ACTIO: Docência em Ciências
ISSN: 2525-8923

http://periodicos.utfpr.edu.br/actio

Development of a modeling activity in a collaborative training space

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ABSTRACT

This text aims to present the process of developing an activity within the scope of a Collaborative Modeling Training Space. This article is an excerpt from a research project entitled "Training Itineraries and Mathematical Modeling in the New High School: perspectives and practices". Supported by qualitative research methodology, data were obtained through the transcription of an audio recording of one of the meetings with Basic Education teachers, scholarship holders of the aforementioned project. The analysis of school data highlights moments of listening, dialogue, collaboration during the elaboration of a Mathematical Modeling activity, in addition to highlighting nuances of praxis in the Collaborative Space constituted there and the need to disseminate and implement environments like this as training instruments permanent. The data reveal that Modeling, as an approach that starts from everyday situations, can contribute to the reading of the world and the constitution of a liberating, emancipating and transformative education.
KEYWORDS: Mathematics Education; Mathematical Modeling; Listening; Dialogue; Collaboration.

Elaboração de uma atividade de modelagem em um espaço colaborativo de formação

RESUMO

Este artigo é um recorte do projeto de pesquisa intitulado “Itinerários Formativos e Modelagem Matemática no Novo Ensino Médio: perspectivas e práticas” e visa apresentar o processo de elaboração de uma atividade de Modelagem no âmbito de um Espaço Colaborativo de Formação em Modelagem. Com base na metodologia de pesquisa qualitativa, os dados foram obtidos por meio da transcrição de uma gravação em áudio de um encontro com professores da Educação Básica, bolsistas do referido projeto. A análise dos dados evidencia momentos de escuta, diálogo e colaboração durante a elaboração de uma atividade de Modelagem Matemática. Ademais, salienta nuances da práxis no Espaço Colaborativo ali constituído e a necessidade de difundir e implementar ambientes dessa natureza. Como resultados, destacamos que a Modelagem, como abordagem que parte de situações do cotidiano, pode contribuir para leitura do mundo e na constituição de uma educação libertadora, emancipadora e transformadora. Diante disso, entendemos que a abordagem de atividades de Modelagem Matemática nas escolas deve se configurar como um instrumento de formação permanente.

PALAVRAS-CHAVE: Educação Matemática; Modelagem Matemática; Escuta; Diálogo; Colaboração.

INTRODUCTION

Pre-service teacher training is a recurring theme that has received considerable attention in academic productions and educational events. Specifically, in the context of teachers who teach mathematics[1], field in which we are inserted, we perceive a movement in the community of mathematics educators searching for courses that "favor initial teacher training in which knowledge for practice is integrated into the different curricular disciplines" (Omodei & Almeida, 2022. p. 2). As far as we are concerned, practice is intrinsically linked to the various pedagogical approaches that exist for mathematics classes, including Mathematical Modeling [2], an approach we advocate using because of its potential for promoting an emancipatory, humanizing and liberating education, eventually contributing to reading the world through mathematics (Malheiros; Forner & Souza, 2021).

In a move towards convergence between teacher education and modeling, Barbosa (2001, p. 14) is still very contemporary. For the author, teacher education should be based on "two inseparable fronts: modeling itself and the practical knowledge resulting from its approach in the classroom". In this sense, we agree with the author as we believe that the articulation between theory and practice is fundamental in Modeling education.

In the context of our experiences, as Pre-service trainers for educators who teach mathematics, we have sought, in research and practice, different training possibilities that focus on modeling. We are not limited to the theoretical aspects of this approach, but also involve at its core the elaboration, development and reflection on its practice in the classroom (Forner, 2018; Souza, 2022; Forner & Malheiros, 2020; Souza & Malheiros, 2023). In this sense, it is important to note that, inspired by Paulo Freire, Forner (2018) proposed the Collaborative Modeling Pre-Service Training Spaces[3]. Forner and Malheiros (2020) expand on this reflection. Such spaces[4] are perceived by us as environments in which theory and practice permeate the entire training process, and in which collaboration between teachers results in the development of modeling activities that present aspects that would not be thought of individually, i.e., an activity developed by more than one teacher can bring different visions and contemplate different points of view. Thus, those who are immersed in this space are both training other colleagues and being trained by them.

Inspired by these reflections on the potential of the Collaborative Space, considering Modeling can contribute to the teaching of Mathematics and in line with the changes proposed for High School, the objective of our research[5] is to understand the possibilities for students to develop Formative Itineraries (FI) through Modeling. To this end, a Collaborative Space was elaborated with teachers who have been awarded grants by the project, which aims to provide training in Modeling.

The aim of this article is to present the process of developing a modeling activity within the framework of a Collaborative Modeling Pre-service Training Space. The following sections present the characteristics of what we call the Collaborative Modeling Pre-service Training Space; our methodological option, including a presentation of the research context and its founding aspects; and a

discussion regarding an activity that was planned during one of the Collaborative Space meetings. Finally, we share our considerations for future developments.

