U.7 – Our planning was what supported the experience. The simulated lessons and the experience we developed in the Project also helped a lot, because we saw that there were things that could be improved for our teaching practice.	Anticipating planning and experience offers reflective elements for improving aspects of practice.		
U.8 – Since it was an activity that we created ourselves, there were some points that needed to be adjusted. [...] So I consider that the entire planning process and the simulated lessons were necessary supports for this.	The development of the activity precedes actions that support its viability in practice.		
U.12 – Well, the students participated more because we observed some other classes and noticed that they weren't very participative. So with Modelling, we were able to make them feel more willing to participate, partly because they looked up some data.	Since modelling involves the production of data, it requires student participation.		
U.15 – What helped was that we brought something that was of interest to them; we analyzed and saw that they were active online and really liked Instagram. So this helped, because we chose a topic we planned for them to engage with—it was part of their everyday life.	Modelling practice focuses on something that is of interest to the students.		
U.16 – Since we did the co-participation, we did a good analysis of the class. We knew, more or less, their performance and interests, so we had to bring something different to grab their attention.	Analysis of class characteristics to propose a modelling practice.		

Units of meaning	Formative aspects	Axes/descriptions	
U.1 – Dealing with the students was difficult, because although it was a cool topic, I was afraid they wouldn't like the activity and wouldn't respect us. The hardest part was these insecurities.	Managing teacher insecurity about the planned activity.	Pedagogical axis - Teaching Mathematics	This axis brought together pedagogical understandings, of a broader nature, that express the role of a teacher who teaches mathematics through Mathematical Modelling.
U.5 – Math classes had to be completely lecture-based until I entered university and got to know the methodologies that exist. [...] I didn't know it was possible to teach math without the blackboard and chalk! So, it was a novelty. I completely changed my concept of math classes.	Understanding another way of teaching math, other than lectures, is new.		
U.6 – Math class for us now goes far beyond using the blackboard, chalk, and textbooks. We can analyze the class, see what they like, and from that, create an activity for them to achieve better results (performance/learning). So, this really changed our perception.	The change in perception of math classes.		
U.9 – For sure, thinking more about the students' interests, because now I know this pays off; they will participate, something that will be innovative; I will achieve the goal, which is for them to learn math.	This experience gives credibility to the practice as something innovative, given the class's interest in learning mathematics.		
U.10 – This internship was rewarding; it exceeded expectations.	The experience exceeded expectations.		
U.11 – It was a turning point because I didn't want to be a teacher. I always thought I wouldn't be able to handle it, I was very afraid. Now I see that it's possible and that I want to "teach classes."	As a turning point, the experience showed that it is possible and feasible to teach mathematics, confirming it as a viable profession.		
U.14 – We realized that it doesn't have to be a traditional class (shouting, blackboard, and chalk); we were able to bring a different activity. We didn't know much, but we were able to learn through Modelling during the Internship.	A moment of learning with mathematical modelling in a traditional classroom.		

Source: The authors (2024).

We chose not to present the dialogue established between the authors of the text, as we agree that its absence does not compromise the reflections, since the ideas will be articulated through the presentations and interpretations of each of the emerging **axes**, with the U.s manifested and representative of those ideas. It is also important to clarify that the interns agreed to participate in the research, revisited the interview they provided, and, therefore, are co-authors of this text from the initiation of their reflection on their own practice, with writing also serving as a formative tool.

Having outlined the path we followed and the articulation of the formative aspects manifested by the future teachers in the **guiding axes of their formation**, we will reflect on each of them in the following section, exposing the possible meanings.

GUIDING AXES OF TEACHER TRAINING: REFLECTIONS PROMPTED BY THE PRACTICE OF MATHEMATICAL MODELING IN SUPERVISED TEACHING INTERNSHIP

> [...] knowledge about teaching does not come before practice, as established by the paradigm of technical rationality, but begins with the questioning of practice; supported by theoretical knowledge, it is a product of understanding the problems experienced and the creation of new solutions aimed at overcoming them (Ghedin et al., 2015, p. 179).

This section fulfills the interpretations relevant to the previous citation. The reflective movement established in a dialogical manner with the interns aimed to provoke them, so that the practice, previously described, would be subjected to analysis — that is, inviting them to the "questioning of practice." The openness provided by the dialogue revealed unique formative aspects from teaching with Mathematical Modelling during the Supervised Internship, and when reflected upon in an articulated way, these aspects became **guiding axes of the formation**, which are now being discussed and reflected upon.

