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Mathematical modelling and early childhood education: perceptions of primary teacher training programme students, participating in a complementary training

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ABSTRACT

This paper discusses part of a doctoral research that highlights the understandings of students in a Primary Teacher Training Programme of secondary degree, in relation to Mathematical Modelling, before and after the course offered on Mathematical Modelling in Early Childhood Education. This research is qualitative, with the participation of 30 students from the 3rd year of the Primary Teacher Training Programme, secondary degree. Data were obtained through observations, field diaries, questionnaires, interviews, video recordings and activities developed (written reflections and planning) by the participants during the course on Mathematical Modelling in Early Childhood Education. For data analysis, Discursive Textual Analysis (DTA) was used, in which categories were created from the corpus. For this work, the discussion is based on the emerging subcategory: Understandings about Mathematical Modelling. The results found in relation to the subcategory showed that most of the participants who had already heard something about Mathematical Modelling had the idea of building a model, and none of them knew about the possibilities of this approach in Early Childhood Education classes. The data indicated more positive than negative aspects related to the development of children in the understanding of the participants when studying, thinking and reflecting on the procedures for working with Mathematical Modelling as a pedagogical practice at this stage of education. It is considered that the participants learned about a new approach, realized its potential and its effectiveness for the integral development of the child, in addition to enabling them to become protagonists of their knowledge.

KEYWORDS: Knowledge; Mathematical Education; Teacher Training; Pedagogical Practice.

Modelagem matemática e educação infantil: entendimentos de estudantes de formação de docentes, nível médio, participantes de um curso

RESUMO

Neste artigo aborda-se parte de uma pesquisa de doutorado, cujo objetivo é evidenciar os entendimentos dos estudantes de um Curso de Formação de Docentes, nível médio, em relação à Modelagem Matemática antes e após o curso ofertado sobre a Modelagem Matemática na Educação Infantil. Trata-se de uma pesquisa qualitativa, na qual participaram 30 estudantes da 3ª Série do Curso de Formação de Docentes, nível médio. Os dados foram obtidos por meio de observações, diário de campo, questionários, entrevistas, gravações em vídeos e atividades desenvolvidas (reflexões escritas e planejamentos) pelos participantes durante o curso sobre Modelagem Matemática na Educação Infantil. Para a análise de dados foi utilizada a Análise Textual Discursiva (ATD), em que a partir do *corpus* foram criadas categorias. Para este trabalho a discussão se dá a partir da subcategoria emergente: Entendimentos sobre Modelagem Matemática. Os resultados encontrados em relação à subcategoria mostraram que a maioria dos participantes que já ouviram algo sobre a Modelagem Matemática tinham a ideia da construção de um modelo, e nenhum deles conhecia sobre as possibilidades desta abordagem em turmas de Educação Infantil. Os dados apontaram mais pontos positivos do que negativos relacionados ao desenvolvimento das crianças na compreensão dos participantes ao estudar, pensar e refletir sobre os procedimentos para trabalhar com a Modelagem Matemática como prática pedagógica nesta etapa de ensino. Considera-se que os participantes conheceram uma nova abordagem, perceberam as potencialidades dela e sua eficácia para o desenvolvimento integral da criança, além de propiciar que elas se tornem protagonistas dos seus conhecimentos.

PALAVRAS-CHAVE: Conhecimentos; Educação Matemática; Formação de Professores; Prática Pedagógica.

INTRODUCTION

From the perspective of Mathematics Education[1], Mathematical Modelling, in its different understandings, whether seen as a pedagogical alternative (Almeida; Silva & Vertuan, 2012), a learning environment (Barbosa, 2001), as a teaching-learning strategy (Biembengut & Hein, 2013) or as a teaching methodology (Burak, 2019), has proven effective for the development and learning of students at all stages of Basic Education (Early Childhood Education, Initial and Final Years of Primary (elementary) School and Secondary (high) School).

Mathematical Modelling has recently gained space in research aimed at Early Childhood Education classes (children aged 0 to 5 years and 11 months), since it contributes to the development of children's autonomy, creativity and logical thinking, as can be seen in the research by Belo (2016), Abbeg (2019), Coutinho (2020), Zampirolli (2020), Rezende (2021), Santos (2021) and Dalvi (2023). Regarding the initial training of teachers who will work in this stage of teaching, who, according to the Brazilian Law of Guidelines and Bases (LDB) nº 9394/96, must be trained by the Teacher Training Programme, secondary degree and/or Degree in Pedagogy (college course of education), there are only four studies that address Mathematical Modelling and initial training courses in these modalities: Braz (2014), Bastos (2018), Silva (2018) and Belo (2023). Of these, Belo's research (2023) emphasizes Primary Teacher Training and Mathematical Modelling in Early Childhood Education.

According to Mizukami (2004) and Shulman (2014), it is during initial training that future teachers learn and systematize the content and the different ways of approaching it with students, as well as the laws that govern education, learning theories, school organization, and the assessment system. Thus, working with Mathematical Modelling in the initial training of teachers, presenting the theory, its possibilities, and how to develop them in Early Childhood Education classes, becomes relevant so that it is increasingly included in this stage of teaching. Furthermore, according to Belo and Zimer (2023), Mathematical Modelling as a pedagogical practice in Early Childhood Education respects the child's opinion and allows them to become autonomous and protagonists in the construction of their knowledge, meeting the pedagogical proposals that should be provided to children, in accordance with the Brazilian Curricular Guidelines for Early Childhood Education (DCNEI) (Brazil, 2010) and the Brazilian Common Curricular Base (BNCC) (Brazil, 2018).