COLLABORATIVE SPACE FOR PRE-SERVICE TEACHER TRAINING IN MODELING: SOME CHARACTERISTICS

In what regards education, pre-service training is a fundamental issue if its development is to be emancipatory, liberating and transformative. According to Freire (2013), this training must be permanent. In this sense, it "presupposes that the trainer and the trainee understand themselves as inconclusive beings and that this is a human condition that impels man to curiously embark on the search for knowledge of himself and the world" (Saul & Saul, 2016, p. 25). Thus, this position becomes essential for a critical educational practice, based on the awareness that we are constantly developing and on the ontological vocation of individuals to "be more" (Freire, 1992).

In addition to content knowledge, teacher training should provide an understanding of the connections between this knowledge, the different pedagogical approaches and possible difficulties that can be faced in the school context with their students. In this regard, Freire (2013, p. 19) points out that in training there should be no imposition or transmission of knowledge from the trainer, arguing that:

> If, in the experience of my formation, which must be permanent, I begin by accepting that the formator is the subject in relation to whom I consider myself the object, that he is the subject who forms me and I, the object formed by him, consider myself as a patient who receives the knowledge - content - accumulated by the subject who knows and which is transferred to me. In this way of understanding and experiencing the training process, I, the object now, will have the possibility tomorrow of becoming the false subject of the "training" of the future object of my training act.

Thus, teacher training should not be seen merely as a means of providing teachers with certifications and mere updates in scientific and didactic knowledge. But, above all, training that considers the educator as a subject, through situations that enable them to critically reflect on the social context in which they operate and the possibilities for it (Freire, 2013, 2015a). In this sense, to promote permanent training in Mathematical Modeling, based on the Freire's legacy, Forner (2018) proposed the creation of Collaborative Pre-service Training Spaces in Modeling. He considers that such a Space

> […] can be a fruitful place for training, in the sense that the exchange of experiences between teachers, combined with theoretical perspectives from reading texts related to the topics under discussion, can give new meaning to the teacher's practice, and this can happen through the teacher's awareness, in the sense of reflecting on what they want for their students (Forner, 2018, p. 74).

In this Space, as opposed to the traditional approach of unilaterally imposing the knowledge presented by the trainer, educators are invited to contribute their experiences, perspectives and solutions to the challenges of educational practice. For us, teacher training in Mathematical Modeling requires an approach that values the active listening of educators who work directly on the "school ground". We believe that this approach is essential for fostering authentic and transformative educational relationships (Malheiros et al., 2021).

For this to happen, we agree with Freire (2011) that listening is a central element in the educational process. He points out that "it is not by speaking to others, from the top down, above all, as if we were the bearers of the truth to be transmitted to others, that we learn to listen, but it is by *listening* that we learn to *speak to them*" (Freire, 2011, p. 88, emphasis added). According to the author, listening is not just hearing sounds, but understanding the true meaning of words, a deep, empathetic and loving understanding of the experiences and perspectives expressed by others (Freire, 2011). Freire also points out that listening means being open to the other person's speech and their differences, which does not mean that

> listening requires those who really listen to reduce themselves to the other who is speaking. That wouldn't be listening, but self-annulment. True listening in no way diminishes my ability to exercise the right to disagree, to oppose, to position myself. On the contrary, it is by listening well that I prepare myself to better position myself or better situate myself from the point of view of ideas. As a subject who is open to the discourse of others, without prejudice, the good listener speaks and says his or her position with ease. Precisely because they listen, their dissenting speech, being affirmative because they listen, is never authoritarian (Freire, 2011, p. 93).

Thus, we believe that listening is fundamental throughout teacher training in Modeling to provide a critical reflection on the context that educators experience on a daily basis, so as to enable them to understand their difficulties and interests, as well as those of their students. Based on this understanding, we see that it is possible to work together towards a liberating and transformative education.

However, listening is not merely an act of passive receptivity, but represents a gesture of respect for the other, enabling the emergence of an authentic and enriching dialog (Freire, 2011, 2013). For the author, dialogue is the meeting of subjects who join in solidarity to reflect and act on the world to be transformed, not reduced to the act of depositing ideas from one subject to another, nor the simple exchange of ideas (Freire, 2011, 2013). He also points out that dialog

> must be understood as something that is part of the very historical nature of human beings. It is part of our historical progress, the path to becoming human beings! Is that clear? In other words, dialog is a kind of necessary posture, as human beings become more and more critically communicative beings. Dialogue is the moment when humans meet to reflect on their reality as they make and re-make it (Freire & Shor, 2013, p. 120).

