ACTIO: Docência em Ciências
ISSN: 2525-8923

http://periodicos.utfpr.edu.br/actio

An analysis of heuristics in strategic learning in a mathematical modeling activity

Suzana Lovos Trindade
stlovos@gmail.com
orcid.org/0000-0002-3389-2979
Universidade Tecnológica Federal do Paraná (UTFPR), Londrina, Paraná, Brasil

Karina Alessandra Pessoa da Silva
karinapessoa@gmail.com
orcid.org/0000-0002-3389-2979
Universidade Tecnológica Federal do Paraná (UTFPR), Londrina, Paraná, Brasil

ABSTRACT

In this article, it was essential to look at strategic learning in the development of a mathematical modeling activity by students in the final years of elementary school. To this end, mathematical modeling supported the research as a pedagogical alternative as well as strategic learning, which aims to value procedures over the product. The arguments are based on the description and analysis of an activity entitled "Pastry," developed in a private school located in northern Paraná in an 8th-grade class. To this end, the data collected came from the students' written records, the teacher's logbook, photos, videos, and audio recordings. Through the qualitative and interpretative analysis of the data, it was evidenced that heuristic strategies are employed throughout a mathematical modeling activity and can appear repeatedly in each instance with different meanings. However, they are strictly related to strategic learning, one of the five components of learning, characterized by the importance of the process, implying the interpretation of students' actions and decisions during the resolution of a problem situation.
KEYWORDS: Learning by Modeling; Learning Strategies; Teaching Methodology.

Uma análise das heurísticas na aprendizagem estratégica em uma atividade de modelagem matemática

RESUMO

Neste artigo, fez-se imprescindível o olhar para a aprendizagem estratégica no desenvolvimento de uma atividade de modelagem matemática por alunos dos anos finais do Ensino Fundamental. Para isso, a modelagem matemática subsidiou a pesquisa enquanto alternativa pedagógica, bem como na aprendizagem estratégica que tem como objetivo valorizar os procedimentos em detrimento do produto. As argumentações são pautadas a partir da descrição e análise de uma atividade intitulada “Pastry”, desenvolvida em uma escola privada localizada no norte do Paraná, em uma turma do 8º ano. Para tanto, os dados coletados foram provenientes de registros escritos dos alunos, diário de bordo da professora, fotos, vídeos e gravações em áudio. Por meio da análise qualitativa e interpretativa dos dados, foi evidenciado que estratégias heurísticas são empreendidas ao longo de uma atividade de modelagem matemática, podendo aparecer repetidas vezes, em cada uma, com significados diferentes. Contudo, estão estritamente relacionadas à aprendizagem estratégica, uma das cinco componentes da aprendizagem, caracterizada pela importância do processo, implicando na interpretação das ações e decisões dos alunos durante a resolução de uma situação-problema.
PALAVRAS-CHAVE: Aprendizagem por Modelagem; Estratégias de Aprendizagem; Metodologia do Ensino.

INTRUDUCTION

Considering concerns about the teaching and learning of mathematics and the emergence of new methodological approaches over time, this article focuses on mathematical modeling as a pedagogical alternative. In this approach, an initial situation is used to seek solutions to problems arising from reality through the deduction mathematical models. From this understanding, the aim is to highlight that strategic learning is mobilized in developing a mathematical modeling activity.

According to Barbosa (2003), mathematical modeling is a learning environment that invites students to question and investigate real-life situations, sometimes originating from other fields of knowledge, requiring the formulation of a mathematical model that solves non-strictly mathematical situations. Bliss and Libertini (2016, p. 12) state that in a modeling problem, there is "[...] room for students to interpret the problem and make choices in the solution process."

In this sense, we agree with the author and consider mathematical modeling a pedagogical alternative that enables learning through research, knowledge construction, exploration of new paths and experiences, communication, and autonomy, among other actions. According to Almeida, Silva, and Veronez (2021, p. 22):

> [...] thinking about the characteristics of a mathematical modeling activity encompasses not only the representation of students' and teachers' actions but also specificities regarding the domains with which they need to engage in their activities, such as the organicity and the back-and-forth relationships in terms of the perceived reality and the mathematical system in use, as it highlights the elements with which they come into contact, such as experiences, theories, and data necessary for the investigative process that culminates in solving a real-life problem through mathematics (Almeida, Silva and Verenez,2021, p. 22).

Given the inherent concerns regarding students' learning when implementing mathematical modeling in mathematics classes, the considerations covered in this study are based on the insights of Pinilla (2010), who addresses what students learn in mathematics and the causes of their mistakes, elucidating five components of learning in her studies to make interventions: conceptual, strategic, algorithmic, communicative, and semiotic. In this article, the goal is related to strategic learning, which, according to the author, is the most difficult because it focuses more on the processes than on the products, requiring attention to the procedures and strategies that the student uses to solve the problem situation.

In order to achieve the objective of this research, it is based on the studies of authors (Stender 2017; Stender & Kaiser 2017; Almeida 2020; Koga 2020; Koga & Silva 2020) who, in the context of mathematical modeling, suggest that heuristic strategies are like "[...] tools of mathematical thinking to guide students in the search for a solution" (Almeida 2020, p. 224). These tools are related to trial and error, problem-solving, hypothesis formulation, freedom to navigate through the mathematical modeling activity cycle as needed, problem subdivision, and the search for basic mathematical concepts, which align with the characteristics of strategic learning.