In this sense, Belo (2023) proposed an online course on Mathematical Modelling and Early Childhood Education for thirty (30) students in the 3rd year of the Teacher Training Programme, secondary level, enrolled in the Training Practice discipline. The question that guided the doctoral research was: “what are the contributions of Mathematical Modelling in Mathematics Education for Early Childhood Education teachers in the initial training process?” (Belo, 2023, p. 29). With the objective of “analysing the contributions of Mathematical Modelling in Mathematics Education in the training process of future Early Childhood Education teachers” (Belo, 2023, p. 29).

This paper is an excerpt from the doctoral research, in which the corpus analysed, through Discursive Textual Analysis (DTA), constitutes the initial and final questionnaires proposed to the participants of the aforementioned research. Thus,

the guiding question for this article is: What understandings about Mathematical Modelling do future teachers, students of a Primary Teacher Training Programme, secondary level, participants of an online course on Mathematical Modelling in Early Childhood Education have? The objective is to highlight the understandings of students of a Primary Teacher Training Programme, secondary level, in relation to Mathematical Modelling before and after the course offered on Mathematical Modelling in Early Childhood Education.

Therefore, it is necessary, before going ahead, to clarify some conceptions of Mathematical Modelling that have appeared more prominently in research involving Early Childhood Education and teacher training. In addition, it is worth elucidating Mathematical Modelling as a pedagogical practice in Early Childhood Education, which was the focus of the course offered to the participants. Next comes the research and analysis methodology, as well as the final considerations.

MATHEMATICAL MODELLING FROM THE PERSPECTIVE OF THE MATHEMATICAL EDUCATION

For Burak (2019, 2023), Mathematical Education is the interaction of Mathematics with other areas of Education, such as Anthropology, Philosophy, Mother Tongue, Psychology and Sociology, based on the teaching and learning of students. According to the author, Mathematical Modelling from the perspective of Mathematics Education covers all these areas, seeking to train students so that they become autonomous and capable of making decisions. However, Mathematical Modelling has different conceptions and understandings. For this paper, the perspectives that appear in bibliographic reviews, articles, journals and events, dissertations and theses related to Teacher Training, as well as Early Childhood Education, were highlighted, so that they can be seen in Table 1:

Table 1

Concepts and procedures of Mathematical Modelling

Authors	Conceptions	Perceptions	Procedures
Almeida, Silva & Vertuan (2012)	Pedagogical alternative	"Mathematical Modelling is a pedagogical alternative in which a problem that is not essentially mathematical is addressed through Mathematics" (p. 9).	Integration – first contact with the problem-situation in order to know more about it; Mathematization – moment of transforming the natural language from the problem-situation to mathematical language; Resolution – construction of the mathematical model to respond to the problem-situation; Interpretation of results and validation – analysis and validation of the results found through the mathematical model.

Authors	Conceptions	Perceptions	Procedures
Barbosa (2001)	Learning environment	"Modelling is a learning environment in which students are invited to inquire and/or investigate, through mathematics, situations arising from other areas of reality" (p. 6).	Elaboration of the problem situation; simplification; qualitative and quantitative data; and resolution. These can be: Case 1 – the teacher presents the problem situation, the necessary information and the formulated problem, and the students look for the solution; Case 2 – the teacher presents the problem, and the students look for the information to find the solution and solve it; Case 3 – with the help of the teacher, the students, based on a non-mathematical theme, formulate the problem, look for the necessary information to solve it and find the solution.
Biembengut e Hein (2013)	Teaching and learning strategy	"Mathematical Modelling is the process that involves obtaining a model" (p. 12)	Interaction – recognizing and becoming familiar with the problem-situation; Mathematization – formulating and solving the problem; Mathematical model – interpret and validate the model.
Burak (1992, 2019)	Teaching methodology	"Mathematical Modelling is a set of procedures, whose objective is to establish a parallel to try to mathematically explain the phenomena present in the daily life of human beings, helping them to make predictions and decisions" (Id., 1992, p. 62).	Choosing a topic – based on the students' interests; Exploratory research – time to learn more about the topic, which can be done through internet research, field research and/or talking to an expert on the subject; Problem(s) survey – based on the data collected in the previous stage, questions are raised to be answered that still generate curiosity in students; Problem(s) resolution and development of mathematical content related to the topic – answers to the problems raised are sought. In this stage, mathematical and non-mathematical knowledge (ethical, social and political aspects) is developed; Critical analysis of the solution(s) – time when the results of the problems raised are discussed.

Source: Authorship (2024) based on Almeida, Silva and Vertuan (2012); Barbosa (2001); Biembengut and Hein (2013) and Burak (1992, 2019).

Thus, it is perceived that there are different ways of perceiving and understanding Mathematical Modelling, such as pedagogical alternative, learning environment, teaching and learning strategies, teaching methodology, among others. These diversities occur in relation to the theoretical and epistemological basis, the vision of Mathematics, the conception of teaching and learning of each author, as well as in the procedures to be carried out with students during their development. However, in all of them, the focus is on bringing everyday situations and reality to the classroom.