As for the **Theoretical Axis – Mathematical Modelling**, this axis brought together theoretical understandings of Mathematical Modelling, according to the characteristics presented in the literature on Mathematical Modelling. The research revealed two formative aspects, namely: i) understanding of the stages of Modelling manifested in relation to practice; and ii) the theoretical-practical deepening of Mathematical Modelling.

Units U.4 and U.13 suggest, respectively, a theoretical interpretation of the practice developed, indicating a deepening in Mathematical Modelling facilitated by the experience.U.4 – *"First, we did the interaction, we presented some influencers, had a conversation with them, and then we introduced the worksheet with the activity, which took what we did in the interaction to mathematization — we asked them to look for data about the influencers and then work on the solutions."* U.13 – *"I liked it because it allowed us to go deeper into Mathematical Modelling."*

Both meanings expressed by the units overlap when, in light of the research question, we articulate the theoretical axis of the formation. When the student-teachers suggest a deepening that we interpret as both theoretical and practical in Mathematical Modelling, they themselves link the content of this deepening to the interpretation of the steps they took in practice. This interpretation reveals itself as an attribution of meaning not only to the process of "doing" Mathematical Modelling (Almeida, Silva & Vertuan, 2013) but also to aspects of pedagogical practice (Schrenk & Vertuan, 2022).

In our understanding, this axis, while permeating the entire formation and giving meaning to what is being experienced, also has a character of incompleteness and partiality, since the same meanings and interpretations that are being established are reworked into new understandings that, successively, express another way of perceiving and comprehending (the practice of) Mathematical Modelling.

Regarding the **Didactic Axis – Mathematical Modelling Practice**, this axis gathered didactic understandings about the practice of Mathematical Modelling

and the attitudes of both teachers and students governed by the theoretical-practical movement.

Six formative aspects were manifested, namely: i) guiding the interpretation of data to construct a mathematical representation; ii) mediating and suggesting paths without offering ready-made answers, valuing the ongoing investigation conducted by the groups; iii) anticipating and experiencing the lesson as support to improve aspects of the practice; iv) stimulating student participation in the data production within the practice of Modelling; v) thematizing something of student interest through the practice of Modelling; vi) analyzing class characteristics to propose a Modelling practice.

As revealed by the units, this axis expressed aspects related to the attitudes of both teachers and students in Mathematical Modelling practices, from the choices made by the teacher to the way in which they propose and conduct the practice in the classroom, considering the student attitudes that are manifested in a dialogical movement.

In general, regarding teaching attitudes, the guiding role of the (future) teacher stands out in terms of how to model, without indicating procedures that could be adopted in conjunction with the mathematical content, which had already been established by the classroom teacher for the teaching practice, as indicated by the unit, U.3 – *"We see, we have to ask some questions to lead them to solve the problems. Because they developed the other parts, that was easier. To ask these questions, we looked at the solutions and thought, 'What can I say so that they understand, without giving them a direct way to do it, closer to what we wanted, to our goal.'"* Above all, it is possible to highlight the recognition by the future teachers of the importance of the guidance provided by the teacher in promoting autonomous modelling work by the students, considering the learning objectives of the activity.

The actions of seeing, reflecting, and guiding manifest themselves as successive movements of practice in a more problematizing and investigative approach to teaching. In this process, the practice that the future teachers wish to develop is not "dressed" by the theory they studied, as this could risk stiffening the practice and stifling the students' actions, thus losing potential learning opportunities that could emerge in the classroom. By appropriating the theory, the future teachers develop their actions considering the uniqueness of their practice context, preserving and valuing individuality, which demands openness to embrace and deal with unforeseen situations that are potentially relevant for the students' learning. This configuration of the theoretical-practical movement can be characterized as an experience of professional vulnerability (Oliveira & Cyrino, 2011).

This vulnerability destabilizes the future teacher but does not leave them unsupported; it allows for the suspension of their certainties and convictions (for moments that may be short or long, infrequent or frequent) (Oliveira & Cyrino, 2011). The questioning and reflection on professional experiences and the meanings produced for them are what we consider essential for promoting the learning of (future) teachers (Cyrino, 2018; Lasky, 2005; Oliveira & Cyrino, 2011), especially in the context of the Supervised Internship.

