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The specialized knowledge of the (future) teacher manifested in a mathematical modeling activity

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ABSTRACT

Research has revealed opportunities that Mathematical Modeling offers for learning, skill Research has revealed opportunities that Mathematical Modeling offers for learning, skill development, and the manifestation of teacher knowledge. This study aims to investigate the teacher's specialized knowledge manifested in a mathematical modeling activity. Following a qualitative approach, empirical research results from analyzing written records, audio transcriptions, and the researcher's notes. The subjects of data collection are three students from a Mathematics degree course, scholarship holders of an extension project, in the context of developing a mathematical modeling activity. From the investigation carried out, we infer that group work, the real context, simplification, the mathematical resolution of a real problem, and the validation of the answer obtained are characteristics of the modeling that most contributed to the manifestation of the teacher's specialized knowledge, both the mathematical and the pedagogical content. Furthermore, it is worth considering that, sometimes the modeling activity requires specific knowledge, and sometimes the knowledge infers consequences for the activity.

KEYWORDS: Mathematics teaching; learning through modeling; initial teacher training; systematization of knowledge.

O conhecimento especializado do (futuro) professor manifestado em uma atividade de modelagem matemática

RESUMO

Pesquisas têm revelado oportunidades que a modelagem matemática oferece para a aprendizagem, o desenvolvimento de competências e a manifestação de conhecimentos do professor. Neste estudo temos como objetivo investigar o conhecimento especializado do professor manifestado em uma atividade de modelagem matemática. Seguindo uma abordagem qualitativa, a pesquisa empírica resulta da análise de registros escritos, transcrição de áudios e anotações das pesquisadoras. Os sujeitos da coleta de dados são três estudantes de um curso de Licenciatura em Matemática, bolsistas de um projeto de extensão, no contexto do desenvolvimento de uma atividade de modelagem matemática. A partir da investigação realizada, inferimos que o trabalho em grupo, o contexto real, a simplificação, a resolução matemática de um problema real e a validação da resposta obtida são características da modelagem matemática que mais contribuíram para a manifestação de conhecimentos especializados do professor, tanto o matemático quanto o pedagógico do conteúdo. Ainda, é válido considerar que, ora a atividade de modelagem matemática requer conhecimentos específicos, ora os conhecimentos inferem desdobramentos para a atividade.

PALAVRAS-CHAVE: Ensino de matemática; aprendizagem por modelagem matemática; formação inicial do professor; sistematização do conhecimento.

INTRODUTION

The literature has pointed out that using mathematical modeling as a teaching and learning strategy is significant and that this strategy has gained importance in the curricula of various countries. Characterized essentially by the use of mathematics to solve real problems, mathematical modeling is a process that allows a movement from the real world to the mathematical world and back to the real world.

This movement implies recognizing, as Almeida, Silva, and Vertuan (2012) and Castro and Veronez (2017) suggest, that mathematical modeling involves the search for a solution to a problem, not essentially mathematical, which may come from a suggestion by the students or the teacher, "a set of procedures, which enables involvement with mathematical structures and concepts and a conscious analysis of the answer obtained for such a problem, which may or may not be recognized as a solution" (Castro & Veronez, 2017, p. 96). In this process, teachers and students have the opportunity to mathematize the situation under study and analyse the solutions, in an attempt to bring mathematics closer to reality.

From this perspective, we recognize, like Almeida et al. (2013), that mathematical modeling requires dynamic and active behavior from teachers and students, as there is a need for both to seek to relate scientific knowledge with school knowledge, taking into account the characteristics of the situation under study, whether they are social, economic, environmental or from other areas.

Therefore, working with mathematical modeling presupposes different actions by teachers and students, since it suggests the study of problems different from those normally found in textbooks (Dias & Almeida, 2004). In this sense, the development of mathematical modeling activities requires that "teachers are prepared to play an active role in organizing, implementing and evaluating mathematical modeling activities with their students" (Dias & Almeida, 2004, p. 6). On the other hand, the insertion of mathematical modeling in the school context can lead teachers to experience, construct, or manifest different knowledge (Ferri, 2018; Wess et al., 2021; Bisognin & Bisognin, 2021).