MATHEMATICAL MODELLING AS A PEDAGOGICAL PRACTICE IN EARLY CHILDHOOD EDUCATION

The pedagogical proposals of Early Childhood Education teachers, according to the DCNEI (Brazil, 2010) and the BNCC (Brazil, 2018), have interactions and play as their guiding axes, aiming at the integral development of children, that is, affective, cognitive, social and physical. The proposed actions should provide children with situations in which they become autonomous, creative and protagonists of their knowledge.

In this sense, there is Mathematical Modelling, which contributes to the development of skills in students "such as observing, exploring and investigating; establish relationships, classify and generalize; making decisions and arguing; conjecture and prove, use imagination and creativity, among others" (Burak, 2019, p. 107).

Trindade and Silva (2024) state that Mathematical Modelling unites the development of mathematical content with the reality of students, providing greater interaction between them and increasing their interests. Thus, "[...] it can be said that mathematical Modelling is an environment that, through inquiries, strategies, concepts, develops new skills based on previous knowledge, enabling learning" (Trindade & Silva, 2024, p. 3).

In addition, according to Zampirolli and Kato (2021), when Mathematical Modelling is inserted in Early Childhood Education, it is possible for children to interact with each other, so that they discuss previous and new knowledge, which are mathematical and non-mathematical. Thus, in agreement with Dalvi and Lorenzoni (2023), Mathematical Modelling, at this stage of teaching, "[...] it provides an interdisciplinary education that looks at the child in its entirety, understanding him as a developing being who thinks, observes, raises questions, has curiosity, feelings, learns and is inserted in the world producing his culture" (Dalvi & Lorenzoni, 2023, p.261).

During the course offered to the research participants, the concept of Mathematical Modelling as a Teaching Methodology was assumed, that is, according to Burak (2019, 2023). However, understanding it as a pedagogical practice when it comes to Early Childhood Education. According to Belo (2023), based on studies focused on the specificities of Early Childhood Education and the understanding that the term "teaching methodology", as much as one is aware that "the teaching methodology must be understood beyond the method, since it is a reflective practice in the teaching action" (Belo, 2023), can be linked to a schooled sense, in which the teacher is seen as the transmitter of knowledge,

which is not consistent with the characteristics of Early Childhood Education. Therefore, when dealing with this stage of teaching, Mathematical Modelling is named as a pedagogical practice, defining it:

> [...] as that practice in which the teacher intentionally provides experiences and experiences for children, seeking their integral development and enabling them to be protagonists of their knowledge, based on their social reality, their previous knowledge and interests, as well as the place where these interests are found (Belo, 2023, p. 124).

Therefore, Mathematical Modelling as a pedagogical practice in Early Childhood Education undergoes adaptations in terms and procedures according to the age group of children, but carries the essence of Mathematical Modelling as a Teaching Methodology, according to Burak (2019, 2023).

In this sense, when developing Mathematical Modelling as a pedagogical practice in Early Childhood Education, it is based on the two principles of Burak (2019, 2023), which are: 1) starting from a topic of interest to students; and 2) search for and collect information at the place of interest.

Also, Mathematical Modelling, for its development with students, has five stages, which are flexible according to the stage of teaching that is being worked on. In Early Childhood Education, the stages, according to the understandings of Belo (2016, 2023), Belo and Burak (2020) and Burak (2023) can be developed as follows: 1) Choice of theme – according to the age group being worked on, the teacher can observe the children's tastes and interests, make suggestions and exchange information with them; In a conversation circle, children can raise topics that interest and curiosity they have and with the mediation of the teacher can choose a theme, it is also possible to hold a vote to find which theme arouses the greatest interest in children, which can be related to drawing, music, story characters, games, among others. 2) Exploratory research – it is the stage that seeks to know more about the chosen theme, based on songs, stories, images, videos, conversations with family members or people who understand more about the subject, and/or a walk in a place about the theme. 3) Survey of the problem(s) – based on the information collected in the previous step, the teacher seeks to understand what are the curiosities and questions that the children still have about the theme. 4) Problem(s) resolution and the development of mathematical content related to the theme – it is the moment in which solutions to the problems raised are sought, through the development of actions on the theme, which can be based on a song, a story, among others. In this way, from this, children develop mathematical and non-mathematical knowledge (cultural, economic, political and social aspects) according to the theme and age group of the children, as exposed in the DCNEI (Brazil, 2010) and in the BNCC (Brazil, 2018). 5) Critical analysis of the solution(s) – it can occur during the development of the actions through observations and notes, and/or later, through conversation circles, seeking to perceive what the children have learned about the theme.

For Abbeg (2019), Mathematical Modelling as a pedagogical practice in Early Childhood Education provides children with "[...] to be active subjects in the process of knowledge construction, as they participate in the entire process of Mathematical Modelling, from the choice of the theme, the study of the theme, the formulation, resolution and analysis of problems" (Abbeg, 2019, p.20). This allows the child to develop in different dimensions, namely: affective, cognitive,

physical, and social, as they will interact with each other, give their opinions, listen to the opinion of others, and develop the proposed actions according to the chosen theme (Belo & Burak, 2020).

So far, theoretical aspects have been presented regarding Mathematical Modelling in Mathematics Education and as a pedagogical practice in Early Childhood Education. Such references supported the analysis of the fieldwork developed in the research, as can be seen below.

