

# Critical meaningful learning in mathematics teaching-learning via problem solving from the perspective of undergraduate students

## RESUMO

The objective of the article is to analyze which principles of Critical Meaningful Learning (CSA) are evidenced by undergraduate students on the Teaching-Learning of Mathematics via Problem Solving (EAMvRP). To this end, a formation process was developed with 23 academics from a public university in Paraná. First, we discussed the 11 principles of ASC and, later, we worked on teaching the critical point content, Differential and Integral Calculus, in the EAMvRP approach. A questionnaire was administered to students to relate the principles of ASC with the actions of EAMvRP. The results reveal that the principles were evidenced to a greater extent in classroom actions, especially when introducing the problem and helping students during resolution. The research revealed that EAMvRP has the potential to promote the principles of Critical Meaningful Learning, especially the principle of not exclusively using the chalkboard. However, the principle of knowledge as language was not observed by the undergraduate students, indicating the need to explore it in future research.

**KEYWORDS:** Prior knowledge; Problem as a starting point; David Ausubel; Mathematics; Graduation

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# Aprendizagem significativa crítica no ensino-aprendizagem de matemática via resolução de problemas sob o olhar de licenciandos

## ABSTRACT

O objetivo do artigo é analisar quais princípios da Aprendizagem Significativa Crítica (ASC) são evidenciados por licenciandos sobre o Ensino-Aprendizagem de Matemática via Resolução de Problemas (EAMvRP). Para tanto, um processo de formação foi desenvolvido com 23 acadêmicos de uma universidade pública do Paraná. Primeiro, discutimos sobre os 11 princípios da ASC e, posteriormente, trabalhamos o ensino do conteúdo de ponto crítico, do Cálculo Diferencial e Integral, na abordagem do EAMvRP. Um questionário foi aplicado aos estudantes para relacionarem os princípios da ASC com as ações do EAMvRP. Os resultados revelam que os princípios foram evidenciados em maior grau nas ações em sala de aula, sobretudo na introdução do problema e no auxílio aos alunos durante a resolução. A pesquisa revelou que o EAMvRP tem potencial para propiciar os princípios da Aprendizagem Significativa Crítica, especialmente o princípio da não utilização exclusiva do quadro de giz. No entanto, o princípio do conhecimento como linguagem não foi observado pelos licenciandos, indicando a necessidade de explorá-lo em pesquisas futuras.

**PALAVRAS-CHAVE:** Conhecimento prévio; Problema como ponto de partida; David Ausubel; Matemática; Licenciatura.

## INTRODUCTION

More and more it has been defended as a process of teaching and learning of Mathematics which provides critical formation by the students (Erdoğan, 2020; Hartmann, Mariani & Maltempi, 2021). It has also been discussed about the importance of the work with their previous knowledge, in a manner of relating them to the new content which is expected to be taught (Biasotto, Fim & Kripka, 2020). Ausubel (1963) presents an theoretical explanation for this relation in his Theory of Meaningful Learning.

With emphasis, Moreira (2010), by deepening in the studies by Ausubel (1963), and considering the importance of a subversive education, that is, proposes, by means of 11 principles, what is called Critical Meaningful Learning – ASC. Since then, his theory has been researched and discussed in the many forms of Mathematics teaching.

In specific, in the problem solving, it is verified that there is a huge potential of favoring these principles when working the problem as a starting point, seeing this approach favors the students' previous knowledge (Assunção, Moreira & Sahelices, 2018; Puhl, Müller & Lima, 2020; Mendes & Proença, 2020). In this perspective, Proença (2018) presents the Teaching- Learning of Mathematics via Problem Solving – EAMvRP, based on five actions, as a manner of aiding teachers who want to work this way.

Considering the ASC and the EAMvRP, Mendes, Proença and Moreira (2022) proposed, in theoretical form, relations between the 11 principles by Moreira (2010) and the five actions by Proença (2018). However, Mendes, Proença and Moreira (2022) consider the principles might be evidenced beyond what was proposed by them. Having this in sight, we ask ourselves about in which actions and moments the principles might emerge in a class in the EAMvRP approach?

Thus, this research has the objective of analyzing which principles of Critical Meaningful Learning are evidenced by graduates about the Teaching-Learning of Mathematics via Problem Solving. In order to do it, a process of training about the ASC was developed with 23 graduates in Mathematics, by means of an activity developed with the EAMvRP to teach the content of critical point of Integral Differential Calculus.

After this introduction, we reflected on the relations between ASC and the EAMvRP. In the third section we explain our methodological procedures; In the fourth section we analyze the data obtained from the research and, lastly, we weave our considerations, in a manner of answering the guiding question.

## APPROXIMATIONS BETWEEN CRITICAL MEANINGFUL LEARNING AND THE TEACHING-LEARNING OF MATHEMATICS VIA PROBLEM SOLVING

Meaningful Learning is a cognitivist theory of teaching developed by Ausubel (1963) in which searches, in its essence, discuss the relation between previous knowledge with the new knowledge. In search of favoring its development in the classroom, Moreira (2010) proposed, by means of 11 principles, what he called Critical Meaningful Learning (ASC). These principles aim to guide the teacher for the development of a class in a subversive way. This term, subversive, is

understood here in the same meaning of Moreira (2010), which refers to the subjects allowing themselves to be part of their culture at the same time in which, outside of it, the students are part of their culture, but not subdued by it.