Villa-Ochoa et al. (2021) suggest the need to provide learning spaces that enable teachers to address possible limitations in their professional knowledge about the nature of mathematical modeling and thus be able to effectively integrate it into the teaching and learning process. The authors emphasize that the development of successful mathematical modeling activities in school practices requires complex pedagogical skills and competencies, given that activities of this nature constitute open and challenging learning environments. In addition, they stress the importance not only of a solid command of mathematical and contextual knowledge but also of familiarity with the specific mathematical modeling activities selected.

Studies on aspects related to the initial and continuing training of teachers in mathematical modeling have already been carried out by researchers such as Almeida and Dias (2004), however, the number of publications that discuss specialized knowledge from the perspective of MTSK - an acronym whose meaning is Mathematics Teachers' Specialized Knowledge and translated into Portuguese as

Conhecimento Especializado do Professor de Matemática - proposed by Carrillo-Yañez et al. (2018), the focus of interest of our investigation, is still limited.

Like Padilla-Escorcia et al. (2023), we believe that it is possible and necessary to integrate aspects of mathematical modeling with the categories of knowledge proposed in the MTSK. According to these authors, such integration is possible because the MTSK model delves into disciplinary aspects of mathematics and didactic-pedagogical aspects which, in turn, are also present when you want to develop mathematical modeling processes in the classroom. Furthermore, this integration is necessary to provide feedback to the MTSK model itself (Padilla-Escorcia et al., 2023).

Furthermore, our interest is justified because we believe that an analysis of teachers' knowledge and the relationship between this knowledge and their training is extremely relevant. In this work, we seek to focus attention precisely in this sense: to discuss teachers' specialized knowledge, from the perspective of the MTSK, as they plan the development of mathematical modeling activities within a training context. With this in mind, this study aimed to investigate teachers' specialized knowledge as manifested in a mathematical modeling activity.

MATHEMATICAL MODELING: A LOOK AT TEACHERS' KNOWLEDGE

Mathematical modeling can take on different conceptions and different educational approaches (Kaiser; Sriraman, 2006), so the implementation of activities of this nature in the classroom can have different objectives, whether for the teacher or the students, according to the different levels of schooling. However, Ferri (2018) states that there is a consensus that mathematical modeling can be described as an activity whose essential characteristic lies in the fact that it involves the transition between reality and mathematics.

In the context of the project as well as this investigation, we assume, like Almeida et al. (2013, p. 9), that mathematical modeling is “a pedagogical alternative in which a non-essentially mathematical problem is approached through mathematics”. In this understanding, the development of a modeling activity can be described in terms of an initial situation (problem), a desired final situation (solution to the problem), and a set of procedures and concepts needed to move from the initial situation to the final situation (Almeida et al., 2013). This process of moving from the initial situation to the final situation can be guided by what the authors call the phases of mathematical modeling, which are: integration, mathematization, resolution, and interpretation of results and validation.

> The ideation phase involves choosing a topic and searching for information about it. In mathematization, the information and the problem are associated with mathematical language. Variables are defined, simplifications are made and hypotheses are defined to write the problem down mathematically. Aspects of the mathematization phase lead those involved to build a mathematical model capable of describing the situation and making it possible to answer the problem defined. The phase of finding a solution using the model is called solving. Finally, it is necessary to interpret the answer obtained and assess whether it is appropriate, both from the point of view of the situation and the mathematics used. This phase is called interpretation of results and validation (Almeida et al., 2021, p. 385).

Although these phases do not necessarily happen in a linear manner, but imply a back-and-forth movement (Almeida et al., 2013), characterizing them helps to understand how the students carry out the activity, as well as guiding relevant intervention by the teacher. Nevertheless, the teacher's intervention in mathematical modeling activities, to guide the students, enables them to construct or mobilize knowledge of the situation under study, of mathematics or the relationship between the two. To do this, the teacher plays a different role, which requires diverse knowledge.

Villa-Ochoa et al. (2021) state that mathematical modeling offers opportunities for learning, the development of skills, and the manifestation of knowledge for teachers involved in activities of this nature.

For Wess and Grefath (2019) the teacher's professional competencies include the ability and knowledge to: (a) solve a modeling task in multiple ways, (b) analyze modeling tasks and (c) develop modeling tasks. In addition, according to the authors, modeling activities require the (future) teacher to understand the pedagogical potential of this type of task, as well as "the ability to design and use tasks to cognitively stimulate students and assess their learning performance, as well as the ability to analyze the work done by students in these tasks" (p. 5).