METHODOLOGY

According to Bogdan and Biklen (1994), qualitative research "[...] brings together several research strategies that share certain characteristics. The data collected is called qualitative, which means rich in descriptive details regarding people, places and conversations [...]" (Bogdan & Biklen, 1994, p. 16). This article is an excerpt from Belo's doctoral research (2023), which included the participation of 30 students from the 3rd Grade of the Primary Teacher Training Programme, secondary degree, who attended the Training Practice discipline, in 2021, in the municipality of Irati, Paraná State, Brazil (Irati/PR). An online course was offered during the discipline on Mathematical Modelling in Early Childhood Education, and data were collected through: video recordings of classes, notes of observations in the field diary, questionnaires, transcripts of interviews and development of activities (written reflections and individual and group planning) carried out by the participants. The research was authorized and approved by two ethics committees: from the Federal University of Paraná (UFPR), Certificate of Presentation of Ethical Appreciation (CAEE): 41559020.1.0000.0102 and by the Regional Centre of Irati/PR, co-participant in the research, CAEE: 41559020.1.3001.5539.

The class was divided into two parts. According to the structure of the discipline, participating a part on Wednesday and the other part of the class on Friday, both had access to the same content of the Course on Mathematical Modelling in Early Childhood Education. The meetings were divided into seven (7) lasting 2 hours/classes and were distributed as follows (Table 2):

Table 2

Meeting descriptions

Meetings	Procedures
1st	Initial questionnaire; Jamboard; Beginning of the explanation about Mathematical Modelling. Presenting that there are different conceptions, but that we would assume Burak's (2010, 2019). And, the stages of Mathematical Modelling, used for development as pedagogical practices in Early Childhood Education, followed by examples.
2st	Continuation of the explanations about the stages of Mathematical Modelling and examples of its use in developments as pedagogical practices in Early Childhood Education and the realization of an experience with Mathematical Modelling as a pedagogical practice with the large group.
3st	Individual reading by students of an article on Mathematical Modelling in Early Childhood Education, followed by written reflections.

Meetings	Procedures
4st	Explanation about the planning and practices of Mathematical Modelling. Organization of students in groups to choose a theme to propose an action of Mathematical Modelling as a pedagogical practice thinking about Early Childhood Education classes and elaboration of a plan.
5st	Presentation of the themes selected by the groups and the planned actions. Delivery of the planning.
6st	Preparation of an individual plan.
7st	Presentation and delivery of individual planning. Final questionnaire.

Source: Authorship (2024).

In this way, it was sought to know the perceptions that the participants had, through an initial questionnaire, in conversations with the class and making notes on the virtual platform *Jamboard,* in relation to Mathematical Modelling, who heard about it, as well as what they thought it was. Then, during the course presenting, discussing and bringing information about Mathematical Modelling as a Teaching Methodology, the paths for this approach as a pedagogical practice in Early Childhood Education classes were presented in all meetings of the course. Finally, a final questionnaire was proposed to understand the modifications and new perceptions of the participants in relation to Mathematical Modelling.

The analysis was carried out through the Discursive Textual Analysis of Moraes and Galiazzi (2016) with "the purpose of producing new perceptions about phenomena and discourses" (Moraes & Galiazzi, 2016, p. 13). It can be understood:

> [...] as a self-organized process of comprehension construction in which perceptions emerge from a recursive sequence of three components: the deconstruction of the texts of the "corpus", the unitarization; the establishment of relationships between the unitary elements, categorization; the capture of the emerging in which the new understanding is communicated and validated (Moraes & Galiazzi, 2016, p. 34).

According to the authors, a *corpus* is understood as a set of documents. For this article, the *corpus* consists of the participants' answers to the initial (before the course) and final (after the course) questionnaires. After the definition of the *corpus*, the analysis begins from its deconstruction and unitarization, this moment "implies examining the texts in their details, fragmenting them in order to produce constituent units, statements referring to the phenomena studied" (Moraes & Galiazzi, 2016, 152 p. 33). In this way, based on the question and objectives, the researcher will fragment the texts, diving in-depth into the data, respecting the voices of the subjects in search of new understandings. In order for the researcher to know, in relation to the whole, which part that unit is about, it is necessary to codify it. For this article, the following codes are used: IQ – Initial Questionnaire, QF – Final Questionnaire, P – Participant 1 to 30, Q – Question, 1 to 11 referring to the questions of the initial questionnaire and 1 to 17 for the questions of the final questionnaire (Table 3).

Table 3

Example of coding and deconstruction of the corpus

Text code	Unity of meaning	Rewrite
QIP1Q11	I believe that these are **practices adapted** to **the inclusion of mathematics** in a more **playful** way.	He has never heard of Mathematical Modelling, and believes that they are adapted practices to include mathematics in a playful way.

Source: Authorship (2024).

Observing Table 3, it can be seen that code QIP1Q11 refers to the initial questionnaire (IQ) of participant 1 (P1), in question 11 (Q11). One hundred and ten (110) units of meaning were obtained in the subcategory analysed in this article.

Next, according to Moraes and Galiazzi (2016), there is categorization, in which units that have similar elements are grouped. The categories can be, a *priori,* defined before the beginning of the analysis based on the theories studied; and emergent, based on the in-depth dive into the analysis of the data, with the objective of the phenomenon to be studied as a guide. It can also have mixed categories (a *priori* and emergent).