The first, *principle of previous knowledge*, is, maybe, what has more relation to the Meaningful Learning itself, once it implies in valuing the students' previous knowledge. Ausubel (1963) highlights that if he could summarize his theory, he would say it is important to take what the student already knows and work based on it. In this sense, the previous knowledge of the students on Mathematics itself must be valued in the classroom.

The second, *principle of social interaction and of questioning*, is considered by Moreira (2010, p. 9) in the sense of "teaching/learning questions instead of answers". In specific, when the teacher transmits answers to the students and then, later, transmits the tests, it comprehends a non-critical teaching, but a mechanical one (Moreira, 2010). In this manner, when the students ask interesting questions, this is revealed as a sign of meaningful learning. Otherwise, it is fundamental that the teacher and the students have a dialogical posture and that there are interactions between them, and, also, students to students.

The third, *principle of the non-centrality of the textbook*, cannot be understood in a literal manner, as if it were to banish the textbooks ( didactic books, theoretical books, reference books, as those of Calculus, Algebra, etc.) from school, but of the need of using, beyond this material, other materials. Moreira (2010, p. 10) comments that using only the textbook in the classes "is a deforming teaching practice, instead of forming, both for the students and the teachers".

The fourth, *principle of the apprentice as perceiver/representative*, refers that every student perceives the world and the information which are given to them in an unique manner and, eve, differently from other students. Moreira (2010, p. 11) comments "we see the things not as they are, but as we are". In this form, teacher and student must search to perceive in similar ways what is worked from the content.

The fifth, *principle of knowledge as a language*, refers that the language if a manner of perceiving reality, in a manner that everything which is knowledge might be considered as language, as an example, how Mathematics itself. For Moreira (2010, p. 12), learning this new language "[...] in a critical manner is perceiving this new language as a new way of perceiving the world".

The sixth, *principle of semantic awareness*, refers that "[...] the meaning is in the people and not in the words" (Moreira, 2010, p. 12). In specific, the students give meaning to the thighs based on their previous knowledge; However, if they cannot give meaning to some content, this reveals the learning was mechanical. Besides this, it is important the students understand the words are used to name things, but are not the thing itself, they only represent them.

The seventh, *principle of learning through mistakes*, refers to the normalization that we make mistakes many times and it is alright about it. However, it is necessary to learn from these mistakes. In specific, Mathematics is treated in an exact form, as an absolute truth and that, when we make mistakes, we have a punishment for it. But, according to Moreira (2010), in the perspective

of ASC, the most adequate occurs when we learn from this mistake, overcoming it.

The eighth, *principle of unlearning*, cannot be understood in a literal manner, in the sense that people are going to unlearn something, but yes that they can advance in the line of thought to a new one. As an example, this occurs in the courses of Mathematics when up to a certain moment for the students there was only Euclidean Geometry as knowledge. But, when they start to see Non-Euclidean Geometry, which is something that changes the perception of Geometry, it does not mean they are going to unlearn Euclidean Geometry, but that they are going to widen their concepts and focus on this new perception.

Besides this, this principle also has other characteristics, that the people might be selective about what they are learning, having in sight that, currently, we have thousands of information at every moment. Thus, Moreira (2010, p. 16) points out that “learning and unlearning is learning to distinguish between the relevant and irrelevant in the previous knowledge and freeing yourself from the irrelevant”.

The ninth, *principle of the uncertainty of knowledge*, is related to the idea that by means of definitions, questions and metaphors are the forms which we build the view of the world. In this sense, Moreira (2010) comments that:

The meaningful learning of these three elements is only going to be the manner which I am calling the critic when the apprentice perceives the definitions are human inventions, or creations, which all we know has origin in questions and all our knowledge is metaphorical (Moreira, 2010, p. 16).

In this case, for the learning to be critical, the students must have the understanding that the knowledge is our construction, in a form that it might or might not be wrong and also depends on how we built it.

The tenth, *principle of non-usage of the chalkboard*, also cannot be understood in a literal manner, in the sense the teacher cannot give classes writing with chalk on the board. Its understanding is related to the fact the teacher might give classes this way, but not only this way. Moreira (2010) considers important other materials and teaching strategies are used in this process.

The last, *principle of abandonment of the narrative*, implies in a direct form the teachers must let the student speak in their classes. The focus of learning is on the students, in how they understand it, how they apply it and how they ask it. Therefore, it is essential in a classroom in the ASC perspective, that the students talk more and the teacher mediates the process.

This way, these are 11 principles which, according to Moreira (2010), favor the development of a class which values the ASC. These might be present in various teaching approaches. In the case of Mathematics and, in specific, in a teaching with focus on problem solving, it is plausible that these principles are evidenced when working the problem as a starting point, considering that, this way, previous knowledge by the students are valued.

Thinking about it, Mendes, Proença and Moreira (2022) present in a theoretical form what would be the possibilities of ASC being present when using

the Teaching-Learning of Mathematics via Problem Solving – EAMvRP, by Proença (2018), based on five actions. This approach opposes the traditional method, considering it makes possible for the students the active participation in the process of teaching and learning in the construction of concepts. As we know, in traditional teaching the mathematical contents are worked with “presentation of the first content, followed by an example and later application of these subjects by the students in the activities known as exercises and even in those which are not contextualized” (Proença, 2018, p. 11). Contrary to it, the EAMvRP proposes a problem as a starting point to introduce new content, in a manner the students need to mobilize previous knowledge which will be related to the new content to be studied.