Studies such as those by Ferri (2018) and Wess et al. (2021), emphasize that the fundamental competencies for effective teaching of mathematical modeling include Pedagogical Content Knowledge (PCK), to contemplate the specificities of mathematical modeling. This integrated approach of pedagogical training and disciplinary knowledge is crucial for educators to implement innovative teaching practices that promote a deep and applied understanding of mathematics through modeling. That is, the authors suggest highlighting the importance of robust and continuous pedagogical preparation, which enables teachers to implement mathematical modeling effectively.

Still through the lens of initial and continuing teacher training, Zapata-Jaramillo (2023) states that mathematical modeling can help strengthen teachers' knowledge by enabling them to relate a situation in their social context to mathematics. Based on other research, the author states that mathematical modeling provides teachers with the opportunity to expand their knowledge of certain mathematical objects, as well as knowledge related to teaching mathematics.

In this sense, Bisognin and Bisognin (2021) investigate evidence of the construction of knowledge needed by teachers to teach mathematics using mathematical modeling. Based on Matthew David Monfredo Schullman and Debora Ball, the authors list categories of analysis and indicators to analyze whether mathematical modeling favors the construction of knowledge necessary for teaching, particularly with a focus on mathematical knowledge.

Studies on teachers' specialized knowledge in a mathematical modeling environment are still incipient and do not discuss it from the perspective of MTSK, the focus of our investigation. This statement is based on the results presented in the research by Vitalino and Teixeira (2022), which aimed to present an overview - considering the Brazilian scenario - of scientific articles permeated by MTSK and developed from some formative action. As a conclusion, in addition to identifying only one article aimed at training mathematics undergraduates, these authors highlight the absence of research that considers the MTSK model and teacher training actions that are based on some methodological trend, such as problem solving and mathematical modeling.

Thus, seeking to expand discussions on this topic, in this study, in particular, we are interested in investigating the teacher's specialized knowledge, in a mathematical modeling environment, from the perspective of MTSK.

MTSK SPECIALIZED KNOWLEDGE

The MTSK model is a theoretical model for analytical purposes whose basic assumption is that when carrying out the task of teaching mathematics, certain specialized knowledge, useful and necessary for the mathematics teacher and related to the process of teaching and learning mathematics, is mobilized by the teacher (Zapata-Jaramillo, 2023).

Besides covering elements that permeate and define the organization and use of knowledge (which are related to beliefs about mathematics, as well as its teaching and learning), the MTSK model is based on two knowledge domains: mathematical knowledge (MK) and pedagogical content knowledge (PCK). Each of these domains is associated with three subdomains, which are described in Table 1.

Table 1

MTSK subdomains and their descriptions

MATHEMATICAL KNOWLEDGE (MK) **Refers to the teacher's knowledge of mathematics as a scientific discipline in a school context, i.e. in-depth knowledge of mathematics.**	
Subdomains	Subdomain description
Knowledge of topics (KoT)	Associated with knowledge of the content of mathematics itself, i.e. mathematical foundations, definitions, procedures, facts, rules, properties, appropriate vocabularies, theorems, meanings, different registers of representations and models, as well as contexts and problems.
Knowledge of the structure of mathematics (KSM)	It encompasses the teacher's knowledge of the different mathematical topics as sets of connected and related elements, i.e., as interconnected systems that link the subject and allow more advanced content to be approached with an elementary focus and vice versa.
Knowledge of mathematical practice (KPM)	Knowledge linked to how mathematics works (defined as any mathematical activity carried out systematically) and the rules that govern how the discipline generates new knowledge, in other words, how mathematics progresses. This includes knowledge related to definitions, justifications, different ways of demonstrating, making deductions and inductions, giving examples and understanding the role of counterexamples, knowing the difference between a proof and a definition, understanding generalization criteria and different problem-solving strategies, as well as the logic that supports this knowledge.