In Belo's (2023) doctoral research, mixed categories were systematized, in which subcategories emerged from two a *priori categories*, created inspired by the theories studied. Therefore, there is the category: 1) *Mathematical knowledge in Early Childhood Education*, and the subcategories – "Perceptions"; "Knowledge of the content"; "Practices"; and "Reflections". 2) *Mathematical Modelling as a pedagogical practice in Early Childhood Education*, and the subcategories – "Perceptions of Mathematical Modelling"; "Perceptions of the Mathematical Modelling Guidelines"; and "From practice planning to Mathematical Modelling".

In this article, the category *Mathematical Modelling as a pedagogical practice in Early Childhood Education* will be analysed with a focus on the subcategory: *Perceptions of Mathematical Modelling*, in which the participants' perceptions of Mathematical Modelling before and after the Mathematical Modelling in Early Childhood Education course is sought, trying to perceive the students' understandings and modifications regarding this approach.

Finally, there is the production of the metatext in which "structures of categories are constructed, which, when transformed into texts, forward descriptions and interpretations capable of presenting new ways of understanding the phenomena investigated" (Moraes & Galiazzi, 2016, p. 111). Thus, in the following item, the metatext of the emerging subcategory chosen for this section will be presented.

PERCEPTIONS OF MATHEMATICAL MODELLING

Regarding Mathematical Modelling, of the thirty (30) participants, twenty-two (22) said they had heard about Mathematical Modelling, and eight (8) said they had never heard about it.

From the twenty-two (22) participants who had heard of it, two (2) did not explain what Mathematical Modelling was, for them. Thirteen (13) said that it was a model or a pattern to be followed, as shown in the examples:

> **Mathematical models** are forms of a **pattern**, **simulations**, **methods** (QIP3Q10).
> As if it were a **creation** of a **mathematical model** to explain (QIP5Q10).
> It is the **process** that translates language from **the real world** to the **mathematical world** (QIP9Q10).
> Mathematical Modelling is a **standard** to be followed (QIP10Q10, emphasis added).

These understandings are in line with Biembengut and Hein's (2013) understanding of Mathematical Modelling as a teaching strategy, which they understand as the creation of a model.

Participant 12 (P12) answered that "In the discipline of Methodology and teaching of mathematics that we have, I learned that mathematical Modelling is the **set of steps** that **aims to find a solution**" (QIP12Q10, emphasis added).

This understanding is in line with Burak's Mathematical Modelling as a Teaching Methodology (2019, 2023), which from the five proposed steps finds the solution to the problem raised through a theme of interest and reality of the students.

Six (6) participants answered in the sense that Mathematical Modelling is to transform and bring the mathematics content closer to the students' reality in the classroom; they also spoke from the perspective of being a different way of working with Mathematics, in addition to involving Mathematics with other subjects and with everything that involves the area of Mathematics and the formation of mathematical concepts. As shown in the following examples:

> This is when we bring the **student's reality** to the classroom for a better understanding of what is being studied (QIP16Q10).
> **Everything** that **involves** the **area of mathematics** (QIP18Q10, emphasis added).

In this way, it is understood that, for the participants, Mathematical Modelling is to bring the student's reality into the classroom, developing mathematical concepts. Thus, these understandings meet the common point of the different conceptions of Mathematical Modelling, which is to provide mathematical knowledge from the student's reality.

For those who answered that they had never heard of Mathematical Modelling, there was a question that asked them to expose what they believed it could be. Regarding the eight (8) participants who answered that they had not heard about Mathematical Modelling, one (1) did not answer what they thought it was about, and three (3) related it to a mathematical model, as can be seen in the examples:

> It appears to be something like **Modelling** the math content to **make it easier** to understand (QIP6Q11).
> I imagine that it is a **model of mathematics**, a **didactic** perhaps (QIP24Q11, emphasis added).

In these examples, it can be seen that the understandings of P6 and P24 are related to Mathematical Modelling as a model, but with an idea focused on teaching and learning, so that they think it could be an easier way to understand

Mathematics or didactics. Thus, it is possible to relate the understanding of P6, when considering the terms "Modelling content", with the conception of Biembengut and Hein (2013), who understand Mathematical Modelling as the construction of a model to explain and facilitate the perceiving of the content. The answer of P24 can be related to Burak's (1992) conception of a teaching methodology, in which he seeks, through the proposed steps, to facilitate the student's understanding and presents a concern with teaching and learning. However, in the beginning, he placed the model as one of the stages, since the word model refers to model, due to the fact that its trajectory emerged as Mathematical Modelling applied in exact Mathematics and with time and studies Professor Doctor Dionísio Burak was modifying his ideas.

The remaining four (4) participants believe that Mathematical Modelling is related to practices that approach Mathematics in a more playful way; to work on Mathematics showing its importance in everyday life; to understand how children learn; their development and ways of teaching Mathematics. The answer of P7 was the one that came closest, among the four participants, to the understanding of Mathematical Modelling: "I believe that **working** with the **importance** of mathematics in **our daily lives** by showing examples of this" (QIP7Q11, emphasis added). This participant relates Mathematical Modelling to the daily life of students.

Regarding the participants' new perceptions of Mathematical Modelling after the course actions, of the twenty-six (26) participants who answered the final questionnaire, one (1) participant did not answer the question. Seven (7) participants bring the explanation of Mathematical Modelling as a model, in the sense of explaining and comprehending Mathematics, according to the examples:

> It is a **model**, a way of **explaining mathematics** in a **simpler way** to be understood by the student (QFP3Q3).
> It is a **model**, a way of **explaining** and **comprehending** (QFP5Q3).
> A **way/model** that has several **ways of talking** about mathematics (QFP15Q3).
> It is the way we work, a **model** (QFP14Q3, emphasis added).