Proença (2018) still points out that many teachers consider the activities of applying concepts in contextualized situations as problems. However, the author considers it is not a coherent method of using the problem in the classroom. To understand this positioning, to which we believe, we can look for some definitions of the problem.

According to Klausmeier and Goodwin (1977, p. 347), “the individuals face a problem when they find a situation in which they must solve a problem and do not possess information, concepts, principles or specific methods available to reach the solution”. For a task to be considered a problem, Echeverría (1998, p. 48) says that “the people who are resolving this task need to find some difficulty which makes them question themselves about what would be the path which they need to follow to reach the goal”. That is, the definition of a problem is connected to something difficult, challenging, which does not have an immediate path of resolution, which does not happen when using a “problem” for the application of the mathematical concept or formula.

Besides this, it is also considered that in order to solve a problem it is necessary passing through stages of resolution, which involve the mobilization of diverse knowledge, that is why problem solving is configured as a process. In relation to the process of problem solving, Proença (2018) describes in four stages, being: representative, planning, execution and monitoring. The representation refers to the interpretation and understanding of the problem and involves linguistic knowledge (related to the meanings of the terms in mother language), semantic knowledge (related to the meaning of the mathematical terms) and schematic knowledge (related to the nature of the problem: algebraic, geometric, algébrico, geométrico, arithmetic, etc.).

The planning is related to the path thought to solve the problem, the strategies to be used, being with the usage of a chart, diagram, drawing, logical deduction, among other possibilities, which involve the strategic knowledge. The execution consists in putting into practice what was planned in the previous stage, it involves the usage of procedural knowledge, because mathematical procedures, chart assembling, diagram construction, etc, are going to be done. Lastly, monitoring refers to analyzing the answer, if it is adequate to what the problem requires, and reviewing the resolution problem, verifying if there are mistakes to be corrected.

Having as basis this comprehension on problem solving, an important aspect corresponds to the form of organizing the teaching which uses the problem as a

starting point. Aiming to provide guidance to the teachers who want to use this possibility in the classroom, Proença (2018) elaborated a sequence of five actions for the EAMvRP, being: choosing the problem, introduction of the problem, aid to the students during the resolution, discussion of the students' strategies and articulation of the students' strategies from the students to the content.

The *choice of the problem* consists in the planning by the teachers to the choice of a mathematical situation (possible problem), which might be taken as a whole from didactic books or other materials, reworked or elaborated by the teachers themselves. According to Proença (2018), in order to select a mathematical situation for the development of EAMvRP, the teacher must verify some elements, to be known: does it allow being solved by different strategies? Does it make possible the usage of previous knowledge by the students? Does it allow conducting to the introduction of a new subject? Does it make possible relations between previous knowledge and new knowledge? Fitting these characteristics, it is possible to choose the problem, as well as predicting some strategies to be used by the students, in a manner of planning and reflecting on the following actions to be developed.

According to Mendes, Proença and Moreira (2022), this choice of a situation of Mathematics which values the previous knowledge, might favor the 1<sup>st</sup> principle of *previous knowledge*. Beyond this, and maybe, this is the only moment in which the didactic book is used in this approach, because later other strategies are employed, this may value the 3<sup>rd</sup> principle of the *non-centrality of the textbook*. Still in this action, the authors highlight it provides the 4<sup>th</sup> principle of the *apprentice as perceiver/representative*, when it is searched for more than one strategy for the resolution of the situation, because in this manner they can contemplate a larger number of perceptions by the students.

In the *introduction to the problem* is where the chosen mathematical situation can be configured or not as a problem, having in sight the teacher will organize the class, preferably in groups, and present the situation so they solve in a manner they find more adequate. If the resolution for the search for a solution is a challenge for the students, then it is a problem. Concerning this action, Mendes, Proença and Moreira (2022) consider the 10<sup>th</sup> principle, *the non-usage of the chalkboard*, is promoted, once the focus of the class is in the formed groups and in the developed discussion. In the same form, the 5<sup>th</sup> principle, *of the knowledge as a language*, is approached, considering the student develops the process of learning in the discussions and interpretation of the problem.

In the *aid to the students during the resolution*, Proença (2018) indicates the teacher acts as observer, encourager and director of the learning, being able to follow the groups, verifying the strategies which are being developed, the procedures realized, encouraging the students to reach an answer, to argue about the mathematical knowledge realized in an autonomous and participant manner. In this moment, the teacher also realizes the evaluation, identifying the difficulties of the students, being able to give hints to guide them to a possible strategy (thought in the first action), in a manner to encourage them to solve the problem, without giving ready answers.

For Mendes, Proença and Moreira (2022), it is in this moment that the 2<sup>nd</sup> principle., *of interaction and questioning*, is privileged, because it is when the



students are discussing with their groups the mathematical knowledge. Besides this, according to Proença (2018), the teacher must not give ready answers to the students, but mediate the teaching process. It is in this moment the 9<sup>th</sup> principle, *of the uncertainty of the knowledge*, is propitiated. Otherwise, the 11<sup>th</sup> principle, *of abandonment of the narrative*, is also highlighted, having in sight in this moment the teacher has the role of observer, encourager and director, leaving, thus, the students speak and discuss, as pointed out by Moreira (2010).

In the *discussion of students' strategies*, some socialization occurs in which the group, or a representative of each group, goes to the board and presents their resolution to the class. At this moment the teacher may clarify some mistakes, discuss if the answer they found meets the needs of the problem, as well as evaluate the students about the stages of the process of problem resolution.