A qualitative and interpretive research approach (Bogdan & Biklen 1994) was applied to present the results. The data that supported our research came from written records, audio, and videos of 8th-grade elementary school students from a private school in the countryside of Paraná, during the development of a modeling activity on pastries. In this context, the research objective is structured around the question: "What strategic learning is evidenced in the development of a modeling activity by 8th-grade elementary school students?"

In the next two sections, the theoretical framework of this research is presented. The first section covers mathematical modeling, and the following one discusses heuristic strategies and strategic learning. Next, the methodological aspects are addressed to enable the description and analysis of the activity. Finally, the text brings considerations and reflections on the research question.

MATHEMATICAL MODELING

There is significant research on mathematical modeling in various journals and conference proceedings in the field of Mathematics Education. However, some researchers stand out in our studies because their perspectives and conceptions align with ours (Borromeo Ferri, 2007; Vertuan, 2007; Carreira & Baioa, 2011; Almeida, Silva & Vertuan, 2012; Blum, 2015; Pollak, 2015; Almeida, 2018; Stender, 2018; Castro & Veronez, 2018; Carreira, 2021).

Mathematical modeling is understood as a methodological approach that integrates the development of mathematical content with the inclusion of real-world problems, fostering greater interaction among students and increasing their interest. Thus, it can be said that mathematical modeling provides an environment where inquiries, strategies, and concepts develop new skills based on prior knowledge, enabling learning.

We agree with Bassanezi (2002), who suggests that the essence of mathematical modeling is to transform real-world problems into mathematical ones. This allows us to emphasize that, even when addressing various subjects, it is through mathematical knowledge that it becomes possible to solve, even partially and from the investigator's perspective, and understand situations present in the real-world context.

According to Pollak (2015), mathematical modeling consists of

> [...] formulating a problem situation, deciding what to keep and what to ignore when creating a mathematical model, using mathematics in the idealized situation based on a real-world situation, and then deciding whether the results can, to some extent, be useful in understanding the original situation (Pollak, 2015, p. 627).

Stender and Kaiser (2017, p. 2) agree with Pollak (2015) by emphasizing that these original everyday situations are perceived as something that needs to be "[...] solved, simplified, and transferred into a model of the real world." In this case, it is done through the formulation of a mathematical problem that will be solved (not always entirely) using a mathematical model, eventually leading to a solution. Validation is carried out according to the real world, making the problem situation increasingly interesting and relatable to students.

According to Almeida, Silva, and Vertuan (2012), a mathematical modeling activity is characterized by several elements: it begins with the proposal of an initial situation, and through mathematical procedures, a final situation is reached, where a solution to the problem situation is determined, configuring an investigative process.

According to Niss and Blum (2020), a mathematical model needs to be part of this investigative process because mathematics is used in a way not directly related to mathematics itself; in other words, mathematics is employed to solve problems that are not essentially mathematical. The authors emphasize that the mathematical model gives meaning to something else, in this case, representing characteristics of reality that were considered in solving the situation.

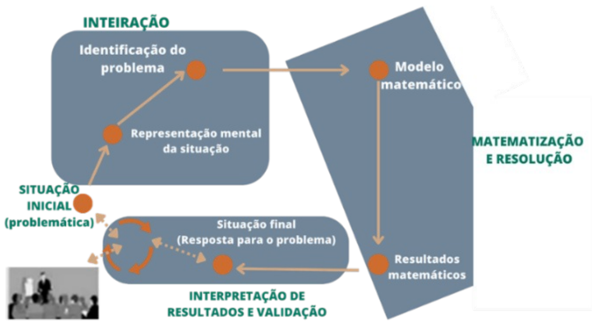
Considering the connections between the real world and Mathematics, some phases characterize the cycle of a mathematical modeling activity, which allows transitions between phases, implying various ways to develop the activities. The phases that characterize the activity, according to Carreira and Baioa (2011), are based on the

> [...] identifying the real-life problem, formulating the mathematical model, producing the mathematical solution or solutions from the mathematical model, interpretating the solutions, evaluating the solutions based on the real-world scenario, and, if necessary, revising the model and carrying a new cycle out (Carreira & Baioa, 2011, p. 211).

According to Almeida, Silva, and Vertuan (2012, p. 4), the phases are characterized as interaction, mathematization, resolution, interpretation of results, and validation, and they can be structured in a cycle as presented in Figure 1. These phases are possible actions for students, but there is no rule indicating that they should be followed linearly.

Figure 1

Mathematical Modeling Cycle



Source: Almeida, Silva and Vertuan (2012)

In the first stage of the cycle, students are presented with an initial situation that encompasses a problem not necessarily mathematical. Through the process of interaction, the problem will be better understood by the students as they seek to identify relationships between the characteristics of the situation. Thus, the modelers create mathematical representations (mathematization) to solve the initial question. Finally, the last stage of the cycle is characterized by the interpretation and validation of the results obtained during the development of the activity. It is important to note that there is no requirement for a linear development of the proposed phases.

Thus, this supports Santana and Gonzales (2019, p. 4) when they emphasize that "[...] using mathematical modeling essentially means using mathematical tools to represent and create a model for everyday situations, thus making mathematical concepts applicable in practice."

It is worth noting that among its characteristics, Mathematical Modeling, as a pedagogical alternative, as well as mathematical modeling activities, enables greater intensity in dialogues, whether between student-student, student-teacher, or teacher-student, which contributes to mathematical learning (Carreira & Baioa, 2018).

Silva and Silva (2021) argue that:

> The interaction between team members can provide dialogues that may serve as a starting point for the development of new knowledge, as well as reflections on the mathematical content involved in the situation, in addition to supporting broader issues that emerge from the defense of viewpoints among students and between students and the teacher (Silva & Silva, 2011, p. 4).