Experiencing this challenging situation and becoming aware of it throughout their own practice indicates didactic movements concerning the practice of Mathematical Modeling. These movements seem to have been complemented, within the scope of the training experienced by the interns, both by designing an activity and by being present in scenarios where the activity was, beforehand, developed and reflected upon, as expressed in U.8: *"Since it was an activity we designed ourselves, there were some points to be adjusted. [...] So, I consider that this entire planning process, the practice in the Project, and the simulated lessons were necessary supports for this."*

Although each experience may have distinct formative meanings, the research indicated that the undertaking of certain actions allowed for adjustments and the anticipation of some necessary steps in the practice of Mathematical Modeling. This was evident both in the design of the activity, with a focus on its structure, which ensured that important elements for the approach to mathematical content were included, and in the attitudes taken toward its implementation in the classroom. This anticipation, also revealed in simulated lessons in Oliveira's (2020) research, indicates an important formative action for working with Mathematical Modeling, especially in addressing potential emerging vulnerabilities.

In this sense, the vulnerability that emerges from the formative action prescribed by the Supervised Internship, the Teaching Internship, does not place future teachers in a situation that paralyzes and/or weakens them. This is because prior actions (lesson planning, simulated lessons) provide means for them to construct and mobilize knowledge and experiences to handle the challenges they face. In other words, these are endeavors that enable the mobilization of professional agency in/for dealing with vulnerabilities. This agency is characterized by its relational, temporal, and subjective nature, being manifested in the possibilities and constraints of the context, and *in* and *through* the engagement of (future) teachers in professional practices that are meaningful to them (Priestley, Biesta & Robinson, 2015).

This space was considered a support recognized by the interns as a necessary aid to the practice of Mathematical Modeling and to the legitimization of this practice as beneficial to the teaching and learning of Mathematics. When we revisit the meaning of support, the experiences seemed to offer sustenance and assistance that provided security for the actions taken by the interns, as they assumed their roles as teachers during the Internship, revealing signs of the mobilization of professional agency mediated by the resources provided by the experiences (Priestley; Biesta; Robinson, 2015). Thus, such endeavors with Mathematical Modeling emerge as formative actions with the potential to bring out vulnerabilities that triggered individual and collective reflections, which, in light of didactic aspects, supported the mobilization of professional agency by the future teachers, enabling the design and development of tasks[2] that enriched their Teaching Internship.

Tied to the previous reflections, the units U.12 and U.16 made explicit other aspects that point to what we have referred to as didactic movements, the perception of the students in earlier stages of the Internship itself, as shown in U.12 – *"Well, the students participated more because we observed some other classes and noticed that they weren't very participative. So, with Modeling, we managed to make them feel more willing to participate, especially since they were*

asked to gather some data"; U.16 – *"Since we did co-participation, we conducted a good analysis of the class. We knew, more or less, their performance, their interests, so we had to bring something different to grab their attention."*

The units revealed that the interns understood Mathematical Modeling as a possibility to develop a practice that, didactically, offered pathways to spark interest and ensure student participation, as they diagnosed some of the class's interests. From the excerpts, the professional commitment of the future teachers is evident, as even in a situation of vulnerability, they sought ways to align their actions with the promotion of student learning, showing signs of their professional agency.

The highlighted units revealed not only the importance of the stages of the Supervised Internship but also how they can be organized by the responsible educators: not as a mere juxtaposition of actions, but as a network of structured and articulated endeavors so that they can be experienced by future teachers with a reflective commitment. This should be done continuously, if we consider it as a field of knowledge about Mathematical Modeling and teaching using it as a foundation (Oliveira, 2020; Omodei & Almeida, 2022; Pereira, 2023).

Regarding the **Pedagogical Axis – Teaching Mathematics**, this gathered broader pedagogical understandings, which expressed the role of the teacher who teaches Mathematics through Mathematical Modeling.

Seven formative aspects were manifested in the dialogue with the interns, namely: i) managing teacher insecurity regarding the planned activity proposal; ii) understanding another way of teaching mathematics, one that is not a traditional lecture, as something new; iii) changing perceptions about mathematics lessons; iv) the lived experience lends credibility to the practice as something innovative, considering the students' interests for mathematical learning; v) the experience exceeded expectations; vi) as a turning point, the experience showed that it is possible and feasible to teach mathematics, making it a viable profession; vii) a moment to learn with Mathematical Modeling as compared to traditional teaching methods.