Thus, dialog is an act in which subjects' experiences, perspectives and ideas intertwine to form a deep and broad understanding of reality (Freire, 2011, 2013). To this end, dialog presupposes overcoming a vertical relationship, enabling a horizontal interaction in which everyone is an active subject in the educational process. Furthermore, it implies a break with authoritarianism and "a fundamental respect for the subjects engaged in it" (Freire, 1992, p. 60).

As far as we are concerned, dialogue is a two-way road in which trainers and educators recognize each other as active subjects in the educational process, enabling acts of creation. We understand that the reciprocity present in dialogue can enable the construction of an environment conducive to the collective creation of knowledge and the development of a critical and reflective conscience, i.e., it is fundamental for the formation of critical subjects who transform their reality. Furthermore, we can see that it is fundamental for training teachers who are committed to an emancipatory, liberating and transformative educational practice.

Furthermore, for Freire (1992), dialogue is an ethical and political attitude that recognizes the voice and experience of others. Through the dialog between trainers and educators in the Collaborative Spaces, there is an incentive to question one's own practice, providing a posture of constant search for improvement, respecting the contexts of the participants.

For us, dialog throughout pre-service teacher training is an essential element in the construction of knowledge, it contributes to developing reflective, critical educators who are committed to social transformation. In this sense, in the Spaces, participants can work together to develop modeling activities according to their realities and educational experiences. In these spaces, there is a movement of listening, dialog and collaboration between the participants, creating a movement of articulation between theory and practice.

In addition to the movement of listening and dialog in teacher training, we understand that praxis is a fundamental element, understood as the dialectical relationship between theory and practice, action and reflection. This is because education is a political act that implies critical reflection on the world and the transformation of reality (Freire, 2011, 2013). For Freire (2019, p. 112), “no one is born an educator or marked to be an educator. We become educators, we train as educators permanently, in practice and in reflection on practice”. Thus, teacher training cannot be reduced to the simple memorization of content by the educator, for later reproduction in their practice, but must be a process of raising awareness and training subjects to become active agents in building a just and equal society (Freire, 2011).

Praxis can be understood as “a dialectical process of action and reflection, contemplating different classroom realities” (Malheiros et al., 2021, p. 1971). According to Freire (2011), praxis is both a process of reflection and action, a fruitful nourishment between theory and practice, as well as an ontological stance towards the world, in addition to being an act of unveiling and constant reflection on the teacher's educational practice.

According to Freire, praxis is a dialectical process of intervention in the world, in which subjects are active in transforming their reality through reflection and criticism (Freire, 2011). Praxis is an inseparable unity, “it is the reflection and action of men on the world in order to transform it” (Freire, 2011, p. 40). For us, it is fundamental in teacher training as a means of understanding their surroundings, questioning and reflecting on their practices, stimulating the search for creative alternatives to challenges in their practice, as well as enabling them to act consciously and with a commitment to social transformation. This is because, as Freire (2013, p. 31) points out, “it is by critically thinking about today's or yesterday's practice that the next practice can be improved”.

We believe that one of the principles of Collaborative Spaces is to value participation, discussion and reflection on modeling through praxis. In other words, the purpose of these environments is for participants to engage in dialog about modeling and collaboratively design and develop activities in their educational practice, as well as reflecting on it, in a dialectical movement aimed at transformation. This action can make it possible to confront ideas and give new meaning to knowledge, based on what they experience in their realities and the discussions that come from the process of listening and dialog between the

participants. In addition, the Collaborative Spaces can enable critical reflections on modeling in the classroom, in which participants are encouraged to question and analyze its possibilities and challenges in their teaching practices, seeking paths towards a transformative and liberating education.

Based on the theoretical ideas discussed here, a Collaborative Modeling Training Space was set up in the context of the project "Training Itineraries and Mathematical Modeling in the New High School: perspectives and practices", with the aim of providing reflections and practices about modeling in FI classrooms of the New High School. In this way, after explaining the methodological approaches, we will describe and analyze the development of an activity in this context.

METHODOLOGY

The objective of this article is to *present the process of developing an activity within a Collaborative Modeling Pre-Service Training Space*, i.e., it aims to understand a particular phenomenon, one of the main characteristics of the qualitative paradigm, and is not concerned with generalizations, principles or laws (Alves-Mazzotti, 2001; Creswell, 2014).

A qualitative approach was chosen for the development of the research. We relied on the voices of the participants, in this case, the teachers who participated in the project and tried to understand how people in a given context think and act. To produce the data, we used audio recordings, which were transcribed, and entries in a field diary. The idea of using audio recording was to reduce the aspects that could interfere with the reliability of the data observed (Belei et al., 2008).

The data presented in this study was collected at a meeting in 2024, in a city in the countryside of São Paulo, in a school linked to the Mogi Mirim Board of Education. The meeting was attended by teachers, who are collaborators in the research, and scholarship holders from the project. It should be noted that these participants come from two participating schools, five of whom teach FI related to the exact sciences and one teaching supervisor. Modeling was gradually introduced to them by the researchers. To this end, weekly meetings were held during the first semester of 2024, involving discussions and the development of activities related to Modeling.