PEDAGOGICAL CONTENT KNOWLEDGE (PCK) This refers to the teacher's knowledge of aspects related to mathematical content as an object of teaching and learning at each school stage, i.e. didactic knowledge of the content.	
Subdomains	Subdomain description
Knowledge of mathematics teaching (KMT)	Theoretical knowledge specific to the teaching of mathematics that leads teachers to identify the most appropriate ways of presenting certain mathematical content (whether through metaphors, situations, or explanations, or including examples, tasks, and activities), as well as critically evaluating and selecting different teaching materials, resources (manipulable or digital), teaching strategies and techniques, recognizing the possible limitations and potential of the choices made. These aspects take into account the situation and the student profile in the established context.
Knowledge of the Characteristics of Mathematics Learning (KFLM)	The teacher's knowledge of different aspects of learning, more specifically the process of understanding and the construction of knowledge by students when dealing with certain mathematical content. This subdomain includes knowledge about the pathways used by their students to understand mathematical content and its characteristics, taking into account their reasoning, emotions, learning styles, and mathematical procedures. Therefore, this knowledge encompasses the strengths or difficulties they may have, the obstacles associated with concepts, errors, mistakes, potential strategies, mathematical perceptions, terminology and language used to refer to specific content, the emotional aspects of learning mathematics (motivation, math anxiety), etc. Also included is knowledge of learning theories, whether personal or from research in mathematics education.
Knowledge of mathematics learning standards (KMLS)	Teaching knowledge regarding curricular standards and other official documents that govern teaching at different levels, as well as research in the area of mathematics education.This subdomain also includes knowledge of sequencing proposals for different mathematical topics, or knowledge of the level of conceptual or procedural development that a student is expected to have.

Source: Prepared by the authors based on Flores-Medrano et al. (2016), Carrillo-Yañez et al. (2018) and Padilla-Escorcia et al. (2023).

By highlighting relevant domains and subdomains, the MTSK model presents a clear and specialized framework for understanding and improving teaching practice in mathematics, which can make a significant contribution to the continuous professional development of teachers and, consequently, to improving the teaching and learning of mathematics. In this way, the model not only maps the teacher's mathematical knowledge but also allows an in-depth analysis of how this knowledge is applied in educational practice.

In short, the MTSK model "provides an understanding of what Math Teachers know, how they know it, what this knowledge enables, and what they need" (Zapata-Jaramillo, 2023, p. 28).

THE STUDY'S METHODOLOGICAL APPROACH

In order to meet the objective we set ourselves, to investigate the teacher's specialized knowledge manifested in a mathematical modeling activity, we took a qualitative research approach for this study, with an interpretive paradigm.

Referring to Fiorentini and Lorenzato (2012), we understand that qualitative research in mathematics education should consider the case study as a complex and interdependent system, seeking to understand its internal dynamics and its relationship with the broader social context.

From the same perspective, Ferreira (2016) corroborates the assertion of the aforementioned authors by stating that the interpretive paradigm - to which qualitative research is aligned - values understanding and explanation, to develop and deepen knowledge of a given situation, in a given context. This author also clarifies that, in its methodological dimension, this paradigm uses a plurality of qualitative methods (such as case studies, interviews, diaries, participant observation, focus groups, etc.) to interpret reality and the exchange between theory and practice.

Therefore, within the qualitative approach, we opted for participant research, since in this type of investigation, “the researcher is integrated into the activities, and may or may not intervene in them” (Fiorentini and Lorenzato, 2012, p. 224). We would like to point out that one of the researchers, the first author of this article, designed the modeling activity that supports our analysis and followed its development with the sample which, in turn, was made up of three teachers in their initial training, students on a Mathematics degree course, in the context of an extension project at a public university.

The extension project in which the teachers in training are involved, entitled “Mathematical modeling for teaching mathematics”, has the general objective of “promoting mathematical modeling as a pedagogical alternative for teaching mathematics in Basic Education, based on the development of ‘mathematical modeling days’”. To this end, one of the activities of the teachers in training, the project's scholarship holders, is the study and planning of mathematical modeling activities for the implementation of “modeling days” in basic education schools. It is worth noting that the fellows had already carried out theoretical studies on mathematical modeling.

In this article, in particular, we analyze an activity with the theme “Church Clock”, proposed by the project's coordinating teacher to the teachers in training.

As analysis material, we used the digital records and audio recordings of the teachers in training as they carried out the proposed activity. Based on the material collected, we organized the records, identifying the process of solving the problem, and proceeded with the transcriptions of the teacher in training’s speeches, organized into dialogues.

The dialogues presented are representative of the moments in which significant actions and strategies were identified for the development of the activity. In order to maintain the anonymity of the teachers in training, we refer to them as PF1, PF2, and PF3. The speech of the teacher-researcher is indicated by the name Teacher. The dialogues are numbered in ascending order (although they are not necessarily presented linearly during the development of the activity) and represent excerpts that portray the main discussions of the teachers in training during the development of the mathematical modeling activity.