Among the aforementioned aspects, we highlight the satisfaction of the interns with the engagement and involvement of the class in the activity, especially the behavior they exhibited during the Teaching Internship, as it was different from what occurred during the classes in which we conducted co-participations. This satisfaction stems from the evaluation they made of their teaching sessions and was driven, according to them, by the positive results they observed, despite their initial uncertainty about how the students would respond to the proposal—something that caused them some insecurity, as revealed in the unit: U.1 – *"Dealing with the students was difficult because, even though it was a cool topic, I was afraid they wouldn't like the activity and wouldn't respect us. The hardest part was these insecurities."*

It is interesting to reflect that, generally, when teaching is not based on Mathematical Modeling, but rather on expository lessons, this insecurity, to a certain extent, seems to be less pronounced. The insecurity expressed by the future teachers is not a reflection of a lack of commitment; on the contrary, it is a feeling that emerges from their dedication to the actions of the Supervised Internship. When they take on the challenge of Teaching with Mathematical

Modeling, this commitment (which involves decisions, (un)knowns, beliefs, and conceptions) becomes subject to questioning (by themselves or others), which generates concerns inherent to the teaching role. These concerns also resonate with those previously discussed by Oliveira (2010) and those presented in the research by Ceolim e Caldeira (2017).

Furthermore, if we focus on the formative experience of the interns, which in some way emerges between the lines of units U.5, U.6, U.11, and U.14, respectively, we reflect that for both of them, there seem to be no references from prior experiences, as students in Basic Education, that give them any idea of the work they can develop when they become teachers, aside from expository practices:

U.5 – Math lessons had to be totally expository until I entered university and learned about the existing methodologies. [...] I didn't know it was possible to teach math without using a blackboard and chalk! So, learning about Modeling was a novelty. I completely changed my concept of math lessons. U.6 – *Now, math lessons for us go far beyond using a blackboard, chalk, and textbooks. We can analyze the class, see what they like, and from that, design an activity for them to achieve better results (performance/learning); so, this really changed my perception*. U.11 – *It was a turning point because I didn't want to be a teacher. I always thought I wouldn't be able to handle it, I was very scared. Now I see it's possible, and I want to "teach classes."* U.14 – *We realized that it doesn't have to be a traditional lesson (shouting, blackboard, and chalk); we were able to bring in a different activity. We didn't know much, but we were able to learn through Modeling in the Internship.*

Changing the concept of math lessons, the perception of planning and execution, based on validation through a positive experience, self-identifying with the profession, and recognizing aspects rooted in paradigms opposed to those of traditional teaching, are some indications of the pedagogical possibilities revealed in these units of meaning, when the interns experienced Mathematical Modeling during their Supervised Internship. Recent research on Teacher Education in Mathematical Modeling, and studies related to this theme, have recorded these possibilities of re-signifying math lessons, especially when there are experiences with formative practices, particularly those that are more long-lasting and permanent, as noted by Klüber (2017), Mutti e Klüber (2018), Cararo (2022), among others.

We interpret that modifying the concept or perception of aspects that shape lessons involves a shift in the attribution of meaning to teaching activities, inherent to one model or another. The incommensurability between traditional approaches and Mathematical Modelling seems to have ensured a self-identification with the teaching activity when the interns indicated that the experience was "a turning point." On one hand, this expression may signify two distinct ways of understanding the teacher and their professional practice, impacted by something that was broken or ruptured—in this case, potentially facilitated by the experience with teaching approaches that diverge from the traditional paradigm. On the other hand, being a "turning point" does not attribute something disruptive to the experience, as the defense is always for methodological plurality in pedagogical practice.

As it seems to us, this pedagogical axis signals broader understandings of the practice that occurs in the school, developed by the interlocutors who inhabit or visit it. These understandings are articulated through the activity that was designed based on a theme that resonated with the students, in this case, involving the internet, digital influencers, and Instagram, but not only that—also through the dynamism fostered by the practice itself, meaning that it was guided by a teaching perspective different from the one with which they were familiar.