It is important to note that the teachers, except for the teaching supervisor, had little knowledge of modeling. One of the teachers had heard of it briefly in her initial training, and the others did not know what it was when the Space meetings began. Thus, we chose to discuss modeling, at first with examples from the literature, as well as with the development of activities that could be developed in the classroom, in exercises inspired by the concept of pedagogical imagination (Skovsmose, 2015; Lima, 2022), which can be understood as a way of thinking about possibilities based on a given situation, in this case, the design and development of modeling activities for work in the FIs. With this dynamic, our idea was to articulate theory and practice, in a praxis movement, so that while the teachers were getting to know the possibilities of working with Modeling, they could plan ways for it to be enhanced in their classrooms.

Throughout the meetings, we noticed some difficulty on the part of the teachers in drawing up the problem to be worked on in the modeling activity. For

them, this would be the most difficult part of planning, corroborating Setti, Waideman and Vertuan's statement (2021, p. 960):

> [...] among the actions undertaken by subjects when developing Mathematical Modeling (MM) activities from the perspective of Mathematics Education, is that of elaborating "the modeling problem", i.e., a problem that covers aspects of a situation that, for some reason, they are interested in answering. This action, sometimes carried out by the student, sometimes by the teacher, can become a hindrance to the dissemination of modeling, especially in basic education classrooms, where developing problems, not only in the context of modeling, has not been a recurring action.

Faced with this challenge, we thought we would explore the development of an activity with a view to overcoming it. An article[6] from a magazine was presented and possibilities for creating a modeling activity inspired by it were discussed collaboratively. The activity was then collaboratively designed based on what was proposed in the article. These discussions and the activity are analyzed hereinafter.

Once this data had been produced, we carried out the analysis, based on the phases described by Yin (2016). Initially, we transcribed the audio from the meeting in which the activity was developed. Next, the data was systematized and organized based on the ideas presented and debated here and that converged with the objective of this article. The data was then reorganized based on certain emerging patterns and our research interests, and then interpreted.

DIALOG, LISTENING AND COLLABORATION IN THE DEVELOPMENT OF A MODELING ACTIVITY

The activity described in this study was proposed in the Collaborative Space composed of teachers who had been awarded grants by the project. They were in the process of getting acquainted with modeling and having difficulties with the elaboration of the problem, as mentioned hereinbefore.

With this in mind, we proposed the collaborative development of an activity based on a theme related to the school context. It began with an article entitled "Clinging to the Screen". This approach was taken since in modeling activities, the choice of topic is based on situations from everyday life and/or of interest to students (Malheiros, 2012; Forner, 2018; Silva & Silva, 2021; Souza, 2022). To select the topic, we considered the dialogues and discussions held during previous meetings with the teachers, which also aimed to provide an insight into the context in which they teach.

During the meeting, the proposal was presented as follows:

> *Trainer: We're going to read this here today [the magazine article][7] and think, based on the information here, the infographics and so on, how we could take a modeling activity into the classroom. So, thinking again, theme, problem, doing that exercise...*

In the trainer's speech, the idea of resuming actions for the elaboration of a Modeling activity is clear, considering stages that should be considered by teachers (Almeida et al., 2013; Silva & Silva, 2021). These phases, far from establishing a method, aim to have a conducive potential in the constitution of a dialogic environment between the participants of the Collaborative Space.

Thus, immersed in this dialogical environment, the participating teachers read the article and were invited to share their opinions so that we could talk about it and about possible themes for developing modeling activities. To prompt the dialog, the trainer proposed the following question:

> Trainer: What kind of themes could we address after reading this? I think we could address more than one. Apart from the issue of excessive use of screens, which I think is the central theme. But what other topics, based on what we've read, do you think we could think about?

In the subsequent dialogues, everyday themes were discussed, such as the time dedicated to reading books as opposed to the time dedicated to screens, the increased use of social networks due to hectic lifestyles, the lack of leisure and the role of social networks in meeting this need, financial status, debt and others. These topics were related to the participants' experiences, which we consider to be natural when we think about modeling and its potential. (Malheiros et al., 2021).

We chose contexts related to everyday life because, inspired by Freire (2019), we believe that this creates an environment conducive to problematization. For Freire, problematizing is anchored in "exercising a critical analysis of the problem reality" (Freire, 2019, p. 229). Thus, our intention in establishing dialogue, based on everyday life, is to analyze situations, think of strategies and ways of building knowledge with students, in a collaborative way. With this focus, problematization becomes extremely important and we understand that starting from situations that are completely decontextualized may not be as successful as those that are closer to the students' context.

Following this proposal, after half an hour of discussion on the different topics, the trainer continued the dialog with the participants of the Space and considering the objective of the meeting:

> Trainer: So, I think we've come up with a lot of possible topics here that could be tackled using this report. Shall we choose one, so we can try to put together an activity? We can try... Which one?