The data collected provided support for identifying, classifying, and analyzing the knowledge of teachers in training resulting from their involvement in mathematical modeling activities. In addition, they served as a basis to understand how intrinsic aspects of the development of such activities lead to the manifestation of different Math Teacher knowledge and, similarly, how such knowledge influences this development.

This process is supported by the six types of specialized teacher knowledge proposed by Carrillo-Yañez et al. (2018): knowledge of topics (KoT), knowledge of the structure of mathematics (KSM), knowledge of mathematical practices (KPM), knowledge of the characteristics of mathematical learning (KFLM), knowledge of mathematical teaching (KMT), knowledge of mathematical learning standards (KMLS).

DESCRIPTION OF THE ACTIVITY CARRIED OUT, ANALYSIS AND DISCUSSION OF THE RESULTS

The activity with the theme “Church Clock”, proposed by the project's supervising teacher and first author of this article, has the configuration shown in Figure 1.

Figure 1

Summary of the mathematical modeling activity “Church Clock”

Source: Self-authored (2024).

The proposal for the activity was presented by the project's coordinating teacher to trigger discussions on the development of the mathematical modeling activity and the characteristic elements of a mathematical modeling activity.

Dialogue 1

PF2: I was thinking that the students were going to propose the problem. I hadn't noticed it could be in that format.
PF1: And it's something they can see, they know the place. It will be interesting.
PF2: But they can choose the problem, right? We're going to propose it because it's going to be a specific activity, there's not much time and we don't know the class well.

When PF2 explains her initial understanding of mathematical modeling for her work in the classroom, she suggests that her knowledge of mathematics teaching (KMT) is in process, while, based on her experience, she is broadening her theoretical vision of the possibilities for designing activities of this nature.

Based on the proposal, the teachers in training carry out a joint reading of the problem situation, the information and the problem, in order to, as illustrated in Dialogue 2, outline the first resolution strategies.

Dialogue 2

PF1: To know the height of the clock, we need to know the diameter.

PF2: But isn't the center of the clock in the middle of the wall? If we find out the radius, we can also solve the problem.
PF1: Yes. And the wall is 8 meters wide.
PF2: How can I solve it? You have the information that the tower is 50 meters high, 8 meters wide. Where do I use this 50?
Teacher: What for?
PF2: To calculate the radius of the clock.
PF1: I think this is information about the situation, but we're not going to use it in the mathematical solution. Just the information about the width is enough.

From the teachers in training contact with the real situation and the presented problem, it is possible to see signs of knowledge about topics (KoT) in a context associated with the concept of circumference and its elements, i.e. in such a way that the teachers in training show awareness of its uses and applications, as in the statement that “the diameter is the height of the clock”. KoT can also be observed in the statement that “the information about the width is sufficient”, in which the teacher in training (PF1), when faced with the natural language in which the information is found, manages to interpret it in the mathematical language needed to solve the problem, thus establishing an application of the topic in question to a real-life situation.

In fact, we argue that the use of mathematics to solve real problems, which characterizes a mathematical modeling activity, can enhance KoT. Given that this specialized knowledge is strengthened to the extent that it requires the subject involved in an activity to contextualize mathematical applications with real situations and, consequently, with content from other disciplines, as Carrillo-Yañez et al. (2018) point out.

In Dialog 3, the teachers in training continue with the resolution, trying to establish simplifications for the problem.

Dialogue 3

PF2: Isn't the radius half the wall? Like the diameter is half the wall and the radius is a quarter?
Teacher: On what basis can we say that?
PF1: I don't think so (measuring the image). Because the radius is a little over a quarter of the wall.
Teacher: And if the students think that, what will you argue?
PF2: We could ask them to measure it, just like PF1 did.
PF1: I'll enlarge the image to make it easier. I'm going to leave the measurement at 8 cm, because that way the scale is 1 by 1 (cm per m). That's almost 4 and a half cm.