According to Mendes, Proença and Moreira (2022), it is in this action the 7<sup>th</sup> principle, *of learning through the mistakes*, occurs, seeing the strategies are discussed with the class and the teacher makes the indication of possible mistakes. Then, the students might verify if and where they made mistakes and learn through it. The authors also comment that the 8<sup>th</sup> principle, *of unlearning*, is worked in this part, when the teacher leads the students to a rationality of the answer.

In the *articulation of the students' strategies to the content*, the teacher highlights one of the strategies presented in the discussion and, from its main points, relate it to the new subject to be taught, making is possible a connection between previous knowledge mobilized by the students to solve the problem and the new knowledge, making it easier the understanding of new concepts and mathematical expressions.

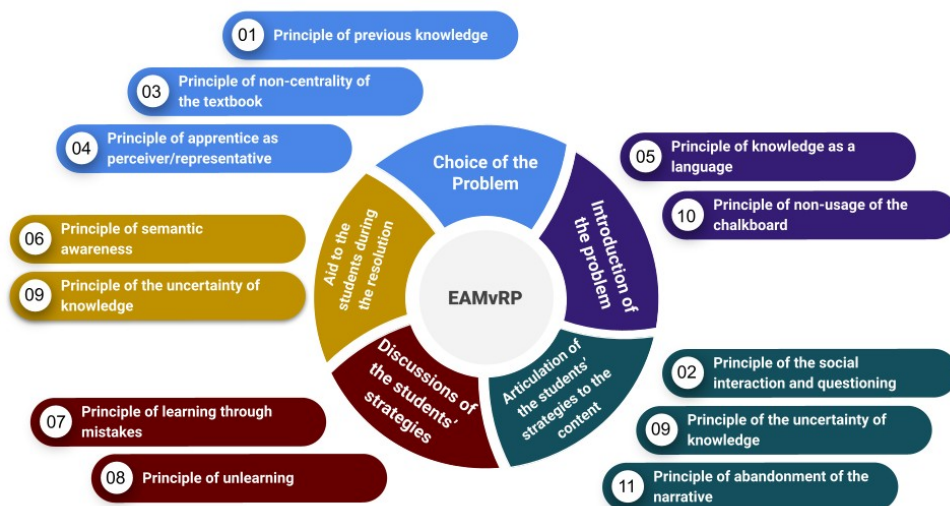
In this last action, Mendes, Proença and Moreira (2022) comment that the 6<sup>th</sup> principle, that *of semantic awareness*, is favored when the teacher makes the process of articulation of the new knowledge. Once the students did not know what subject was to be taught, it is also valued the 9<sup>th</sup> principle, *of the uncertainty of knowledge*. To evidence in a more dynamic manner this relation, we highlight in Figure 1 the organization proposed by Mendes, Proença and Moreira (2022).

The data/results description is clear in charts and figures, preceded by the respective chart/figure. The charts and figures are numbered with arabic numbers in font calibri 11, bold (spacing of 12pts before and 6 pts later) then, right below, the title font calibri 11, italic (spacing 6pts after). Below the chart/figure it is indicated the source in font calibri 9, bold (spacing 12pts before and 6 pts after).

The illustration must be cited in the text and inserted the closest possible to the excerpt to which it refers. See as an example, Figure 1.

**Figure 1**

*Approximations between Critical Meaningful Learning and the EAMvRP.*



Source: Mendes, Proença and Moreira (2022)

In this manner, this association between Critical Meaningful Learning, proposed by Moreira (2010), and the Teaching-Learning of Mathematics via Problem Solving, defended by Proença (2018), is theorized in the study by Mendes, Proença and Moreira (2022).

It is important to highlight that they are not evidenced in other studies which proposed relations between the ASC and the problem solving. However, we evidenced relations in research between Meaningful Learning – ML and PS or the ASC and Mathematics, which we consider might contribute to the discussion. Having this in sight, the research by Sangoi, Isaia and Martins (2011), by searching to work concepts of Derivation in a subject of Integral and Differential Calculus, it was verified a process of teaching-learning with more meaning as there was more communication between teacher-student. Besides that, the process differs as the academics use their own strategies instead of following only what is given by the teacher, what favored their previous knowledge.

Puhl, Müller and Lima (2020) by relating the Meaningful Learning to the problem solving, when the problem is the starting point for the teaching, verified confluences among the. The authors stress that:

Both proposals are based on the student as protagonist and understand it as the propellant of cognitive and intellectual development, consider the student's previous knowledge for the construction of meaning; and have as potential to provide the development of the knowledge, skills and competences for a citizenship training (Puhl, Müller & Lima, 2020, p. 125).

In this case, according to the authors, these points must be deepened about the development of didactic strategies.

The research by Assunção (2015) proposed strategies to work PS and the ML, in a manner which favors the process of teaching and learning. Thus, the authors



consider as a really important point, the teacher mediation in the sense of directing the learning. In the same form Assunção, Moreira and Sahelices (2018, p. 43) also explored the concept of function by mean of the PS and ML, and considered that in this perspective it is “[...] needed for the teacher to interact, question, answer doubts and, lastly, aid the students the whole time so they do not get despondent”. Besides this, the authors stress the previous knowledge is the starting point for the learning of new mathematical concepts by means of problem solving.

About what concerns this same content, that of functions, the study by Dias (2019) aimed to evidence the principles of the ASC with a High School class. Between them, the most present in the process of teaching were the principle of previous knowledge, of the non-centrality of the textbook, of knowledge as a language, of the non-usage of the chalkboard and the abandonment of the narrative. The others appeared with less emphasis, according to the participants of the research.