It can be observed in the answers that, even using the term "model", the participants' understanding is focused on ways of working with Mathematics with students.

Four (4) participants presented an understanding of Mathematical Modelling related to the way of working with Mathematics in order to provide mathematical knowledge to students, as in the following examples:

> You will develop **a** different form that will apply **to students** a **different way of applying** (FPF1Q3).
> **Differentiated way** of passing mathematics concepts (QFP12Q3, emphasis added).

It is perceived that the understanding was generated in the participants that Mathematical Modelling will make it possible to approach mathematical knowledge in the classroom in a different way, what can be assumed is that the participants used the terms "different" and "differentiated" in the sense of the approach they are often used to, the traditional one.

There were also two (2) participants who expressed their understandings in relation to Mathematical Modelling, which were focused on how to apply and learn to use Mathematics, as can be seen:

> For me it is learning to **use** mathematics, it would be a **learning field** (QFP6Q3).
> The **way** to **apply** mathematics (QFP23Q3, emphasis added).

P6 relates Mathematical Modelling as a way of learning Mathematics, in the sense of how to use it, and P23 understands the approach as an application of Mathematics.

There were three (3) participants who answered that Mathematical Modelling is a way of explaining Mathematics through everyday life and reality and that it involves other disciplines. As in the examples:

> It is the way in which **we apply** mathematics in everyday life (QFP9Q3).
> It is to bring the **student's reality** to school and **try to mix the subjects** (QFP10Q3, emphasis added).

Thus, it is perceived that there is an understanding that Mathematical Modelling has the essence of bringing everyday situations and the student's reality to the classroom. But there is still a need for maturation, since the participants have the understanding of application and of "trying to merge" Mathematics, since Mathematical Modelling, based on the theme of reality and interest of students, by itself involves other disciplines, as it is possible to work with economic, political and social aspects.

In the course, the approach of Mathematical Modelling assumed was as a teaching methodology, bringing its possibilities as a pedagogical practice in Early Childhood Education. Thus, there are two (2) participants who bring this definition:

> It is the **working method/a set of stages** of mathematics teaching (QFP20Q3).
> **set of steps** whose ultimate objective is to provide a mathematical description of a given real-world phenomenon (QFP22Q3, emphasis added).

Four (4) participants answered that Mathematical Modelling is a way of solving problems and that it is: a theme that involves Mathematics; a recreational way for students to learn Mathematics and helps in the construction of knowledge:

> [...] an **easier way to solve problems**, **solve answers** (QFP4Q3).
> It is a **topic** that involves mathematics (QFP17Q3).
> it is **to make** students **learn** in a lighter and more **recreational** way (QFP19Q3).
> Something that helps in the **process of constructing** the individual's knowledge (QFP21Q3, emphasis added).

The common points are in providing a school environment where students are encouraged to build knowledge.

In this path of knowledge construction, there is P24, which understood Mathematical Modelling as a strategy for the construction of children's knowledge: "Mathematical Modelling is a **teaching strategy** that can contribute to the **process of building mathematical knowledge**, **logical reasoning**, **language development** and **autonomy** in the face of the resolution of situations" (QFP24Q3, emphasis added).

It can be seen that P24 understood that Mathematical Modelling provides the integral development of children, that is, affective, cognitive, physical and social aspects. As much as she calls it a teaching strategy, two facts can be highlighted to understand that she understood Mathematical Modelling as a pedagogical practice in Early Childhood Education: 1) she mentions that Mathematical Modelling contributes to the construction of knowledge in various aspects; and 2) I had never heard of Mathematical Modelling, building this understanding during the course offered.

There were two (2) participants, P11 and P16, who had already heard about Mathematical Modelling before the course and expressed their new understandings. P11 points out that:

> After delving into the subject during the course, I can say that my conception of the concept of mathematical Modelling **consists of approaching mathematical concepts** through themes that are pleasant to children, whose **choice is made from discussions with children**, with **their families**, among other **methods of approaching the content** (QFP11Q3, emphasis added).

P11 understood that by developing Mathematical Modelling as a pedagogical practice in Early Childhood Education, the theme will start from the interest of children, will involve the family and develop mathematical concepts. Thus, P16 mentions that:

> It is much broader than I expected, because students can have the chance to **choose what they want to work on** is very important because **mathematics** can be worked in **many ways** and fits into **several themes** and **activities at the same time** (QFP16Q2, emphasis added).

P16 modifies its perceptions of Mathematical Modelling, realizing the possibilities of working with Mathematics with different themes and actions with children. Thus, she understood that in Mathematical Modelling as a pedagogical practice in Early Childhood Education, children choose the theme and work on mathematical notions and knowledge in different ways. This is in line with the literature, in which there is Belo and Burak (2020) who highlight that based on a topic of interest to children, there is the possibility of developing various experiences with them, providing not only mathematical knowledge, but also the integral development of children.

The participants were also asked what positive and negative points they consider regarding the development of Mathematical Modelling as a pedagogical practice in Early Childhood Education.