The dialogue undertaken suggests signs of knowledge of the structure of mathematics (KSM), given that the teachers in training make connections to simplify the mathematics involved in the situation, such as assuming the measurement of the radius to be a quarter of the measurement of the wall or projecting a scale of 1:1 (cm:m). Furthermore, there seems to be a relationship between the concepts unit of measurement and fraction, as well as the notion of unit proportion, units of measurement, and scale. In this case, the intrinsic characteristic of a modeling activity, which is a simplification, is what caused the KSM to manifest itself.

The manipulation of the scale, carried out by the teacher in training (PF1), also carries with it signs of knowledge of math teaching (KMT), which allows him to work with the enlargement/reduction of the photo using, exploring a technological resource, as it was a digital image.

Also, when PF1 takes the measurements of the image, she performs a proof to test the truth value of PF2's proposition (the radius is a quarter of the wall), justifying it with a mathematical argument, a fact that denotes aspects of the knowledge of mathematical practices (KPM).

While dialog 3 shows an attempt at simplification, resulting in the emergence of KSM and KPM knowledge, in dialog 4, the teachers in traininig explain ideas that could lead to solving the problem using a more complex approach.

Dialogue 4

PF2: Only this image isn't very good, it's in perspective, a bit crooked. Could we use another image, a photograph?
Teacher: You can look for more information if you think it's necessary.
PF2: There's a Google Maps image of the street behind the church, but it's also in perspective.
Teacher: Are you thinking of working with perspective? In activities of this nature, we often need to make some simplifications.
PF3: What do you mean in perspective?
PF1: You know those pictures like "holding the sun in your hand" or the person holding the other person who is far away?
PF3: Oh, yes. It's as if the distance reduced the size, so the distance or separation would need to be considered in the calculation. I think we could do it, but the students (in Basic Education) would have more difficulty, because we usually work with flat figures. [...]
PF2: So let's disregard perspective.
PF1: I think so. Because, let's see here. From the first part of the wall to the middle of the clock is 3.2 cm, and from the middle of the clock to the second part of the wall is approximately 3 cm. The difference is not that significant.
PF2: How did you do it?
PF1: So, I'm considering that the clock is exactly in the middle of the wall, it's the midpoint of the wall measurement segment, so it must have the same measurement from the middle to both ends.

Thus, we understand that by suggesting working with the concept of "perspective" knowledge of the mathematical structure (KSM) is evidenced since they propose an increase in complexity in the interpretation and planning of resolution, i.e., the suggested concept seems more advanced than initially expected by the coordinating teacher. This issue of increased complexity, established by the KSM, can be recurrent in mathematical modeling, given that, in activities of this nature, although the teacher suggests the theme, and the problem and provides information, students may take different approaches from those anticipated by the teacher, as suggested by Castro and Veronez (2018).

To carry out an explanation, by means of analogous contextual situations, or as Carrillo-Yañez et al. (2018) describe as using a metaphor (pictures like 'catching the sun in your hand') PF1 denotes knowledge of mathematics teaching (KMT).

While, as a result, PF3 points out that “then the distance or remoteness would need to be considered in the calculation” denotes knowledge of mathematical practice (KPM), presenting an understanding of the logic that support their understanding of the concept of perspective. In this sense, KPM also manifests itself when PF1 argues about the negligible influence that the notion of perspective would have on mathematical resolution, in other words, the teacher in training validates the mathematical idea, which leads to refuting the use of the concept. Thus, since validation is an inherent element of a mathematical modeling activity, as it makes it possible to articulate the information needed to confirm how reasonable a mathematical answer or result is for the real situation (Almeida, 2022), we understand that, at this moment too, the teacher's knowledge, manifested by the KPM, was enhanced because it was a mathematical modeling activity.

Also in dialog 4, knowledge of the characteristics of learning mathematics (KFLM) is manifested when PF3 points out possible difficulties that his students would have with the mathematical concept, which indicates that the future teacher, when planning the activity, starts to consider how students think and construct knowledge when dealing with mathematical tasks.

Dialogue 5, an excerpt from the discussions about the mathematical resolution of the problem, brings new elements that lead us to highlight the teacher's specialized knowledge and shows that this task enhances the development of this knowledge.

Dialogue 5

Teacher: So let's think. We have information about the actual width of the tower, and what else?
PF1: From its measurement in the photo.
Teacher: Right.
PF2: I don't understand.
PF1: If in the photo it measures 6 cm and in reality, it measures 8 meters, then if the clock measures 3.4 cm in the photo, in reality, it will measure x.
PF2: I'll record it here. Real, photo. Can you work it out by the rule of three? I'll use the calculator.