Considering that this article intends to bring reflections on the question "What strategic learning is evidenced in the development of a modeling activity by 8th-grade students?", the goal was to highlight the presence of strategic learning in the student's actions during the phases of developing the mathematical modeling activity. To analyze the students' actions, it is necessary to focus on the strategies they use. In this case, the analyses are supported by theoretical frameworks on heuristic strategy and strategic learning, as covered in the next section.

HEURISTIC STRATEGY AND STRATEGIC LEARNING

The term "strategy" originated in Ancient Greece, with a connotation related to warfare, meaning "the art of the general," referring to how a general led/conducted their army (Meirelles, 1995). The word has always been used in the fields of law and management, with its meaning being directly related to "[...] a path to be followed" (Oliveira, Grzybovski & Sette, 2010, p. 3).

Moreover, according to the Etymological Dictionary, the initial meaning of the word strategy was directly related to wars. However, over time, creating methods, plans, or maneuvers used to achieve a goal or result can be related to various fields, including politics, military, psychology, and education.

When it comes to the word "strategy" in the context of Mathematical Education, there are few discussions on the topic. George Polya, recognized for his contributions to problem-solving, provides in his studies insights into strategies, considering them as tools used to achieve mathematical thinking (Polya, 1945), in this case, during the problem-solving process where students seek a solution.

However, the author offers many considerations on the use of heuristic strategies in problem-solving, focusing on "[...] the study of methods and rules in the problem-solving process, particularly mental operations" (Cavalheiro & Meneghetti, 2020, p. 65).

It can be highlighted that the use of heuristic strategies helps shape students' thinking during the problem-solving process. It can be considered as a toolbox that enables and encourages students, indicating that there is more than one possibility to choose from. Hoon, Kee, and Singh (2013) emphasize that:

> The use of heuristic approaches cultivates students' abilities to think about solving mathematical problems. This means that these skills induce subconscious cognition to trigger ideas, stimulating creativity without using fixed algorithms. The heuristic approach encourages the communication of mathematical thoughts through discovery, drawing diagrams, examining special cases, specializing in the solution, and generalizing the solution (Hoon, Kee & Singh, 2013, p. 863, translation ours).

This study aligns with Pinilla's (2010) intentions, considering strategic learning as one of the five components of learning, with the main goal of questioning the origins of students' errors and misconceptions during mathematical activities. It is worth noting that misconceptions are considered errors made by students—incorrect conceptions—and as they are explored by the teacher, they can lead to discoveries about students' learning difficulties (Mendes, Souza & Almeida, 2017).

According to D'Amore, Pinilla, and Iori (2015), strategic learning is the most complex among the components of learning, due to its intersection with semiotic learning; the use of semiotic registers to express objects is strictly necessary. Consequently, to evidence its presence, the teacher needs to be attentive to the interpretation consistent with the treatment or conversion made by students from one representation to another.

It is understood that, although finding solutions is the most visible "focus" of a problem from the student's perspective, they must recognize that the process is more important than the product. The process refers to the paths taken to reach the final solution, which does not need to be correct at the first attempt, as long as steps are taken to correct any potential errors (Almeida, 2020). When a student knows which path to follow and how to act in solving a problematic situation, the presence of strategic learning is evident (Pinilla, 2010).

Regarding problem-solving, some characteristics function as "norms" for solving a problem. However, Pinilla (2010, p. 96) states that "[...] although applying previous rules (norms, experiences) is important, it is necessary to note that the problem-solving process also generates, above all, new learnings."

Given this, it can be concluded that the most significant factor is the strategies a student uses to solve a problem, not the pre-established rules for its resolution. Being able to analyze and observe the creation of concepts during problem-solving promotes learning, which may have been guided by the teacher but is primarily the result of the student's actions (Pinilla, 2010).

Learning through problem-solving encompasses the components of learning, where the search for solutions spans various significant moments so that the development of more thoughts about the studied subject is directly related to the set of strategies the student uses to solve the problem, emphasizing the importance of the process.

Pinilla (2010) ensures that the fact that problem-solving generates thoughts, reflections, creations, and formulations of questions is directly related to the student's learning. However, strategic learning is present in each of the student's actions during problem-solving, with the records of semiotic representations providing this evidence, whether through writing, speech, gestures, or the way the student felt comfortable sharing their ideas.

Duval (2012, p. 269) states that:

> "Semiotic representations are productions constituted by the use of signs belonging to a system of representations that have their specific issues of meaning and functioning. A geometric figure, a statement in natural language, an algebraic formula, and a graph are semiotic representations that exhibit different semiotic systems. Semiotic representations are generally considered merely as a means of externalizing mental representations for communication purposes, that is, to make them visible or accessible to others. However, this view is misleading. Representations are not only necessary for communication purposes, but they are also essential to the cognitive activity of thinking."

However, it is important to emphasize that it is through the students' semiotic representation records that strategic learning and heuristic strategies can be evidenced, as previously mentioned.

Considering the interest in demonstrating how strategic learning emerges as students develop mathematical modeling activities, this is anchored in the studies of Stender (2018), who, based on some results obtained, suggest actions that reveal heuristic strategies in the development of problem-solving skills.

STRATEGIES IN THE FIELD OF MATHEMATICAL EDUCATION

Some authors, considering that mathematical modeling begins with a problem situation, have researched the use of heuristic strategies in mathematical modeling activities (Stender & Kaiser, 2017; Stender, 2018; Almeida, 2020; Koga, 2020; Koga & Silva, 2022).