Chirone, Moreira and Sahelices (2021) searched to explore the Critical Meaningful Learning in the teaching of numbers and their groups. The principles were analyzed in relation to the application of a summative test, among which were more present the semantic awareness, learning through mistakes, knowledge as a language and abandonment of the narrative. In this sense, based on this principles, the authors stress that:

77.5% of the students understand two or more of the following items: they perceive what is taught to them, they built mental representations; they aim to discover and correct their mistakes; as well as they use the verbalization in written form to demonstrate the acquired knowledge (Chirone, Moreira & Sahelices, 2021, p. 18).

This reveals the importance of these practices in the teaching of Mathematics. In the same line of thought, Carvalho (2012) searched to analyze the relation between ASC and Mathematics, with students from the 9<sup>th</sup> grade from Elementary School to work the Thales Theorem. In his study were manifested, according to the students, the principles of the non-usage of the textbook, of the apprentice as perceiver/representative, of the knowledge as language, semantic awareness, of unlearning, of uncertainty of knowledge and the abandonment of the narrative.

In this manner, it is evidenced that Meaningful Learning has been discussed and evidenced when working with problem solving. Otherwise, the ASC, by means of its principles, is manifested in processes of teaching and learning of Mathematics.

## **METHODOLOGICAL PROCEDURES**

This study is supported in the assumptions of the qualitative research, because “it is developed in a natural situation, is rich in descriptive details, has an open and flexible plan and it focuses the reality in a complex and contextualized form” (Lüdke & André, 1986, p. 18). In specific, it is characterized as a descriptive research, which [...] has as main goal the description of the characteristics of determined populations or phenomena or the establishment of relations between variables” (Gil, 2008, p. 27).

The participants of the research were 23 academics who were studying the third year of the Mathematics graduation course, of a public state university in the State of Paraná. To do so, it developed a training process on ASC and an activity with the usage of EAMvRP to work the content of critical numbers, which is discussed in the subject of Integral and Differential Calculus. In Table 1, it is presented the organization of how the research was developed.

**Table 1**

*Organization of the development of the activities done in the research.*

Class	Activity	Development	Time
1 <sup>st</sup>	Discussion and reflection on the ASC	– Discussion about Meaningful Learning	1h50
		– Presentation and reflection on ASC (11 principles)	
		– Presentation and discussion of Mathematical Examples	
		– Synthesis of the ideas	
2 <sup>nd</sup>	EAMvRP	– Retomada da ASC	1h50
		– Realization of the activity using EAMvRP for the teaching of the calculus content	
		– Reflection between the practical activity and the five actions by Proença (2018) in theoretical form	
	Questionnaire Application	– <i>On-line</i> Questionnaire via <i>Google Forms</i>	

Source: The authors (2024).

According to Table 1, in the activity with the EAMvRP, was approached the following Mathematical situation: “A farmer has 1.200 m of fence and wants to cover a rectangular field which is at the margin of a straight river. He does not need to circle along the river. What are the dimensions of the field which has the largest area?” (Stewart, 2013, p. 294). This Mathematical Situation was worked to approach the concept of critical number in which “a critical number of a function  $f$  is a number  $c$  in the domain of  $f$  thar or  $f'(c) = 0$  or  $f'(c)$  does not exist” (Stewart, 2013, p. 251).

It fits stressing this activity was developed by the first author of this research as a professor. In this process, his role constituted in favoring the students to experience the EAMvRP in his five actions. Thus, the professor also searched only to mediate the process and develop everything Proença (2018) recommends.

After the development of the process of training and practical activity, a questionnaire was applied, containing 11 open questions. Each question involves an ASC principle with the following question, as an example: “Did you identify at any moment the principle of previous knowledge? If yes, comment about it”. The answers to these questions were analyzed in qualitative form, with the intent of verifying at what moment of the practical activity (EAMvRP actions) each principle was evidenced. In this manner, we aimed to group similar responses to the same principles, in a way that the 11 principles consisted of our categories

(P1, P2, ... P11), given *a priori*. The data obtained are presented in the next session.

## DATA ANALYSIS AND DISCUSSION

The first, the principle of previous knowledge (P1), was considered by 21 of 23 participants of the research.

A1 – Yes, it was the basis to start the resolution.

A8 – Yes, because from the second degree equation the student developed a new area of Calculus.

A13 – Yes, knowledge over the area and perimeter.

A15 – Yes, quadratic functions and study of area.

A21 – Yes, by being realized a general approach of the answer were used diverse previous concepts as function, derived, among others.

Concerning the first principle, the answer by A1 demonstrates it was approached in the action of *introduction of the problem*, when the student starts solving the situation. On the other hand for A21, this previous knowledge got clearer of need in the last action by Proença (2018), when the relation between previous knowledge and new knowledge must be done. Furthermore, as the answers by A8, A13 and A15, the contents of second degree equations, area, perimeter and quadratic functions were used as previous knowledge to teach the new subject.

The second, principle of social interaction and questioning (P2), was indicated by 19 of 23.

A1 – Yes, in the end, to evaluate the resolutions, we opened for social interaction and a debate.

A3 – Yes, after the wrong interaction I did, we discussed in a group ideas which opposed what was thought.

A7 – Yes, because the professor allowed various students to present their resolutions.

A15 – There was dialog and interaction all the time.