The meeting was based on dialog, giving everyone the opportunity to speak up, put themselves forward and present their opinion, a characteristic of a dialogic and collaborative environment emanating from the Collaborative Space. Thus, instigated by the trainer's speech, teacher Israel sets out his perspectives and, based on the infographic presented in the article. He poses a provocative question, as we can see in the excerpt below, which is complemented by Dante and Fausto:

> *Israel[8]: So I've been wondering about time. What's the excuse everyone gives? Not to go to the gym, not to eat well, not to travel...*
> Dante: Having no time.
> Israel: Everyone gives that excuse. But is it true? No, it's everyone's priority. Everyone has 24 hours. Now... Ah, I don't have an hour to go to the gym. But then, in the table [pointing to an infographic in the report], I'm saying that everyone has 3.5 hours. What do you do in those 3.5 hours, then, statistically? With 3 and a half hours, you could be an hour at the gym, an hour at...
> Fausto: Traveling to São Paulo.
> Dante: Oh, I cook so well, I'm not even in the kitchen...

From the excerpt, we can see that there is a dialog between the teachers, established from a dialogic environment (Freire, 2011, 2013), in which they not only speak, but also listen attentively to each other. In this sense, we reiterate that listening "means the permanent availability on the part of the listening subject to open up to the other's speech, the other's gesture, the other's differences" (Freire, 2011, p. 93).

In this process of active listening, we see a willingness to understand each other's perspectives and the collaborative construction of knowledge, as they interact with each other, contributing to reflection and critical analysis of the situation (Freire, 2011). The teachers' comments focus on the importance of critical awareness and reflection on the use of time, raised by the article explored at the meeting.

Continuing the dialog, the trainer leads the discussions and highlights the main theme: the time spent using cell phones, which Fausto confirms. Continuing with the dialog, the trainer listened to and instigated the participants about the topic, as we can see in the following excerpt:

> Trainer: So, the question of the topic, you're talking about the issue of time spent on cell phones.
> Fausto: Yes.
> Trainer: Would that be the theme? Is that it?
> Fausto: I like the theme.
> Trainer: What do you think?
> Rafael: I think it's the most popular, right?
> *Israel: I think, (...) Everyone spends three and a half hours on their cell phones. So the cell phone isn't the problem. But what is it consuming? With 3 and a half hours on his cell phone, he could be doing a postgraduate course. He's taking a degree; he could be learning gastronomy. He's reading a book on his cell phone, an audiobook. So what? 3 empty hours equals empty calories.*

The interaction between the participants throughout the meeting shows an open and participatory dialog (Freire, 2011), in which they collaboratively tried to determine the theme for the Mathematical Modeling activity. It also shows that the relationship is not limited to agreeing with the other, but to reflecting on the context and experiences of the participants. There is a movement to bring the situation under investigation ever closer to the students' context, in the sense of problematizing it.

After the trainer's questioning and the dialogue between the teachers, the group decided that the central theme for the modeling activity would be the time spent on social networks using cell phones. Then, to instigate the teachers, the trainer asked:

> Trainer: So, I think that's fine. So, we've come to look at the time spent on cell phones, on social networks. Yes, on social media. And what's the problem? That we want to present. [laughs because of the difficulty in coming up with a problem]
> Dante: We were already making fun of the problem.
> Bernardo: We were talking. I said, tomorrow is problem day.
> Trainer: It is problem day. I came here to... To give you a problem. What problem could we present? To the students.

The question gave another direction to the dialogue, which until then had been focused on the choice of the theme to be problematized, which is expected

for the development of a Mathematical Modeling activity. This is because modeling activities have a multiple character, in other words, there is a diversity of questions that can emerge from the theme (Souza, 2022; Barbosa, 2001).

Thus, in the above excerpt, we see two nuances that deserve attention: the action of the trainer in the Training Space to coordinate the dialogue, never imposing or influencing (Freire, 2015b), but instigating everyone to speak up and for listening to prevail, characterizing truly collaborative work; and the issue related to the difficulty in obtaining a problem (Setti et al., 2021).

This aspect of developing a problem is emblematic since our education is based on answers rather than questions. This statement is in line with what educator Antonio Faundez said in a dialog with Paulo Freire:

> what teachers should teach - because they should know it themselves - is, first and foremost, to teach how to ask questions. Because the beginning of knowledge, I repeat, is to ask. And it's only from questions that one should go in search of answers, and not the other way around [...] (Freire; Faundez, 1985, p.28).