Regarding the positive points, in general, the participants pointed out that: it is something good and different to work with; learning from the interaction with the reality and interest of children; it is dynamic and playful; Mathematical Modelling prevents children from growing up understanding Mathematics as something difficult and exemplifies facilitating the understanding of concepts; works with various mathematical concepts; enables the integral development of children and them, as researchers, as well as teacher/child interaction; and it makes it possible for children to learn to make decisions based on the choice of theme. In addition, Mathematical Modelling as a pedagogical practice in Early Childhood Education: "**It**

stimulates creativity and **imagination**, helps in **solving problems** and **awakens motivation**" (QFP6Q8, emphasis added).

And, P7 highlights that "To carry out the practical activities, fine motor coordination, broad motor coordination, manual glasses, oral and visual language, laterality, balance, association, classification... These are **points that I consider positive** because they will be **developing** their **physical** as well as **cognitive parts**" (QFP7Q8, emphasis added).

In this way, the participants understood that Mathematical Modelling as a pedagogical practice in Early Childhood Education allows children to develop fully from the choice of the theme of their reality and interests, enabling actions to instigate children to learn. These understandings are in line with Belo (2016), Abbeg (2019) and Belo and Burak (2020) who highlight in their works that the participation of children in all stages of Mathematical Modelling, from the choices of the theme to the analysis of solutions, enables the integral development (affective, cognitive, physical and social) of children.

Another positive point pointed out is the fact that with the development of Mathematical Modelling as a pedagogical practice in Early Childhood Education:

> It is **possible to make children aware** of **some issues** that need to be questioned through **themes** that can be worked on in the classroom.
> It is **a way** to make children **learn by playing** and having **fun** with a **topic that is of interest to them**.
> Through Modelling, children can **relate what they learn in the classroom to their daily reality** (QFP11Q8, emphasis added).

P11 raises the issue of addressing issues that need to be worked on to generate awareness. In Early Childhood Education they can be related to different cultures, hygiene, environmental issues, waste separation, care and the importance of water, among others. These issues are in line with Burak (2019, 2023), who highlights that the choice of the theme may come from some social, economic, and/or political issue, and it is important that the teacher knows about the situations of the regions of the school community in which he works. P11 also highlights that children learn by playing based on the theme of their reality and interest, corroborating what is exposed in the DCNEI (Brazil, 2010) and the BNCC (Brazil, 2018), whose pedagogical proposals should have interactions and games as guiding axes, providing children with experiences and experiences based on their previous knowledge, enabling them to be protagonists of their knowledge and to develop themselves integrally.

Regarding the negative points, fourteen (14) participants said they did not have or do not see negative points regarding the development of Mathematical Modelling as a pedagogical practice in Early Childhood Education. Five (5) participants raised the issue of time, as they considered that for the development of all stages it will require more time, in addition to the number of children in the classes. As in the examples:

> I think that a point that stands out the most is the **time spent** by the teacher when he **decides to apply the Modelling**, if he goes to see a **normal lesson plan** and another with the Modelling, the **Modelling will take much longer** to be done because he needs to think about several concepts (QFP10Q9).

> The **number of students** in the room and often the **lack of time** that hinder student learning (QFP21Q9, emphasis added).

In Early Childhood Education, the issue of time is not very consistent, due to the fact that at this stage of teaching it is not divided by disciplines and the pedagogical proposals should focus on interactions and games, in order to promote experiences in which children develop in several dimensions, such as: affective, cognitive, physical and social aspects. What may have led the participants to think about the issue of time was the fact that even in CMEIs and preschools, there is a routine with arrival times, snack times, park times, among others. In this way, even though they are not divided by subjects and have greater flexibility in time, compared to the classes of the other stages of education, there are still times when it is fragmented. This fact may have been the reason why some participants pointed out as a negative point the time in the development of Mathematical Modelling as a pedagogical practice in Early Childhood Education. On the other hand, the issue of the number of children in the classes can generate insecurity in the teacher who does not have much experience and who is interested in developing these practices, as he may feel difficulties in mediating the actions.

Three (3) participants pointed out as negative points the teacher's lack of knowledge in relation to Mathematics, in addition to the little affinity, as he will work with the children in such a way that they do not attract their attention. As in the examples:

> If the **teacher doesn't like** math, it will be **very bad** because he will teach in a way as if it were boring, this would make the children think it is boring too (FP6Q9).
> Many teachers **do not know how to apply** mathematics, and end up frustrating children (QFP23Q9, emphasis added).

Thus, the participants related the development of Mathematical Modelling as a pedagogical practice in Early Childhood Education to the fact that the teacher likes Mathematics or not, raising some questions: Do the participants think that if the teacher does not like Mathematics, it will hinder the development of this approach? Did they comprehend that the theme will come from the interest and curiosity of children? What caused this relationship with the teacher liking or not Mathematics?

In addition, there were three (3) participants who related the negative points with the children's learning. As can be seen in the examples:

> I believe that the only **bad part** is that **each student learns** in a different way, so you **have to apply it to something** that everyone learns (FP1Q9).
> If **it is not well organized**, the mathematical Modelling process **may not promote** the full development and learning of children, they may become agitated with a certain theme addressed, or conflicts may occur in the choice of the theme to be worked on, for example (QFP11Q9, emphasis added).

In relation to P1, she may not have comprehended that from the moment the theme is chosen by the children, through their reality and interest, the teachers will provide them with the opportunity to learn and be protagonists of their knowledge.