Based on the answers above, what transpired refers to the moment the academics were working in a group, as demonstrated in the answers by A3 and A15. This refers to the actions of *introduction of the problem* and *aid during the resolution*. Furthermore, for A1 and A7, the social interaction occurred in the action of *discussion of the students' strategies*, when they presented and discussed with the class their resolutions. The social interaction was also a pertinent factor in the research by Sangoi, Isaia and Martins (2011), in a way the discussions between professor-student and student-student provided a bigger understanding of the subject.

The third, principle of the non-centrality of the textbook (P3), was considered by all academics, as demonstrated in the following answers:

A4 – Yes, the professor demonstrated the situation was taken from the didactic book of calculus, but he worked practically without using it.

A19 – The problem is from the book, but it was not used.

A23 – The situation was taken from the didactic book, but the situation was taken from the didactic book, but the class was not focused on it.

It is perceived by means of the responses that the textbook was used by the 23 academics to choose the mathematical situation the professor used in the class, but it was not the center. In this sense, this principle is evidenced in the first action, the *choice of the problem*, the moment in which the professor in fact used the book to pick the mathematical situation which would work in the classroom. In the research which discussed ASC, this was one of the principles which were more evidenced (Dias, 2019; Chirone, Moreira & Sahelices, 2021).

The fourth principle, of the apprentice as perceiver/representative (P4), was noticed by 13 of the 23 participants. The answers of the students who considered this principle are presented.

A2 – Yes, each one understood and it was through a different path of resolution.

A9 – Yes, each one had a form of development and reached the same result.

A19 – Yes, while the professor gave the idea and explained his own method of resolution.

A23 – Yes, each one explained the problem in our own experiences.

Based on the answers by A2, A9 and A23, it is possible to understand that the apprentice as perceiver/representative occurs when each group has some understanding about how to solve the problem and, from this, follows a path. This is revealed then in the second and third actions from the EAMvRP. Otherwise, A19 also highlights the perception/representation of the professor when explaining how to solve the content. This occurs in the process of articulation of students' strategies to the content. For Carvalho (2012), this principle also occurred this way, when there was the explanation of what the student was learning.

The fifth principle of knowledge as language (P5), was not evidenced by any academic. Only A19 justified the answer when stressing that "No, because we did not need to interpret the words". In fact, in all the process there was not presented a new word or mathematical symbol. But, Moreira (2010, p. 12) highlights that "practically everything we call 'knowledge' is language". In this sense, the academics did not present an explanation that the new knowledge on the critical point could be a new language. They focused only on the existence of this principle if they learned a new word or mathematical symbol. It fits highlighting that Moreira (2010) stresses Mathematics itself is a language. In the research by Chirone, Moreira and Sahelices (2021), the principle of knowledge as language was evidenced when they perceived what was taught to them. However, in the case of our research, we considered the academics perceived what is taught to them, but did not know how to relate this to P5.

The sixth principle, of semantic awareness (P6), was considered by five academics of 23. The justifications by the graduates who considered this principle were:

A3 – Yes, we worked in various forms and languages on the subject.

A10 – Yes, if we repeated as the teacher the form of thinking this would end limiting us in certain cases.

A21 – Yes, because it was the way we understood how to solve after attempts and failures.

A22 – Yes, asking the students to explain how they solved it.

The focus of the principle of semantic awareness, according to Moreira (2010), is that the meaning is in the people and not in the words, that is, each person creates the meaning and his or her own view of the Mathematics they are learning. This ends up being represented in the answers by A3, A10, A21 and A22, in the sense that in the moments they were solving, they did it their way, not following a previous form, as it is when it happens in traditional education. In this sense, the EAMvRP desconcretizes the learning of Mathematics as something immutable, ready and exact. Thus, this principle was present in the second and third actions of the EAMvRP, when there is the process of problem solving. It is possible to stress that in the case of the research by Chirone, Moreira and Sahelices (2021), this was one of the most evidenced principles by the students.

The seventh principle, learning through the mistakes (P7), was highlighted by 17 of 23 students, when considered that:

A3 – Yes, considering we made mistakes once and needed to reflect to find the correct path.

A11 – Yes, in the form of problem solving itself, in the development of the problem through trial and error.

A15 – Yes, we were Learning with our mistakes and searching for new manners to solve the problem.

Based on the academics' answers, the learning through mistakes occurred, mainly, while they were realizing the problem solving in the second and third actions of EAMvRP. In specific, the main strategy used by the academics, as explained by A11, was of trial and error. This reveals the approached mathematical situation was constituted as a problem, at least to those 17 students, having in sight they did not have an algorithm or previous formula to execute this resolution. In specific, these academics had already seen or should have seen this content. However, possibly, the subject was not internalized and meaningful to them.

The eighth principle, of unlearning (P8), was evidenced by two academics. A3 stressed that "Yes, at the moment it is perceived the perimeter is not fixed, then we cannot say the square is the largest area of the rectangle". This reveals some unlearning, when observing squares and rectangles which possess the same area, but different perimeters. Thus, this principle was evidenced in the second and third actions of EAMvRP. On the other hand A19 answered "Yes, because there

were other forms of solving the matter we need to let go". The other forms of solving were discussed in the fourth action, when the groups shared their resolutions. Unlearning was also important when approached the subject of Thales Theorem in the research by Carvalho (2012).

The ninth principle, of uncertainty of knowledge (P9), was considered by 9 students, who stressed:

A10 – Yes, when the professor opened to different methods of resolution, in which different groups went to the board to demonstrate different methods.