In this sense, we understand that you cannot teach what you do not know (Freire, 2020) and, therefore, learning to ask questions is important for teachers. Still about complexity, continuing the dialog, there is confusion on the part of the teachers regarding the problem in a Modeling activity, as demonstrated in the following dialog:

> Dante: Here's a suggestion of how you could use the time you spend on your cell phone.
> Fausto: What's wrong with that?
> Trainer: No, no. The problem that we would take away, thinking about the activity (...) We came up with the theme for our activity, is the time spent on cell phones and social networks. That's going to be the theme, okay? Then we have to think about... familiarizing ourselves with the theme, but before that I want to think about the problem. Which was the most...
> Bernardo: Intricate.
> Trainer: Intricate last time.

In other words, they understood 'problem' as something that can arise from the excessive use of social networks on cell phones. Listening attentively to what the teachers were saying, the trainer realized the mistake. Talking to the teachers, she clarified that she was referring to the problem in the modeling activity. The trainer was referring to the problem that should be the guiding thread of the activity, aimed at understanding a situation related to the chosen theme and which should be explored throughout the math classes (Souza, 2022). From the excerpts, we can see that listening and dialogue between the participants in the Space were essential for clarifying the teachers and, as a result, knowledge about aspects of a modeling activity was developed in a collaborative way.

When asked by the trainer what the problem would be for the Mathematical Modeling activity, Israel argued about the consequences of spending too much time on his cell phone, which led to the following dialogue:

> Israel: If you're at home and you're spending three and a half hours on your cell phone, you're giving up three and a half hours doing something in your house. Cleaning it, washing dishes...
> Trainer: Yes, but are we going to include this information in the problem?

> Israel: Let's think [silence] Is the problem time? Is the problem the lack of time? I don't know, a title. The problem is lack of time?
> Trainer: But is that the title or the problem?
> Israel: Let's decide. [silence] I see it as a problem. Because the class says...
> Trainer: But how do we get to the student and arrive at this problem: The problem is lack of time?

The listening and dialog between the participants show collaboration between the subjects to clarify and define the problem for the Modeling activity, corroborating Freire (2013, p. 166) who assumes that "dialog, which is always communication, is the foundation of collaboration, which, in our view, presents relevant aspects for the teacher training process, because when a participant in the Collaborative Space shows their point of view, they can contribute to other nuances on a given topic. This is in line with the ideas of Alrø and Skovsmose (2006, p. 127) when they state that "the greatest gain a teacher can have is that by observing, reflecting and expressing their world view in a cooperative process, they can change and come to know things in a new way".".

Under this approach, the participants discussed the possibility of tackling problems related to the use of cell phones in contrast to the time spent on them, on social networks. They also articulated their points of view in relation to what they observed the students doing:

> Fausto: We have students who even give up sleep to be on social media.
> Israel: Wow, playing games? Play.
> Fausto: It's just that there's more social media here, right? But there are students who stop sleeping because of their cell phones. They're on their cell phones, on WhatsApp until late, on video calls, on Instagram. So I think this idea is great. What could I do with the time I spend on social media??

The dialog then broadens the discussions about what could be done, thinking about the development of a problem for the modeling activity. The trainer summarizes the discussions held so far by proposing two problems for the participants to consider, pointing out that we often overcomplicate matters:

> Trainer: I've put two problems here [pointing to her notes]. So you can see that it's not complicated. And I think that's our problem. This really is our problem. We complicate things. We've come to the conclusion that the problem is the social network. Not necessarily the computer, the cell phone. Because I can be on the computer too, right? I can be on the tablet. The problem is the social network. That's what we're talking about here. The report even talks about the screen. Clinging to the screen. We took it to the cell phone. But it's the screen, right? So we could ask the students the following: How much time do you spend on social networks? And what could you do with that time?
> Israel: That's the problem? Guys, it seems like such a difficult thing.
> Trainer: That's a problem. It's not the problem. It's a possibility. It's just to show you that it's not that complicated. We complicate things...

Israel's account shows astonishment at the simplicity of the problematizations - summarized by the trainer, based on the dialogue that had taken place up to that point - that could be addressed through the article with the students. For us, as mentioned earlier, this may be related to an education based on answers rather than questions, in which students and teachers are involved (Freire & Faundez, 1985).

In our view, through dialog, listening and collaboration, the Collaborative Space can be seen as a means of overcoming some of these difficulties, with the aim of understanding how Modeling can be developed in the context of classrooms and, in the case of this Space, the FIs.

Following the development of the activity, possibilities for familiarization with the topic were discussed (Almeida et al.2013) and the way it could be developed was also discussed, in an exercise of pedagogical imagination (Skovsmose, 2015; Lima, 2022):

> Rafael: He said that there's a graph that grows over time. Have you ever seen this graph? I find it very interesting. The countries as the years go by. Have you seen it? It goes on and on. Who uses it the most and so on.
> Fausto: There's a lot of that on YouTube, Tik Tok.
> Dante: But there must be a lot of videos on YouTube that talk about this. How long it takes, I don't know, for a tree to grow. For you to plant a tree and for it to grow. How long it takes to... And do you know how to make time comparisons? There are a lot of videos about that. What we can do with our lives. Look at things like this.
> Trainer: Let me see if I... I think it's the growth of social networks. [...] This video here that you mentioned? [showing the video to the teachers]
> Fausto: Yeah, that's the one.
> Dante: That's cool.
> Trainer: Show this to the student. It shows the year. The growth [of social networks].
> Fausto: When it emerged and its growth
> Trainer: This one will do up to 2021.