The use of calculators as a resource for solving mathematical calculations efficiently indicates knowledge of mathematics teaching (KMT). While the perception of the relationship between quantities and the recurrence of the “rule of three” indicates knowledge of mathematical practices (KPM) on the part of teachers in training.

In mathematical modeling activities, as has already been pointed out in the literature, it is not always possible to predict the content that will be needed to solve the problem, because the “mathematics to be used may not be chosen in advance, but it emerges from the mathematization of the situation; different mathematical contents can arise from the mathematization of a situation” (Almeida, 2022, p. 136). This unpredictability resulting from mathematical modeling reveals the manifestation of some specific knowledge in this group of teachers in training. If other teachers had developed this same activity, the content and resources could have been different, as well as perhaps the knowledge mobilized.

In addition, the context in which the teachers in training find themselves may have led them to make specific choices, because, at the same time as they develop the modeling activity as an active subject, they consider how it can be developed in the classrooms where they will carry out the extension intervention. In dialog 6, this consideration becomes more evident.

Dialogue 6

PF1: For which "grade" will this activity be applied? It seems that some would be easier for them to understand mathematics.
Teacher: Which do you think is more appropriate?
PF2: Ninth grade they work with proportional segments. I think this is provided in the BNCC.
Teacher: Is it possible to solve this problem using only the idea of proportional segments?
PF2: Not just this one, we solved it using the rule of three. Directly proportional quantities.
PF1: Do they need to know the content before solving the problem? Because of the rule of three, for example, they can know at any level.

In Dialogue 6, aspects related to knowledge of mathematical learning standards (KMLS) seem to be dominant, which, as described in Chart 1, include aspects related to teachers' knowledge of curriculum standards and official documents governing teaching, as well as the sequencing of content and the level of development (conceptual and/or procedural) expected of the student. In our investigation, this knowledge can be observed when the teachers in training express concern about the stage of schooling of the students who will develop the activity, as well as their level of mathematical ability, guided by the Common National Curriculum Base (BNCC), which deals with curriculum guidelines.

In this way, even though mathematical modeling activities are not guided by specific content, but by a real situation and the problem that gave rise to the activity, teachers in training discuss the students who will develop it, their previous knowledge, and the mathematics that will be useful to arrive at an answer, a solution, but which they will still need to learn. In fact, reflections like this can be made possible and, to the same extent, require KMLS.

The next dialog is based on the mathematical process of solving the problem, which is illustrated in Figure 2.

Figure 2

Mathematical resolution of the problem

Variables
Measurement of the clock in the photo (RF)= 3,4 cm
Measure of the width of the tower in the photo (TF)= 6 cm
Actual width of the tower (TR)= 800 cm
Actual measurement of the clock (RR)= ?

	In the photo (cm)	In actual (cm)
Clock	3,4	x
Tower	6	800

Solving

$$6x = 3{,}4 \cdot 800$$

$$x = \frac{2720}{6}$$

$$x = 453{,}33 \ cm \ ou \ 4{,}33 \ m$$

Source: Self-authored (2024).

The dialogue 7 shows a dynamic when we look at the knowledge expressed.

Dialogue 7

PF1: Mine was 4.33.
PF2: Mine was 4.20.
PF3: A bit higher than the room.
Teacher: Right, what did we find?
PF3: The measure of the diameter.
Teacher: Does that answer the problem?
PF1: No, because we need to answer the length of the frame.
PF3: A circle? Usually, clocks are round.
PF2: But it could also be square. The students can think about that.
PF1: Then we can do both.
PF2: But we have the radius, can we think of the square?
PF1: Yes, because the diameter is the minimum you'll need to draw a frame around the edge of the numbers on the clock.

PF3's strategy of comparing the size obtained, using the example of the height of the room to understand the diameter of the clock, indicates knowledge of teaching mathematics (KMT). Knowledge of topics (KoT) manifests itself when future teachers associate mathematical content and its properties with the real context (circular or square clock, diameter is the limit of the numbers on the clock). On the other hand, when PF2 is concerned about how students can think to solve the problem, it suggests signs of knowledge of learning resources (KFLM).

From these interpretations, we can see that mathematical knowledge and pedagogical knowledge of the content can sometimes coexist. Furthermore, the teachers in training's considerations about the definition of the radius represented the basis for continuing to build the model, as dialog 8 reveals.

