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Students' arguments while solving arithmetic expressions

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ABSTRACT

Argumentation is considered one of the fundamental elements to favor learning in mathematics. Based on this assumption, the article presents the results of a survey carried out with elementary school students to analyze the structural and content characteristics of the arguments developed by students in solving arithmetic expressions. The theoretical contribution was based on the argumentation models of Sales and Toulmin. The present study can be considered qualitative in nature and had a sample of 10 students. Data collection was performed using two instruments, the application of questions and the interaction between the students and the researcher. The analysis was based on the models of arguments in our theoretical foundation, and the results showed that the arguments developed by the students tended to be rational, but we also found natural arguments when dealing with contextualized questions. Furthermore, most of the arguments presented the basic elements of Toulmin Pattern Argument.

KEYWORDS: Argumentation; Arithmetic Expression; Mathematics Education.

Argumentos de alunos na resolução de expressões aritméticas

RESUMO

A argumentação é considerada um dos elementos fundamentais para favorecer a aprendizagem em matemática. Com base nesse pressuposto, o artigo apresenta os resultados de uma pesquisa, de natureza qualitativa, realizada com estudantes do ensino fundamental, a qual teve como objetivo analisar as características estruturais e de conteúdo dos argumentos por eles elaborados, na resolução de problemas envolvendo expressões aritméticas. Os dados constituíram-se nos registros escritos dos estudantes produzidos como respostas às questões propostas e das interações entre eles e a pesquisadora. A análise foi desenvolvida por meio do Modelo de Argumento de Toulmin e de algumas categorias propostas por Sales. Os resultados indicam a habilidade da maioria dos estudantes com as operações matemáticas requeridas nas questões, mas dificuldades, tanto na compreensão do contexto de cada uma delas, quanto na transformação da linguagem natural para a linguagem matemática, por meio de expressões aritméticas. Os argumentos tendiam ao racional, mas foram encontrados também argumentos naturais. Além disso, a maioria dos argumentos apresentou os elementos básicos do Modelo de Toulmin.

PALAVRAS-CHAVE: Argumentação; Expressão Aritmética; Educação Matemática.

INTRODUCTION

In traditional mathematics teaching, memorization and repetition are usually favored over understanding. This practice is very common since the first years of elementary school and is "(...) more frequent when students are encouraged to do the same type of exercises, based on a model provided by the book or by the teacher" [our translation] Pais (2006, p. 36). Students thus tend to get used to oral teaching of contents, copying, training, and repeated activities. This way, their curiosity is not aroused, and begins to disappear.

Research on argumentation in mathematics teaching has the potential to significantly help overcome such a situation. This discursive practice enables students to feel more autonomous and express their views and develop critical thinking (Silva, 2003; H. S. J. Oliveira & R. J. Oliveira, 2018). Students express their reasoning based on arguments concerning mathematical problem-solving; this way, teachers can access their ideas more easily, which favors the process of mediation of knowledge, thus promoting students' reflection and metacognitive thinking. The use of argumentation in the classroom is, thus, a technique in mathematics teaching that contributes to the interanimation of ideas, favoring students' personal and social development.

Van Eemeren and Grootendorst (2004) consider argumentation a verbal and social activity of reason, developed by a speaker or writer who seeks to increase (or decrease) the acceptability of a controversial standpoint for a listener or reader, through a constellation of propositions that aim to justify (or refute) the point of view before a rational judge. E. C. Oliveira. (2012, p.97), in line with the pragma-dialectical theory of Van Emeren and Grootendorst (2004, p. 1), defined argumentation as "the systematic action of organizing facts, ideas or reasons that, when associated with one another, form a unity that is capable of achieving adherence of other spirits" [our translation]. Sales (2010) points out that the act of arguing is the expression of reasoning. Based on the Anthropological Theory of the Didactic (TAD) [1] in the context of mathematics teaching, he notes that argumentation is an ostensive object used for making a non-ostensive object accessible, for example, ideas, concepts, or a chain of ideas and concepts. The author considers that arguing can be a simple explanation or an attempt at persuasion.