P11 emphasizes the integral development of the child, but thinking that it cannot occur if the teacher does not know how to organize and mediate conflicts in

relation to the choice of theme, thus not being able to develop Mathematical Modelling. But, according to Belo (2016), Abbeg (2019) and Belo and Burak (2020), the teacher, when choosing to work with Mathematical Modelling as a pedagogical practice in Early Childhood Education, will observe the tastes and interests of children, seeking to perceive themes that can catch their attention, talk to them and mediate them so that they reach an agreement, using different strategies, such as images, videos, stories, among others.

Finally, one participant pointed out as a negative point her difficulty in understanding Mathematical Modelling: "At first **it was difficult** to understand the concept" (QFP15Q9, emphasis added). Thus, she emphasizes the difficulty she had in relation to the approach.

Thus, in order to systematize and comprehend the perceptions, modifications and expansions by the participants in relation to Mathematical Modelling, some comparisons in relation to the initial and final perceptions can be observed in Table 4, as well as the approximations in relation to the theoretical conceptions:

Table 4

Summary of future teachers' perceptions of Mathematical Modelling

Part.	Initial perceptions	Approx. conceptions	Final perceptions	Approx. Conceptions
P1	I believe that these are practices adapted to the inclusion of mathematics in a more playful way.	Work on Mathematics with playfulness.	You will develop a different pattern that will apply to students in a different way of applying.	Teaching methodology
P4	It would be a way of teaching mathematics with models, shapes, etc.	Teaching strategy	[...] An easier way to solve problems, solve answers.	Teaching methodology
P6	It seems to be something like Modelling the math content to make it easier to understand.	Teaching strategy	For me it is learning to use mathematics, it would be a learning field.	Teaching methodology
P10	It is the process that translates language from the real world to the mathematical world.	Teaching strategy	It is the way in which we apply mathematics in everyday life.	Teaching strategy
P12	In the discipline of Methodology and teaching of mathematics that we have, I learned that mathematical Modelling is the set of steps that aims to	Teaching methodology	Differentiated way of passing on mathematics concepts.	Teaching methodology

Part.	Initial perceptions	Approx. conceptions	Final perceptions	Approx. Conceptions
	find a solution.			
P14	I don't remember well, but it's to realize that we use mathematics in everything, a model.	Teaching strategy	It's the way that we work, a model.	It's the way we work, a model.
P16	This is when we bring the student's reality to the classroom for a better comprehension of what is being studied.	Reality – a common point in all conceptions.	It is much broader than I expected, as students can have the chance to choose what they want to work on, which is very important as mathematics can be worked on in many different ways and fits into many themes and activities at the same time.	Teaching methodology
P24	I imagine that it is a model of mathematics, a didactic perhaps.	Teaching methodology.	Mathematical Modelling is a teaching strategy that can contribute to the process of building mathematical knowledge, logical reasoning, language development and autonomy in the face of solving situations	Teaching methodology /as pedagogical practice.

Source: Authorship (2024).

As shown in Table 4, in relation to the participants who had never heard of Mathematical Modelling, and pointed out what they believed it to be, P1 brings an idea of a playful way of working with Mathematics, and in the end understands it as a different way of working. P6, on the other hand, brings an idea of a model (teaching strategy) and in the end understands it as a learning field (teaching methodology) and P24 approaches the idea as a didactic (teaching methodology), and with the course understands it as a pedagogical practice.

In relation to the participants who have already heard of Mathematical Modelling, P4 initially understands it as a teaching strategy and starts to comprehend it as a teaching methodology. P10 and P14 remain with the initial idea of Mathematical Modelling as a teaching strategy. P12 maintains comprehension

as a teaching methodology. And, P16, which initially presents a broad idea, directs to an understanding of Mathematical Modelling as a teaching methodology.

In general, the participants who knew about Mathematical Modelling understood it as a mathematical model. With the actions in the course, some kept their understandings, but expanded them, and others modified them. There were also those who comprehended Mathematical Modelling as a pedagogical (education) practice in Early Childhood Education and how much it provides experiences for children to develop fully from a topic of interest to them.

FINAL CONSIDERATIONS

This article aimed to highlight the understandings of students of a Primary Teacher Training Programme, secondary degree, in relation to Mathematical Modelling before and after the course offered on Mathematical Modelling in Early Childhood Education. Through the questionnaires, the participants expressed their perceptions about what they knew or thought Mathematical Modelling was and, after the course, exposed their new understandings and highlighted some positive and negative points that they considered important when proposing Mathematical Modelling as a pedagogical practice in Early Childhood Education.

It can be seen from the thirty (30) participants that, of the twenty-two (22) who heard about Mathematical Modelling, most understood it as the creation of a model, meeting the understanding of Mathematical Modelling as a teaching strategy. And none of the participants knew about the possibilities of Mathematical Modelling in Early Childhood Education. In this way, it was provided to the participants to know a new approach and understand its stages for the development of Mathematical Modelling as a pedagogical practice in Early Childhood Education. In addition, they reflected on the numerous possibilities and how much this approach contributes to the development of children at this stage of education, that is, the positive points. However, the difficulties that some participants raised, which can be encountered during the development of actions with the children, were also reported (negative points).

In this sense, it can be understood that the future primary teachers, students of the Primary Teacher Training Programme, secondary degree, participants in the research, were able to understand the potentialities of working with Mathematical Modelling as a pedagogical practice in Early Childhood Education, which allows children to be protagonists of their knowledge and develop integrally (affective, cognitive, social and physical).

NOTES

1. To avoid repetitions, throughout the text, when dealing with Mathematical Modelling in Mathematics Education, it will be referred to as Mathematical Modelling.

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