Dialogue 8

PF2: I don't know if I understood correctly, but these mathematical models, then, would be: for the circular frame, the expression shown there is twice pi

times the radius, which would be the length of the clock, to find out how long the frame would be. And for the square frame, the expression there is that l is the number of sides, which would be the diameter, twice the radius, so 4 times is to find out how many meters the square frame needs to be.
PF3: I understood it like that too.

The model obtained by the teachers in training, which supports dialog 8, is illustrated in Figure 3.

Figure 3

Mathematical model obtained for the length of the watch frame

Mathematical model

Circular frame (RF diameter)

$$C = 2\pi\left(\frac{\frac{RF}{TF}\cdot TR}{2}\right)$$

Square frame (RF diameter)

$$P = 4\cdot\left(\frac{RF}{TF}\cdot TR\right)$$

Source: Self-authored (2024).

Despite the colloquial language used by PF2 in constructing the model, her knowledge of the topics (KoT) is revealed since her interpretation of the mathematical models also reflects her knowledge of mathematical fundamentals and properties.

FINAL CONSIDERATIONS

In view of the aim of this research to investigate the teacher's specialized knowledge manifested in a mathematical modeling activity, we infer some important results.

Collaborative group work, as Almeida (2022) points out, as being inherent to a mathematical modeling activity, in the activity analyzed seems to enhance the manifestation of the teacher's knowledge, to the extent that, in many moments, knowledge is only made explicit/accessible/externalized through the interaction between the group members. For example, in dialogue 2, even though the signs of KoT were identified in PF1's speech, when following the dialogue, it seems that they were provoked mainly by PF2's questions. Therefore, the manifestation of this knowledge is also related to the factors that lead the teacher to externalize it and, thus, are accessible to the researcher's interpretation.

Even when interacting with the group, it can happen that some knowledge comes about in the subject's own particular way, as occurs in dialog 7, in which each teacher in training expresses a particular piece of knowledge that is not directly the result of influences from others. For example, PF3 compares the size of the room, without there being a previous question or statement leading to this consideration.

From the analysis, it is still possible to infer that there seems to be an interrelationship between the knowledge, for example in dialog 3, when the intention to simplify - the radius being a quarter of the wall - (KSM), leads to the verification of the proposition - testing the measurements - (KPM) which, in turn, triggers a new simplification - manipulating the scale - (KSM).

It often seems possible to relate certain knowledge to specific characteristics or elements of mathematical modeling. KoT is privileged knowledge since mathematical modeling requires the problem to come from a real context. Therefore, the need to understand and relate the real context to the application of mathematics favors KoT. Similarly, validation is associated with KPM, taking into account that validation in modeling requires testing/proving mathematical results about the real situation to be answered, which is in line with the understanding of the KPM adopted. Along these lines, simplification heralds the manifestation of KSM, which is normally associated with simplification or complexity connections in the situation, especially in the mathematics involved.

Although some knowledge is not so directly dependent on mathematical modeling, since it could occur in any context that does not involve this type of activity, in a holistic view it is the result of the process as a whole. An example of this is the teacher in training's concern about working on the content following the curriculum guidelines (dialog 6). Although this concern may be recurrent in the teacher's daily practices, in this specific context it is manifested, because the mathematical modeling problem was not designed to meet a content, on the contrary, it arose from the need to answer the problem.

From the analysis carried out, albeit in an initial overview, it is possible to infer that sometimes the mathematical modeling activity requires specific knowledge, and sometimes the knowledge inferred unfolds for the activity. For example, in dialogue 2, it is the real problem, the essence of mathematical modeling, that brings out the KoT; on the other hand, it is the KFLM expressed by PF3 that leads them to decide on the simplification (disregarding the notion of perspective) necessary in mathematical modeling activities.

One limitation of the study is that the knowledge of teachers in training was analyzed in the “learning through” axis (Omodei & Almeida, 2022), that is, from the development of activities, in an initial context of learning and planning, and provides evidence of what Wess and Greefath (2019) call the ability and knowledge to develop modeling tasks. In future studies, we can analyze the specialized knowledge of teachers in situations of implementing activities in the classroom, in the “teaching using” axis, which may provide more differentiated descriptions of the teacher's performance while dealing with student referrals.

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