The authors emphasize that mathematical modeling activities enhance the ability to solve problems, specifically in modeling. However, given the characteristics of these activities, it can be inferred that they lead students towards greater autonomy in seeking solutions, thereby in developing strategies.

According to Almeida (2020, p. 220), "[...] some strategies seem intuitive or associated with students' previous experiences with mathematical modeling. Others, however, appear to be more specific and reflect the characteristic of discovery, and creation associated with the term heuristic."

In defining strategic learning, Pinilla (2010) highlights the importance of decisions made during problem-solving, considering the solution as the result of processes developed from strategic decisions.

Considering the stages of the mathematical modeling activity, Stender and Kaiser (2018), based on Pólya (1973), suggest that heuristic strategies are evidenced—through some actions—in the development of the activity. For Almeida (2020, p. 224), "[...] a heuristic can be something essentially provisional, without a guarantee of producing an effective result in providing an answer to the problem, but it reflects the intentions of the one seeking the answer by outlining a path of inquiry".

In Table 1, inspired by Stender and Kaiser (2017), heuristic strategies present in some of the student's actions during the development of modeling activities are organized. It is worth noting that, in all the actions grouped by the authors, strategic learning is also revealed, which, according to Pinilla (2010), provides important reflections for the learning process.

Table 1

Actions that Reveal Heuristic Strategies

Heuristic Strategies	Actions of Students that Demonstrate the Presence of Heuristic Strategies
Organizing the material and understanding the problem	Discretization: Making the problem accessible through appropriate structuring and organization. Understanding the problem situation. Using available materials to simulate the situation. At this stage, if the student identifies an error, they go back to the previous phase and start a new attempt.
Using work memory effectively	Reducing Complexity: At this moment, the student may reduce the complexity of the problem by seeking transversal concepts. Stender and Kaiser (2018) highlight the use of the "Supersign," which is a "sign that represents multiple signs." At this point, the student may divide the problem into subproblems.
Thinking big	According to Stender and Kaiser (2017), it is important that there are no boundaries to thinking.
Utilizing prior knowledge	The act of combining what the student already knows to enable new knowledge. Making connections with solutions from other exercises to solve new problem situations. The author emphasizes the use of partial solutions to obtain general solutions.
Functional aspects	Koga (2020, p. 30) states that this strategy "[...] depends on functional knowledge. It is performed in various ways (through derivation, tables, iteration, etc.); but the main idea of examining a whole range of values (and not just a special value) and then identifying the best one is always present (functional aspects)."
Organizing the work	For this strategy, Stender and Kaiser (2018) highlight exploring various paths to obtain a problem solution. However, not all paths will lead to a solution, and it may be necessary to change course.

Source: Author's own (2024).

As the importance of each student's action in developing an activity is validated, "[...] we can understand the degree of comprehension by asking them for more detailed explanations of their reasoning" (Pinilla, 2010, p. 148), thereby revealing strategic learning in various ways, whether through oral, written, or gestural communication.

The next section presents the methodological aspects that supported the results of this research. To address the research question "What strategic learning is evidenced in the development of a mathematical modeling activity by 8th-grade students?", analyses of an activity conducted in regular Mathematics classes and its specifics are discussed.

METHODOLOGICAL ASPECTS

To address the research question "What strategic learning is evidenced in the development of a mathematical modeling activity by 8th-grade students?", an analysis was conducted of an activity developed in regular Mathematics classes at a private school located in northern Paraná. On September 20 and 27, 2022, seventeen students were invited to engage in a mathematical modeling activity.

Over four 50-minute classes, organized into groups of four or five members, the students worked on answering the question: How many pastries can be made with a rolling pin of pastry dough? This activity is part of the master's thesis of the first author of this article (Trindade, 2023).

The class had previously engaged in mathematical modeling activities, so they were "familiar" with this pedagogical approach. It was the students who suggested the above investigation, marking the beginning of the third activity developed by the class.

The data supporting this research were collected through written records from the students, presentations made using Canva[2], photographs, audio recordings, and notes taken by the teacher in her journal. A consent form was signed by the institution and the parents, authorizing the development of the activity and data collection. It is worth noting that, in addition to the four classes, the group members also met outside of the school environment.

To achieve this, a qualitative interpretative analysis was chosen, in line with Bogdan and Biklen (1994, p. 16), where the issues investigated "are not established through the operationalization of variables but are formulated to explore phenomena in all their complexity and a natural context."

Additionally, Stender (2018), Stender and Kaiser (2018), and Pinilla (2010) support this approach in their research, comparing strategies to valuable tools for the teacher. These strategies allow for the analysis of the complexity of problems, the stages of resolving a mathematical modeling activity, the diversity of student actions, and extensive discussion by prompting many possible solutions (Castro & Veronez, 2018).

The description and analysis of the activity were anchored in the resolution performed by the group that stood out the most for its involvement in seeking the solution to the problem, generating greater interaction among its members. To maintain the anonymity of the five students in the group, they are referred to as A1, A2, A3, A4, and A5, as described and analyzed in the next section.