Despite the wider debate about the different definitions of argument and argumentation, which explains different dualities, e.g., product-process, individual-social, internal-external, oral-written, formal-informal (Garcia–Mila & Andersen, 2007; Jimènez-Aleixandre & Erduran, 2007), we take into account the fundamental concept of argumentation as the exposition of a justifiable point of view to an audience, as a line of reasoning that is expressed to support and clarify an idea, a claim which one seeks to legitimize. There has been increasing interest in argumentation in the field of Mathematics Education since the 1990s (Sales, 2010), as well as in a sociocultural perspective of teaching and learning. In such perspective, discursive interactions at the social level of the classroom are highly valued, and they foster the cognitive development of individuals as historical socio-historical subjects. Along these lines, the Common National Core Curriculum (BNCC) highlights one's ability to argue as one of the ten general competencies proposed for Basic Education (Ministério da Educação [Brasil], 2018).

In view of these assumptions, the present study, developed during the master's degree program of the first author of this paper (Costa, 2022), aims to analyze the structural and content characteristics of the arguments formulated by 7th grade elementary school students throughout their interactions with the researcher and colleagues, while solving contextualized questions involving arithmetic expressions. Such content is important to solve everyday situations/problems; in addition, it is a prerequisite for studying algebraic expressions, as students will have to deal with letters and numbers (Rosa, 2020). However, there has been little research on arithmetic expressions regarding argumentation (Costa, 2022). We consider, therefore, that by reporting an analysis of such arguments, this study can shed light on the way students assimilate this content in the classroom. Thus, this study aims to contribute more elements for teaching and research communities to reflect on didactic strategies that favor students' learning.

We adopted a qualitative approach to data collection, with a sample of 7th grade elementary school students of a government-funded school in the countryside of the state of Bahia. The interactions between the researcher and the students, as well as the resolution of problems proposed in contextualized questions, occurred in three online meetings (owing to the Covid-19 pandemic) on the *Google Meet* platform. Students could access the questions by clicking the link provided on the platform, which directed them to questions on *Google Forms*. The students' arguments, expressed in their written answers and in the discussions held after the resolution of each question, were analyzed and then classified into the categories proposed by Sales (2011). We also adopted Toulmin's argument pattern (2006), seeking to identify the constituent elements of the constructed arguments and their contents in order to qualify them.

THEORETICAL AND METHODOLOGICAL ASPECTS

TOULMIN'S MODEL OF ARGUMENTATION

Toulmin, together with Perelman, was one of the philosophers who proposed a new approach to the study of argumentation. They advocated informal logic[2] - motivated by dissatisfaction with the type of argumentation previously put forward in books on introduction to logic - from the perspective of analytical argumentation, which originated in North America in the early 1950s (Freitas, 2005). Toulmin sought to develop a theory of argument capable of bridging the gaps pointed out in the Platonic-Cartesian rationality and, thus, propose an instrument capable of replacing the logic brought by Aristotle in the analysis of arguments (M. G. Oliveira, 2017).

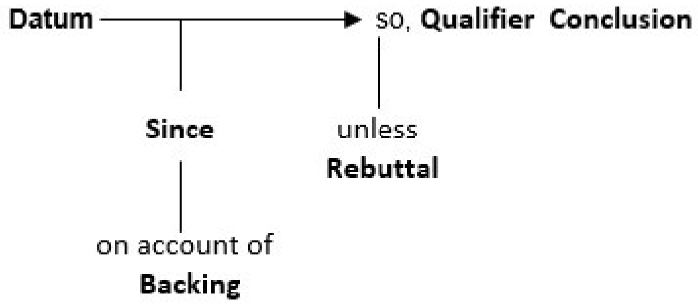
According to Velasco, in the work **The Uses of Argument**, of 1958:

> Stephen Toulmin criticizes the way how some logical categories, such as deduction, are exposed in similar books. According to the author, the usual approach to these categories prioritized certain types of arguments (namely, analytical ones), which are unusual in everyday argumentation [our translation] (Velasco, 2009, p.281).

The scheme for representing the arguments proposed by Toulmin became known as *Toulmin's Argument pattern* (PAT), as shown in Figure 1. This model contains the following elements: data (D), warrant (W), backing (B), modal qualifier (Q), rebuttal (R), and claim (C).