In the above excerpt, we see a movement of listening and dialog between the participants, in which there is an exchange of ideas and knowledge about the videos they know and which could contribute to the development of the activity. The participants in the Space look for ways to bring the students into contact with the problem situation, in a movement of pedagogical imagination (Skovsmose, 2015; Lima, 2022). The dialog between the Space participants continues. Another question arises and the trainer summarizes her thoughts:

> Trainer: Oh. Come on then. We're going to look at these three problems. How much time do you spend on social networks? What could you do with that time? Why do you spend that time on social media? So those would be the three problems. Then we thought that familiarization could be either with a video or using infographics, or both. Everything will depend on the time we have in the classroom. Because we know that the time factor is a determining factor for us to be able to effectively decide what we're going to do in the classroom or not. After this familiarization, we'll present the questions. So, what's the next step?
> Israel: Collecting the data.

One participant mentions that the cell phone has an app that shows how much time the user has spent on the different apps. As a result, everyone starts checking their cell phones and exploring the time spent on social networks, in specific apps for each brand9. The dialog between the participants highlights suggestions, considering their potential development with primary school students (Almeida et al., 2013). In addition, there is a discussion about the classroom context for Space, where students may or may not have access to cell phones, as well as the different ways of accessing social networks.

> Israel: You can do this survey with the students. Go there, check it out.
> Trainer: So you can do the survey with the students.
> Israel: Go in there, check it out...

> Trainer: Then they'll have... No, then they'll have... That's what I'm... I'm trying to say.
> Rafael: Does everyone have a cell phone?
> Israel: 95%
> Rafael: No, I'm thinking like this, because... You could make a comparison and they don't, right?
> Israel: The cell phone?
> Dante: What does he do with his time? Because he doesn't have a cell phone.
> Israel: Wow, but it's rare that he doesn't have one. Yeah. But then...
> Trainer: If he accesses social media... Because he can, at some point, he can access it... Through... Through... On their home computer. But it's more data...
> Rafael: He's got something, right? Because he could talk about how much the difference is for those who have a cell phone. He uses the social network to have a cell phone.
> Fausto: Yes. It's going to be less, right? Probably.
> Dante: Or not.

Dialogue between the participants can be seen as a way of working collaboratively, in which different perspectives and points of view are put forward. For us, this dynamic can, in a way, contribute to teachers' conduct in the classroom, in the sense that there is a confrontation of opinions and, thus, a re-signification of concepts and practices. Their remarks are summarized by the trainer:

> Trainer: Okay, let's go. So we... How could we do it? You've done the familiarization. Then we'd use the cell phone app... And then we'd do a daily check... For a week. Although it [the app] gives me weekly. But for him to take notes every day, at the same time... So, for example... Every day, at 9:30pm, he'd check... Right? Or every day, at a specific time. It's starting today. Thursday. Then tomorrow morning, he'll see how much time he used on Thursday. Then Saturday, Friday... Until Wednesday of the following week. So he'll have seven days. And then he'll be able to make projections... Okay? Thinking about the month, the year... And then I arrive at the mathematical model... In this case... Which is thinking about life expectancy.
> Israel: 75.
> *Trainer: So, if I'm 75... Huh? Thinking that the person starts using the social network... And then we have to assume... At the age of 10. So that's 65 years of using social media. If Mara lost 27 hours on social media in a week... 27 hours on social media... How much will she miss out on in her life??*

The dialog continued with the teachers presenting possibilities for producing data, which could be done in a week or a few months, depending on the objective of the activity, in a movement of pedagogical imagination (Skovsmose, 2015; Lima, 2022). They also questioned whether all the time we spend on social media is, in fact, bad. They concluded that the modeling activity can contribute to reflections that go beyond mathematics.

> Trainer: The way we developed it, we managed to get the students to arrive at the model. And then they realize, right? And that's how we got to the first part of the activity, which is the, shall we say, mathematical part of it, which is the question, how much time do you spend on social networks? The second part, which is what you could do with that time, right? I think it's a more philosophical reflection. And then we can ask them to think, right? Daily what I could do with this time, weekly what I could do with this time, monthly what I could do with this time.
> Dante: And producing videos, right? I like making videos with them.
> Trainer: Yeah, so, I think that's really cool. And I think they give us interesting reflections, thinking beyond. In other words, mathematics helps us to understand the phenomena that happen in our lives. And I think that's the purpose of modeling, right? And then the other question Dante: What could you do with your time?
> Trainer: And why do you spend that time on social media?
> Dante: Why? Are you running away from something?