DESCRIPTION AND ANALYSIS OF THE ACTIVITY

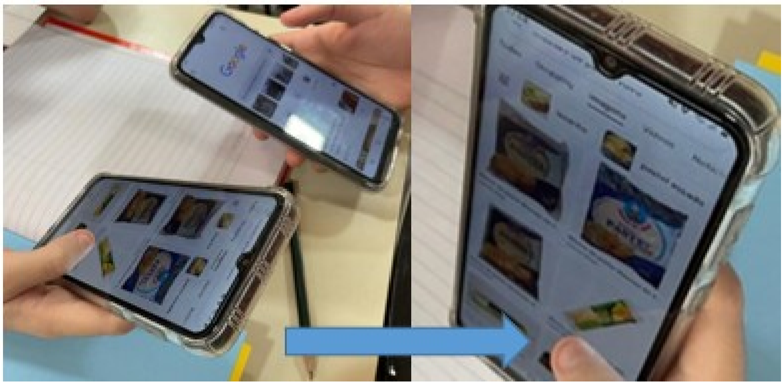
The students were enthusiastic about the topic of Pastel[1], and together with the teacher, they defined a question to investigate: "How many pastries can be made with a rolling pin of pastry dough?" The definition of the question highlights the first strategy of the class as a whole, and according to Pinilla (2010), strategic learning is present when the importance of the processes leading to the product is emphasized.

Stender (2018) states that choosing a theme related to everyday situations reveals heuristic strategies, as does a more detailed study of the topic. Pinilla (2010) notes that seeing various possibilities expands the discussion among peers and reveals strategic learning.

Once the theme and investigation question were defined, the students set out to gather more information using mobile phones connected to the internet (Figure 2), to formulate hypotheses.

Figure 2

Use of Mobile Phones

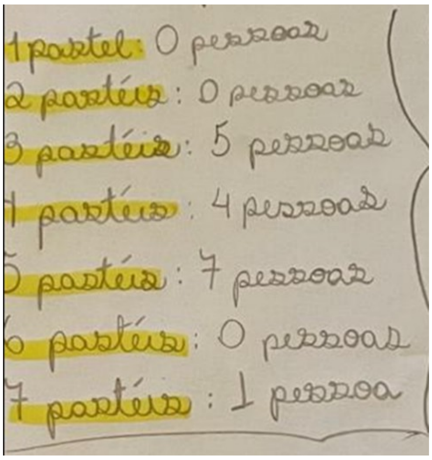


Source: Teacher's Archive (2022).

The group decided to divide the tasks among themselves. While three members researched information about the rolling pin and estimated the dimensions of the pastry—determining that it would be 8 centimeters wide and 10 centimeters long —the others conducted a survey with the class (Figure 3) to determine how many pastries should be made, so they could estimate if one rolling pin of dough would be sufficient.

Figure 3

Data Collection



Source: Students' Report (2022).

However, the strategy outlined by the group is considered a natural and autonomous action in the development of a mathematical modeling activity. The students began to outline procedures and objectives to solve the question, characterizing the organization of material and understanding of the problem, as shown in the excerpt:

A1: “Guys, wait a minute! We don’t even know the dough thickness. From what I’m seeing here, each rolling pin can vary.”

A5: “So let’s decide on the dough thickness first, then we can decide on the size of the pastry, right?”

The students began to question the previously defined actions and realized there was a mistake. It didn't make sense to define the dimensions of the pastries without first knowing information about the rolling pin they were going to buy, demonstrating a trial-and-error approach in their attempt to understand the problem.

After this discussion, the students decided on the type of pastry dough and where it would be purchased, ensuring they chose a supermarket easily accessible to the teacher, who would be responsible for buying the ingredients, as shown in the transcript:

A4: “Teacher, are you going to buy the ingredients?”

Teacher: “I can buy them, or we can ask each person in the class to bring one ingredient. What do you think is better?”

A1: “It’s better if you buy them so no one forgets [laughs].”

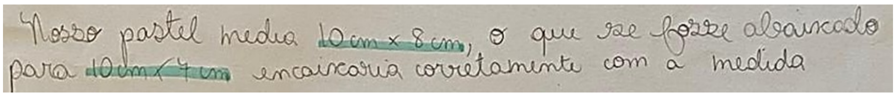
A4: “Which supermarket do you usually go to? Then we can check if they have the dough there.”

strategic learning, as well as the organization of important information.

The students then defined the dough and its dimensions (200 cm x 28 cm), which led them to change the dimensions of the pastries they had previously determined (from 10 cm x 8 cm to 10 cm x 7 cm) to avoid wasting dough and maximize its use (Figure 4).

Figure 4

Choosing the Dimensions of the Pastries



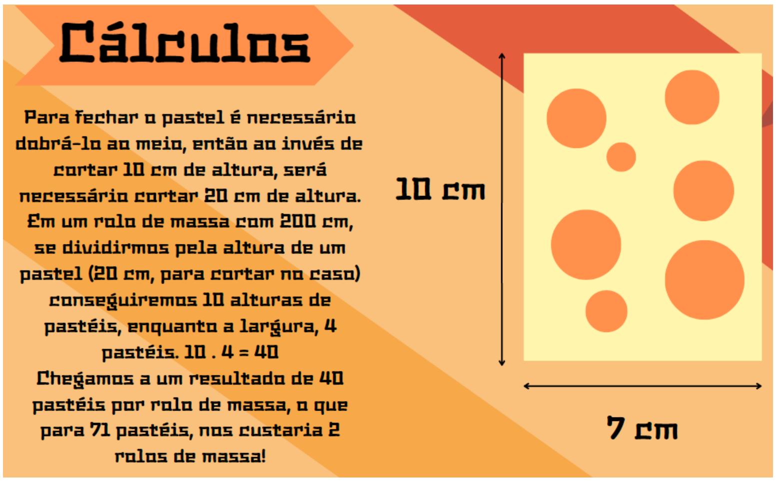
Source: Students' Report (2022).

It is understood that, by opting for these new measurements, the students utilized concepts they already knew, such as multiples and divisors. It can also be concluded that they used their working memory effectively, translating their findings into mathematical language coherently.