A15 – Yes, we had different manners of problem solving.

A23 – Yes, because I did many attempts on the same problem.

Based on the answers by A15 and A23, the uncertainty of the knowledge happened at the moment of the resolution of second and third actions of EAMvRP. On the other hand A10 highlights this uncertainty was evidenced in the fourth action, *the discussion of students' strategies*.

The tenth principle, of non-usage of the chalkboard (P10), was pointed by 12 academics, according to their speech:

A3 – Yes, having in sight we did not use it as an only resource, we used as an example the dialogue.

A10 – Yes, at the moment the professor brought an activity out of the standard.

A18 – Yes, many resources were used, such as the board, slide, notebook to answer, among others.

The speeches by A3 and A18 go in encounter to what Moreira (2010) highlights, in the sense of not stopping using the chalkboard, but of also working with other resources. In specific, the EAMvRP is a form of teaching different from traditional education, as pointed out by A10. Then, this principle is present in all the development of the activity which was realized with the EAMvRP. Based on the evidenced studies which approach the ASC in the teaching of Mathematics. It is verified that working in a critical manner implies the teacher uses strategies and not only the board (Dias, 2019; Chirone, Moreira & Sahelices, 2021).

The last principle, of abandonment of the narrative (P11), five graduates considered it was present in the EAMvRP, as demonstrated by some speeches:

A1 – Yes, by debating the ideas with the professor at the end of the activity.

A2 – Yes, at the end when we debated.

A15 – Yes, we had lots of space to speak and to explain, mainly at the end of the class.

A18 – Yes, the students participated a lot, both them and the professor.

For those academics, it was evident that in this process of teaching they were able to manifest their opinions. Moreira (2010) stresses that in a classroom which favors the Critical Meaningful Learning, the students speak more than the teacher. In specific, the academics highlighted this was more evident in the



fourth and fifth actions of EAMvRP, when they have to explain their ideas. In the studies by Dias (2019), Chirone, Moreira and Sahelices (2021) and Carvalho (2012), this principle was also present, when the focus was on letting the students speak.

With the objective of letting these relations between what graduate pointed out about every principle clearer, Table 2 presents all the answers of the questionnaire. In special, it is colored in green when the academic considers the principle was evidenced and in orange when it was not.

**Table 2**

*Evidences of the principles in the Critical Meaningful Learning.*

Subject	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	TP
A1												6
A2												7
A3												8
A4												4
A5												4
A6												7
A7												5
A8												5
A9												7
A10												6
A11												5
A12												5
A13												5
A14												5
A15												7
A16												2
A17												5
A18												6
A19												6
A20												4
A21												4
A22												7
A23												6
TA	21	19	23	13	0	5	17	2	9	12	5	

Source: The authors (2024).

Based on Table 2, the principle which was evidenced by all academics in the Teaching-Learning of Mathematics via Problem Solving, in specific, in the teaching of the subject central point of Integral and Differential Calculus, was of the non-centrality of the textbook or the didactic book. This might have occurred, because when working with this approach, the first action, the *choice of the problem*, it is something which occurs before starting the class. In this case, the teacher used the book to choose the mathematical situation, but as in a class the situation is presented through slides, the book ended up being only mentioned in the end of the class, but not used in a physical manner. With emphasis, this principle is in not utilizing only the book, but also other materials. In this manner, it is considered it might have become more perceptible to the graduates.

Other principles more evidenced by the licenced were referent to the previous knowledge (P1), to the social interaction and questioning (P2), of the apprentice as perceiver/representative (P4), to the learning through mistakes

(P7) and to the non-usage of the chalkboard (P10). On average, each academic stressed evidencing 5 principles in general.

On the other hand, the principle of knowledge as a language (P5) was not evidenced by any student. This principle is singularly intrinsic to the people, in the sense of perceiving something new to them. In this case, the graduated did not evidence a new language, a new symbol or word. They had new knowledge, of the subject of critical point, but did not consider it as a new language, what made them not evidencing this principle. The principles which were less evidenced were the semantic consciousness (P6), of unlearning (P8), of uncertainty of knowledge (P9) and abandonment of the narrative (P11).

Thus, Table 3 aims to contrast possible relations on what Mendes, Proença and Moreira (2012) considered in a theoretical form about the principles which could be evidenced in the EAMvRP, with what the academics pointed out after the activity with the EAMvRP.

**Table 3**

*Relations between the principles of ASC with the EAMvRP actions.*

Actions by Proença (2018)	Principles identified by Mendes, Proença and Moreira (2022)	Principles evidenced by the academics
Choice of the problem	P1, P3 and P4	P3
Introduction of the problem	P5 and P10	P1, P2, P4, P6, P7, P8, P9 and P10
Aid to the students during the resolution	P2, P9 and P11	P2, P4, P6, P7, P8, P9 and P10
Discussion of the student's strategies	P7 and P8	P2, P8, P9, P10 and P11
Articulation of the students' strategies to the content	P6 and P9	P1, P4, P10 and P11

Source: The authors (2024).

Based on Table 3, the action *choice of the problem* was punctuated by Mendes, Proença and Moreira (2022) as the moment in which three principles could be evidenced. However, only the principle or the non-centrality of the textbook (P3) had relation. In a class in the perspective of EAMvRP, it is in this action the teacher must choose a mathematical situation which might become a problem to the students (PROENÇA, 2018). In this case, the textbook used as basis to work a subject referring to the subject of Integral and Differential Calculus, was of great importance. It also fits stressing the situation itself in the used book was not present in the subject of critical point, but as extra situations in the end of the chapter.