> Trainer: I think these two questions are less mathematical and much more reflective of mathematics. Because I'm sure that many of these students have no idea how much time they spend on social media. We don't. [...] So that's it, right? And then having this control, I think it's important even for us... It's that story, isn't it? I think it's important for us to decide if that's what we want or don't want. But if I don't have this knowledge, how do I decide? Because... Who said it? Fausto said. Ah, I like to watch TikTok videos with exercise solutions. That's not wasting time.

These reflections, even if they have not been effectively taken into the classroom, present elements of Freirean praxis. For Paulo Freire, according to Rossato (2010, p. 325), this "can be understood as the close relationship that is established between a way of interpreting reality and life and the consequent practice that results from this understanding, leading to transformative action".".

In this sense, we noticed an effort on the part of the participants in the Training Space to bring in aspects of the classrooms in which they teach and elements of their daily lives while they were thinking about designing the activity. This indicates a movement that seeks transformation, both in their practice as teachers and in the lives of their students.

Furthermore, the excerpts presented here had as their motto the development of a modeling activity to be implemented at the IFs in the New High School, and this movement took place in a Collaborative Space, which we defend as a collective modeling training environment. For us, working with modeling can contribute to reading the world through mathematics, in search of a transformative education (Malheiros; Forner; Souza, 2021) and training teachers to work with it in the classroom is very important if it is to effectively reach classrooms.

FINAL CONSIDERATIONS

In this article, we are trying to bring elements to the discussion of the process of developing an activity within the framework of a Collaborative Modeling Training Space. We believe that this training practice would be a path towards permanent training, as it starts from the teacher's experience in the classroom, combining theory and practice, towards praxis.

Throughout the text, we provided elements that show that theory and practice go hand in hand through collaborative work, in which dialog and listening prevail. When designing an activity collaboratively, elements such as dialog, listening and collaboration were essential, in a process in which we are formed and are being formed.

This process, far from being static, is in constant movement, which is why it makes sense for it to be permanent, rather than initial or ongoing. In it, convergent or divergent points of view are put forward and, based on them, negotiations are carried out that tend to give new meanings. This contrasts with the one-sided view that prevails in training centers.

We draw attention to active listening, a characteristic of this Collaborative Space, which becomes relevant because it allows us to learn from each other, to delve deeper into experiences and to understand, even if not with all the complexity it deserves, the harsh reality of the classroom. Through listening, dialog

is established and this is a way of making room for everyone to be a participant and not a supporting player in the educational process.

As a final note, we feel that it is urgent and necessary for training courses along the lines of Collaborative Spaces to be implemented, because collaborative work, permeated by dialog and listening, tends to take precedence over individual work, as well as allowing different visions to be presented, discussed and given new meaning. As far as we are concerned, modeling is a way of bringing out the social role of mathematics in the classroom, as well as enabling a liberating, emancipating and transformative education. To do this, teachers need to know about it and realize the possibility of working with it in the classroom, and we understand that Collaborative Training Spaces in Modeling can be a way of achieving this.

ACKNOWLEDGMENTS

We would like to thank FAPESP for funding the development of this article, which is part of a Research Project, Process No. 2022/05760-2.

NOTES

1. The term “teachers who teach mathematics” was used to include teachers who work in all stages of basic education, as well as those who do not have a degree in mathematics, but work with this curricular component in schools.

2. The term “Modeling” was used interchangeably with “Mathematical Modeling” to avoid repetition.

3. The terms Space and Collaborative Spaces were used to designate Collaborative Modeling Training Spaces to avoid repetition.

4. We explain the characteristics of Collaborative Spaces in detail in the next section.

5. The research is entitled “Formative Itineraries and Mathematical Modeling in the New High School: perspectives and practices”, funded by FAPESP, within the scope of the Basic Education Research Program (ProEduca), carried out in partnership with the São Paulo State Department of Education (SEDUC). Process No. 2022/05760-2.

6. Article “Agarrados à Tela”, published in the “Igualdades” section of Revista Piauí, in February 2024. Available at https://piaui.folha.uol.com.br/igualdades-redes-sociais-vicio-celular/, accessed on June 30, 2024.

7. Clarifications have been added in brackets to facilitate full understanding of the excerpts presented.

8. We used fictitious names for the participants to preserve their identities.

9. In the case of Samsung cell phones, for example, the Digital Wellbeing and Parental Controls application, available in the device's settings, provides the user's data

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Received: July. 17st. 2024
Approved: Nov. 12th,. 2024
DOI: https://doi.org/10.3895/actio.v9n3.18849

How to cite:
Malheiros, A. P. S.; Souza, L. B. & Forner, R. (2024). Development of a modeling activity in a collaborative training space. ACTIO, 9(3), 1-20. https://doi.org/10.3895/actio.v9n3.18849

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