After defining the dimensions of the pastries, the students focused on solving the initial question, performing the necessary calculations, and determining that they would make 40 pastries. Since the students presented their solutions using Canva software, it was possible to understand the steps taken to arrive at the answer (Figure 5). It is evident that by choosing a medium for resolution and presentation, the organization of work was present.

Figure 5

Solution of the Activity



Source: Students' Report (2022).

According to the resolution, there are indications of checking functional aspects. During the resolution, the students remembered that to make the pastry, they considered 20 cm instead of 10 cm, as the pastry needed to be folded when closed. Here, the heuristic strategy is revealed, as they use their working memory effectively, solving the problem in parts, reducing complexity, and working with necessary concepts. As a result, they concluded that 40 pastries could be made with the rolling pin they chose; however, they required 2 rolling pins to make the 71 pastries initially defined in the investigation.

What the students overlooked was that the initial question did not ask about the number of rolling pins needed but rather the number of pastries produced with one rolling pin of pastry dough. Therefore, to interpret the results, only one rolling pin of dough was used, from which 32 pastries were made. Figure 6 shows the students' actions in making the pastries, using measuring instruments, and highlighting their concern to maintain the stipulated dimensions.

Figure 6

Interpretation of Results

Source: Students' Report (2022).

In Figure 6, it is evident that the students were consistently concerned with the dimensions of both the pastry dough and the chosen fillings, which needed to be slightly smaller than the dough, according to the transcription:

Teacher: "Everyone, before we start, we need to organize ourselves so it doesn't become a mess."

A2: "Teacher, we need to think about who will do what."

A5: "Yes, we can arrange for each person to do a bit."

A4: "Because we need to cut the dough precisely and divide the filling. Cut it evenly."

A5: "No, it has to be cut smaller so we can seal it with a fork later."

Thus, according to Koga (2020, p. 68), "[...] there is the use of a heuristic strategy, checking functional aspects, as there is a search for connections between the data and the problem for its resolution," since they considered a procedure shared by everyone from one rolling pin of pastry dough instead of buying two, as they had predicted for the solution. At this moment, the students noticed relevant aspects for solving and interpreting the problem as they found relationships between data obtained in the resolution process.

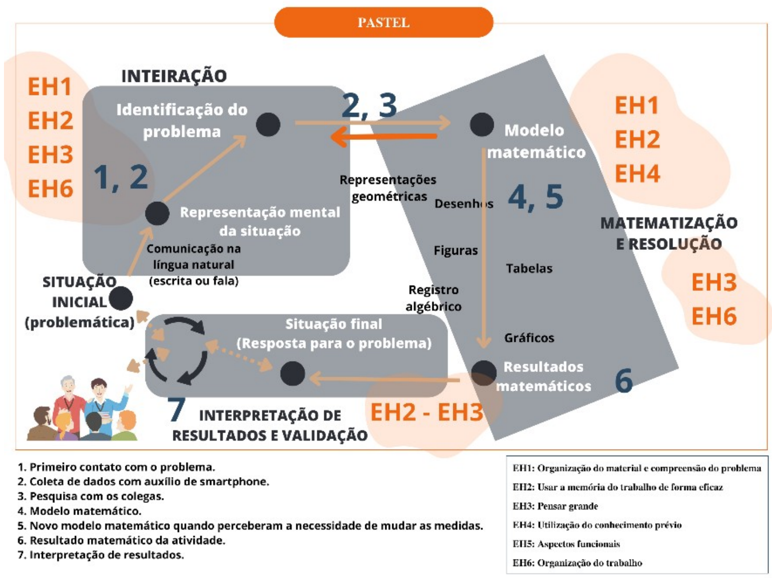
By verifying functional aspects, the student outlines aspects they consider fundamental for solving the activity, such as relating quantities, surface measurements, and division strategies, indicating that the group outlined relationships between them.

The student's actions allowed us to infer that strategic learning was present throughout all phases of the mathematical modeling activity. From the beginning, they showed interest in determining the solution to the problem, but they placed more emphasis on the paths that could lead them to the solution. According to Pinilla (2010), when students understand that the process is more important than the final product, much more can be learned, as it opens up a range of possibilities for the teacher, whether for the development of new activities, for questioning during development, or even for identifying students' difficulties.

Figure 7 highlights the heuristic strategies in the activity cycle. It is worth noting that the figure indicates how learning is revealed in the "Pastry" activity, with codes inserted to show when they are most evident, without excluding their presence at other times.

Figure 7

Heuristic Learning in the Mathematical Modeling Activity Cycle



Source: Source: Author’s own work (2024).

After the pastries were finished and while they were in the oven, the teacher took the opportunity to engage the students in a discussion to explore the difference between the number of pastries they had predicted and the actual number produced. During this conversation, the students suggested several possible explanations, such as the mass not having exactly the indicated measurements or errors in the cutting process.

FINAL CONSIDERATIONS

Understanding how mathematical modeling relates to learning is one of the most important considerations obtained throughout our studies, aiming to highlight the components of learning in the development of a mathematical modeling activity.

This article seeks to understand at which moments and actions strategic learning is present, as well as the Heuristic Strategies outlined by Stender (2018). Therefore, it is dedicated to answering the following question: *"What strategic learning is evidenced in the development of a modeling activity by 8th-grade students in Elementary School?"*

It is recognized that the students were dedicated to seeking connections with reality when defining the problem and, during the investigation phases, showed considerable concern with mathematics. They sometimes applied basic mathematical concepts, demonstrating the importance of prior knowledge for learning new concepts.