The *introduction of the problem* is the action in which Mendes, Proença and Moreira (2022) consider two principles might be evidenced. However in the principle of knowledge as language (P5) it was not pointed out by the academics in the activity. On the other hand the principle of the non-usage of the chalkboard (P10) had relation between theory and practice. However, this principle ends up being used in the EAMvRP as a whole, once the approach is different in its whole from a teaching in which the focus is only on the chalkboard, without any other material.

In the *aid to the students during the resolution*, are highlighted theoretically three principles. Of those, the principle of social interaction and of questioning (P2) and of uncertainty of knowledge (P9) present relations between the ASC theory and the activity with EAMvRP. With emphasis, these two principles were stressed by the graduates in every action they had to develop through the activity, that is, the second, third and fourth actions of EAMvRP. About the principle of abandonment of the narrative (P11), it was present in the last two actions, when there were many discussions on the resolutions and the content.

It is important highlighting the second and third actions of EAMvRP are those which focused the most the evidences of principles of Critical Meaningful Learning. This happened, mainly, because these actions are the moment of problem solving by the graduates, in a manner in which it is in this interaction between the professor and the academics and between the students themselves that we perceive the favoring of an ASC. This implies directly in the posture of the teacher and the format in which the class is developed, in the sense the students have more space to discuss, reflect, think, make mistakes, text and get the right answer.

In the third action, of *discussion of the students' strategies*, the principle of unlearning (P8) had connection between theory and practice. But, the principle of learning through mistakes (P7) pointed by Mendes, Proença and Moreira (2022), in this action, was more present in the attitudes of problem solving of the graduates (2<sup>nd</sup> e 3<sup>rd</sup> actions), mainly, when they made mistakes and had to review their resolutions.

Lastly, the action of *articulation of the students' strategies to the content* did not obtain any association between the ASC theories and the activity with EAMvRP. However, it is pertinent verifying that, besides Mendes, Proença and Moreira (2022) pointing out the moment the ASC could be more evidenced, this work demonstrates this is not an easy task nor unique, once each person has his or her point of view. However, a result which was evidenced is that, in general, the principles are highlighted more than once in a classroom in the perspective of Teaching-Learning of Mathematics via Problem Solving, which stresses its importance as a teaching approach.

## FINAL CONSIDERATIONS

Favoring a process of teaching which works with the previous knowledge of the students has been discussed as something fruitful in the literature. Thinking about it, Moreira (2010) presents the Critical Meaningful Learning, by means of the 11 principles which guide the teachers' work. In specific, Mendes, Moreira and Proença (2022) consider, in theoretical way, these principles might be highlighted when the problem is worked as a starting point to involve the students in problem solving.

With this in sight, this research had the objective of analyzing which principles of Critical Meaningful Language are evidenced by graduates on the Teaching-Learning of Mathematics via Problem Solving. To do it, by means of a qualitative approach, it developed and analyzed a training process with 23 graduates. These, in a first moment, learned about the principles of the ASC. In a

second moment, they got involved in an activity in the EAMvRP approach to work the content of critical point referent to the subject of Differential and Integral Calculus. Lately, they answered a questionnaire, pointing out in which moments they evidenced these principles.

Our results go in encounter to Mendes, Proença and Moreira (2022) pointing to, when highlighting that a principle might appear in more than one EAMvRP action. In fact, this occurred when the principles were evidenced, mainly, in the actions in the classroom (2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup> and 5<sup>th</sup> actions). With emphasis, when the academics received and were solving the problems in the second and third actions, it was the moment the principles were more manifested. This reveals the importance of the actions of *problem introduction* and *aid to the students during the resolution*.

From the associations between the ASC theory and the activity in EAMvRP, our data allow us to highlight the Teaching-Learning of Mathematics via Problem Solving has a huge potential to provide the principles of Critical Meaningful Learning. In particular, mainly, the principle of the non-usage of the chalkboard (P10), which was the most present one in the actions in the classroom. This reveals how distant the EAMvRP is from traditional teaching, seeing that in its development many strategies are used and not only the chalkboard.

Lastly, the fifth principle, of knowledge as language, it was not evidenced by the graduates. This principle treats that not necessarily, it is needed to learn a new symbol or word, but that, yes, all the knowledge is a new language in which you are learning, as Mathematics itself. This might be explored with more attention in future research.

However, our study contributes in the sense of evidencing the associations made by the graduates, once they could get involved in the form of approaching EAMvRP and search to relate the ASC to what they have done. From the scientific point of view, this contributes to strengthening the role the EAMvRP approach might promote if adopted in the classroom. Therefore, future studies might be done in a sense of investigating how the graduates in Mathematics perceive their teaching in the classroom with students when based on the ASC and EAMvRP.<sup>1</sup>

## NOTES

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**Received:** 18 sep. 2023

**Approved:** 28 feb. 2024

**DOI:** <https://doi.org/10.3895/actio.v9n1.17592>

**How to cite:**

Mendes, Luiz Otavio Rodrigues Mendes; Oliveira, Ana Beatriz de; Proença, Marcelo Carlos de. (2024). Critical meaningful learning in mathematics teaching-learning via problem solving from the perspective of undergraduate students. *ACTIO*, 9(1), 1-20. <https://doi.org/10.3895/actio.v9n1.17592>

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