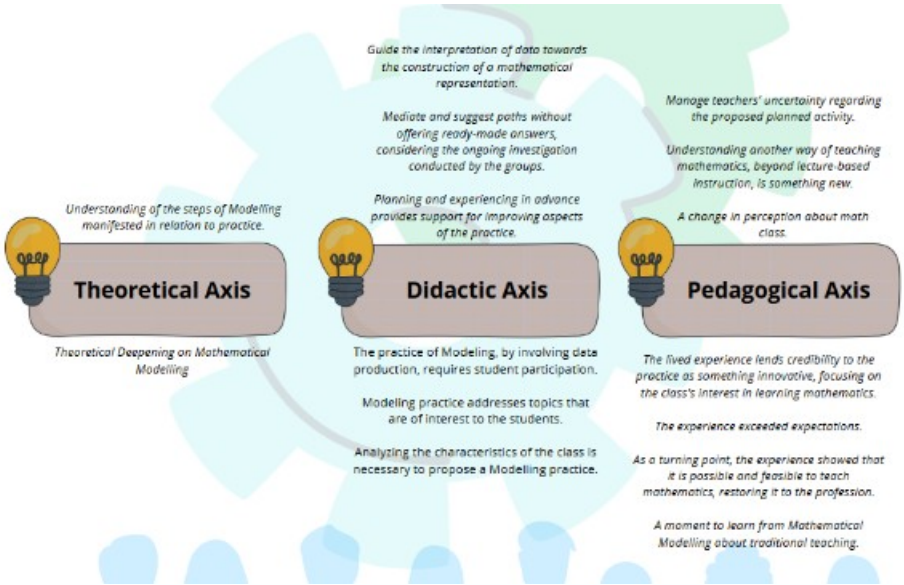
In general, the protagonism displayed during the teaching practice sessions was not observed during the previous classes, as the students were restless and postponed the completion of some activities. Thus, the research seems to reveal that using new teaching approaches, methods, and/or methodologies makes a difference in ensuring student participation and providing evidence of learning. We reflect that this was a distinguishing factor for that group, which, based on the observations made, was used to traditional lecture-based classes.

Thus, with the teaching practice, there are indications of an "initiation" into a different type of professional experience. We use the term "initiation" because it justifies the other experiences that are still expected to be consolidated, not only to refine the theoretical-practical understandings of Mathematical Modeling, but also to develop other practices related to Mathematics Education.

Having presented the guiding axes of the training that brought together the formative aspects expressed by the student teachers, we now seek, in Figure 5, to synthesize the discussions encompassed by these axes.

Figure 5

Representation of formative aspects and axes guiding training.



Source: The authors (2024).

We understand that the **guiding axes of the emerging training** express the relationship between theory and practice in Mathematical Modeling, which arises from the reflective movement that the Supervised Internship provided to the student teachers, as we ventured down the path of "Internship with Research"

(Ghedin et al., 2015). This perspective has been adopted from the outset of the reflective approach to and on practice, involving planning, practice, the production of internship reports, and other actions such as the one we present here.

FINAL CONSIDERATIONS

When considering the research question: **What formative aspects are manifested by future mathematics teachers during their teaching with Mathematical Modelling in the Supervised Internship?**, we reflect that several emerging aspects formed the guiding axes of the teacher education process with Mathematical Modelling, namely: theoretical, didactic, and pedagogical axes.

From these manifestations, we understand that the formative experiences developed within the context of the Supervised Internship, considered as a field for knowledge production and research (Pimenta & Lima, 2006), were structured at the confluence of formative aspects that constituted these three axes. Respecting their specificities, by suggesting understandings manifested about Mathematical Modeling, being a teacher with Mathematical Modeling, and thinking about teaching and learning mathematics, it paves the way for such lived experiences to be structured and articulated with the intention of triggering vulnerabilities, aiming to promote the development/mobilization of future teachers' professional agency, a relevant aspect we consider for the adoption of Mathematical Modeling as a pedagogical practice.

In this sense, the experiences have been promoted by teachers and interns so that they can be experienced by the future teacher as a phenomenon of investigation, analysis, and critical interpretation, and thus, that their experiences during formative processes inspire the possible analyses and understandings of professional practice. Thus, these experiences, as Larossa (2002) proposes, must be "[...] that which 'passes through us,' or touches us, or happens to us, and [that] by passing through us, forms and transforms us. Only the subject of the experience is, therefore, open to its own transformation" (p. 25-26).

As suggested, this lived experience that transcends the reflections raised by this research—that is, with the potential for (trans)formation revealed by the future teachers—suggests the Supervised Internship as an environment for Teacher Education in Mathematical Modelling by, among other things, providing the constitution of formative aspects, which form the guiding axes that support and give meaning to a teacher education process. As discussed by Oliveira (2023), this opportunity resonates with a formative perspective in Mathematical Modelling, when we turn to the intentions of the lived and reflected experience, being the movement of becoming trained in Mathematical Modelling.

Thus, we conclude this text not with the exhaustion of the reflections, but with an invitation to explore theoretical-practical-theoretical movements... that support the refinement of these and/or the constitution of other formative aspects that promote Teacher Education in Mathematical Modelling.

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NOTES

1. Instagram is a free social network that allows the sharing of photos and videos. It enables user interaction through tools such as "follow," "like," "comment," and "share" on posts.

2. Tasks are understood here as a sequence of actions undertaken by the interns and students in the development of the Mathematical Modeling practice. In this comprehensive sense, tasks are not considered synonymous with activities.

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