The students showed interest, organization, autonomy, and greater confidence in solving the problem. It is worth noting that this class had previously participated in mathematical modeling activities, indicating a maturation regarding the moments and stages of the activity.

It is noticeable that the students planned their actions and accepted their mistakes, allowing for the planning of new approaches, as they realized that the dimensions initially defined (10 cm x 8 cm) no longer made sense as the activity progressed. They broke the problem down into subproblems, enabling all group members to actively participate in the process and even devised strategies to involve the entire class in making the pastries.

This study concludes that strategic learning is present throughout the entire mathematical modeling activity. Therefore, in the presence of strategic learning, there are heuristic strategies as outlined by Stender (2017). However, it is important to note that each strategy, when present at different phases of the mathematical modeling activity cycle, serves different purposes regarding students' actions.

For example, when discussing the organization and understanding of the problem, it is a heuristic strategy that is present not only during interaction but also during mathematization, as students notice any errors and feel the need to reorganize their approaches. The same applies to the effective use of memory, which is revealed as students need different signs to represent the solution, organize research in tables, and write in their native language to explain something already written in mathematical language, which is not restricted to any specific phase of the cycle. Thinking big is a strategy that cannot be absent in mathematical modeling activities, as students realize throughout their familiarization with activities that they are given autonomy to navigate the resolution process.

Another moment when strategic learning was emphasized was when students remembered the need to double one dimension of the Pastry so that it could be filled and closed, highlighting the organization of work and the need for a change to solve the problem. Even though they were "at the end" of the resolution, students demonstrated that to give mathematical meaning to the problem situation, it was necessary to reformulate ideas when appropriate, and they did this with great enthusiasm.

Therefore, it supports Koga and Silva (2022, p. 223) when they state that "[...] understanding and identifying heuristic strategies in modeling activities supports the practice of modeling," by evidencing the presence of strategies in the development of the activity titled "Pastry".

NOTES

1. A 'pastel' is a popular Brazilian savory pastry made from a thin, crispy dough filled with a variety of ingredients such as cheese, meat, or vegetables. The dough is folded over the filling and then deep-fried until golden brown. We will refer to it as a pastry.

2. Canva is an online design and visual communication platform used to create publications, presentations, brochures, and others.

3. Text translated by Maria Vitoria R. B. de Carvalho. Education: English Literature, Londrina State University.

REFERENCES

Almeida, L. M. W. (2020, Janeiro). Estratégias heurísticas como meios de ação em atividades de Modelagem Matemática. *Com a Palavra, o Professor*, Vitória da Conquista (BA), 5(11), 220 – 236. https://doi.org/10.23864/cpp.v5i11.563

Almeida, L. W., Silva, K. P. A., & Vertuan, R. E. (2012). *Modelagem Matemática na Educação Básica*. São Paulo, SP: Contexto.

Almeida, L. W., Silva, K. A. P., & Veronez, M. D. (2021). *Elementos Semióticos em atividades de Modelagem Matemática*. 1. ed. São Paulo: Livraria da Física.

Barbosa, J. C. (2003). Modelagem matemática e a perspectiva sócio-crítica. In: SEMINÁRIO INTERNACIONAL DE PESQUISA EM EDUCAÇÃO MATEMÁTICA, Santos. *Anais*... São Paulo: SBEM.

Bassanezi, R. C. (2002). *Ensino-aprendizagem com modelagem matemática*. São Paulo: Contexto.

Bliss, K., & Libertini, J. (2016). What is Mathematical Modeling? In: GARFUNKEL, Sol; MONTGOMERY, Michelle. *GAIMME:* Guidelines for Assessment & Instruction in Mathematical Modeling Education. COMAP, SIAM: Reston, Philadelphia. https://doi.org/10.1007/s10763-021-10232-8

Blum, W. (2015). Quality teaching of mathematical modelling: what do we know, what can we do? In: CHO, S. J. *The Proceedings of the 12th International Congress on Mathematical Education.* Cham: Springer, 76-96. https://doi.org/10.1007/978-3-319-12688-3_9

Bogdan, R. C., & Biklen, S. K. (1994). *Investigação qualitativa em educação* – uma introdução à teorias e aos métodos. Porto: Porto Editora.

Borromeo Ferri, R. (2007). Modelling problems from a cognitive perspective. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.). Mathematical modelling: Education, engineering and economics. ICTMA12. Publishing: Horwood. 260-270. https://doi.org/10.1533/9780857099419.5.260

Carreira, S. (2021). *Elementos Semióticos em atividades de Modelagem Matemática*. Prefacio In: Almeida, L. W.; Silva, K. A. P., & Veronez, M. D. 1. ed. São Paulo: Livraria da Física.

Carreira, S., & Baioa, M. (2011). Students' modelling routes in the context of object manipulation and experimentation in mathematics. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.). *Trends in teaching and learning of mathematical modelling.* Dordrecht: Springer. ICTMA14. 211-220. https://doi.org/10.1007/978-94-007-0910-2_22

Castro, E. M. V. & Veronez, M, R, D. (2018). Diferentes encaminhamentos para um mesmo tema em atividades de modelagem matemática. *ACTIO*, 3(3), 1–18. https://doi.org/10.3895/actio.v3n3.7782

Cavalheiro, G. C. S., & Meneghetti, R.C.G. (2020). Metodologia de Ensino-Aprendizagem-Avaliação de Matemática através da Resolução de Problemas: uma Análise das Perspectivas de Licenciados em Matemática. *Jornal Internacional de Estudos em Educação Matemática,* 13(1), 64-72. https://doi.org/10.17921/2176-5634.2020v13n1p64-72

Damore, B., Pinilla, M. I. F., & Iori, M. (2015). *Primeiros elementos de Semiótica:* Sua presença e sua importância no processo de ensino-aprendizagem da matemática. 1. ed. São Paulo: Editora Livraria da Física.

Duval, R. (2012). Registros de Representação Semiótica e Funcionamento Cognitivo Do Pensamento. *Revista Eletrônica de Educação Matemática*, 7(2), 266-297. https://doi.org/10.5007/1981-1322.2012v7n2p266

Hoon, T. S., Kee, K. L., & Singh, P. (2013). Learning Mathematics using heuristic approach. *Procedia - Social and Behavioral Sciences* 90, 862–869. https://doi.org/10.1016/j.sbspro.2013.07.162

Koga, T. M. (2020). *O fazer Modelagem Matemática em um curso de Licenciatura em Química: análise de estratégias e ações*. 2020. 131f. (Dissertação de Mestrado em Ensino de Matemática) - Universidade Tecnológica Federal do Paraná. Londrina.

Koga, T. M., & Silva, K. A. P. (2022). Familiarização com Modelagem Matemática e as Estratégias Heurísticas dos Alunos. *JIEEM*, v. 15, 214–225. https://doi.org/10.17921/2176-5634.2022v15n2p214-225

Meirelles, H. L. (1995). *Direito Administrativo Brasileiro*. 20. São Paulo: Malheiros.

Mendes, T. F., Sousa, B. N. P. A. & Almeida, L. M. W. (2017). Modelagem Matemática e conhecimento matemático: uma análise de misconcepções. In: ENCONTRO PARANAENSE DE EDUCAÇÃO MATEMÁTICA, 14. *Anais*...Cascavel: SBEM.

Niss, M., & Blum, W. (2020) *The Learning and Teaching of Mathematical Modelling.* 1.ed. Abingdon: Routledge.

Oliveira, J. M. S. R., Grzybovski, D., & Sette, R. S. (2010). Origens e fundamentos do conceito de estratégia: de Chandler à Porter. *Conexão Ciência*, 5(1), 35-48. https://doi.org/10.24862/cco.v5i1.57

Origem da palavra estratégia. *Dicionário etimológico*. (2008). Disponível em: Origem da palavra ESTRATÉGIA - Etimologia - Dicionário Etimológico. Acesso em 29, setembro de 2023. https://www.dicionarioetimologico.com.br/estrategia/

Pinilla, M. I. F. (2010). *Múltiples aspectos del aprendizaje de la matemática: Evaluar e intervenir en forma mirada y específica*. 1. ed. Colombia: Editora Magistério.

Pollak, H. O. (2015). The Place of Mathematical Modelling in the System of Mathematics Education: Perspective and Prospect. In: G. A. Stillman, W. Blum, & M.S. Biembengut (Eds.). *Mathematical Modelling in Education Research and Pratice* (265-276). Cham, Switzerland: Springer. https://doi.org/10.1007/978-3-319-18272-8_21

Polya, G. (1945). *How to solve it.* Princeton University Press.

Polya, G. (1973). *How to solve it. A new aspect of mathematical methods* (2nd ed.). Princeton University Press.

Santana, H. C. P., & Gonzalez, P. L. (2019). Modelagem matemática como metodologia para o processo de aprendizagem de função exponencial. *ACTIO*, anais da III Semana das Licenciaturas da UTFPR Curitiba. https://doi.org/10.3895/actio.v1n1.10870

Silva, R. M.S. & Silva, K. A. P. S. (2021). Diálogos em atividades de modelagem matemática: uma análise à luz da educação matemática crítica. *ACTIO*, 6(2), 1–22. https://doi.org/10.3895/actio.v6n2.14137

Stender, P. (2018, abril). The use of heuristic strategies in modelling activities. *ZDM*: The International Journal on Mathematics Education, 50(1-2), 315–326. https://doi.org/10.1007/s11858-017-0901-5

Stender, P. (2017). The use of heuristic strategies in modelling activities. *ZDM* Mathematics Education. https://doi.org/10.1007/s11858-017-0901-5

Stender, P.; Kaiser, G. (2017, February). The use of heuristic strategies in modelling activities. *CERME 10*, Dublin, Ireland. https://hal.science/hal-01933448

Trindade, S. L. (2023). *Análise semiótica de componentes da aprendizagem em atividades de modelagem matemática no 8º ano do ensino fundamental.* (Dissertação de Mestrado em Ensino de Matemática), Universidade Tecnológica Federal do Paraná, Londrina.

Vertuan, R. E. (2007). *Um olhar sobre a Modelagem Matemática à luz da Teoria dos Registros de Representação Semiótica.* (Dissertação de Mestrado em Ensino de Ciências e Educação Matemática), Universidade Estadual de Londrina, Londrina. APA Publishing Training. (n.d.). *Home* [YouTube channel]. Retrieved February 20, 2020, from https://www.youtube.com/user/PsycINFO/

Received: 21 apr. 2024
Approved: 05 aug. 2024
DOI: https://doi.org/10.3895/actio.v9n2.18473

How to cite:
Trindade, S. L. & Silva, K. A. P da. (2024). An analysis of heuristics in strategic learning in a mathematical modelling activity. ***ACTIO***, 9(2), 1-20. https://doi.org/10.3895/actio.v9n2.18473

Correspondence:
Suzana Lovos Trindade
Avenida João Miguel Caram,n. 1250, Cidade Industrial 2, Londrina, Paraná, Brasil.