Figure 1

Toulmin's argument pattern



Source: Toulmin (2006, p.150).

Data: They are the starting point of the argument, corresponding to facts or even claims that underpin a claim.

Warrant: They are statements that provide complementary information or illustrate the data, and act as bridges that connect the data to the claim.

Claim: It is the statement that one seeks to establish using the argument, that is, statements that one seeks to legitimize as valid.

According to Toulmin (2006), an argument can be developed with its basic structure, containing only data, claim, and warrants. The latter, in turn, can be better understood when we consider their association with basic knowledge or backing. In addition to support, Toulmin considers new elements that make the argument more complex and consistent and explains the role of the new categories in the argumentative model:

Backing or basic knowledge: They are theoretical bases for the warrants that justify or exemplify a piece of data.

Rebuttal: These are claims that oppose data or warrants, indicating circumstances in which warrants do not apply, or conditions of exception to the claim.

Qualifiers: They are a complement to the structure of the argument. They will modulate the reasoning by showing their likelihood, strength, or weakness. Thus, when we use, for example, expressions that are within the areas of human argumentation, such as **probably**, **possibly**, **presumably** to compose a rhetorical structure, we can achieve greater adherence from our interlocutors.

The focus of Toulmin's argument pattern is the structure of the argument, its constituent elements, and the links between them, rather than their content, i.e., it is through TAP that we understand the argumentative coherence. However, from the perspective of teaching and, particularly, mathematics teaching, also focuses on the content that is developed along the structural elements, to provide further insights into how students use mathematical knowledge to justify their views or the solutions that they suggest for the problems that were proposed. Sales (2010) argues that a thought-out, responsible argument, always contains facts and information that serve as warrants that support the argument. In the Anthropological Theory of Didactic (TAD), such warrants are called technologies or theoretical knowledge. Thus, when we emphasize the importance of focusing on warrants and backing, we assume that there lies the core of students' reasoning, as they clearly show the ideas that students use to start from the data and move on to reach their conclusions.

RATIONAL, NATURAL, AND FOLK ARGUMENTS – THE CATEGORIES PROPOSED BY SALES

Based on the TAD, Sales (2011) considers two types of arguments: explanation and justification. The author advocates the latter by considering that it aims to persuade, that is, in the case of mathematics teaching, it intends to show the reason behind the procedure involved in solving any problem or question; on the other hand, an explanatory argument is not intended to convince an individual about the validity of a given procedure, but only to convey it as valid. Therefore, for Sales (2011), to argue "is the action of doing or showing how it is done and it is also the action of justifying why it is done" [our translation] (Sales, 2011, p. 01).

Sales argues that:

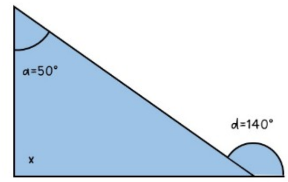
> Doing mathematics is an activity that consists in developing an action justified by theory-based discourse. When resulting from this activity, mathematics will always be new to the one producing it if the didactic moment that is being experienced does not consist only in repeating tasks that use the same technique to consolidate a piece of knowledge [our translation] (Sales, 2010, p. 53).

Taking into account a justification or a justifying argument, Sales (2011) also introduces the following categories: rational, natural, and folk. He thus recognizes that, while developing rational argumentation, students can present other levels of argument. An argument is rational when it is based on a theory, i.e., it is the one whose development is based on some content or mathematical rule. A natural argument is the one based on experience, but it does not involve theoretical-formal systematization. Thus, "one formulates reasoning, a chain of ideas, an integration between the parts of reasoning, but there is no systematization" [our translation] (Sales, 2011, p. 6). A folk argument is often based on naivety, feelings, myths, and desires (Sales, 2010). This folk level is still loaded with jargon, beliefs, traditions, and fads that exist within people's imagination. It is divided into two subcategories: naive and traditional. Naive reasoning contains all arguments that are somewhat childish and simplistic, while in traditional reasoning, an argument will be based on experience, on observed and unquestioned facts (Sales, 2011).

Table 1 shows some examples:

Table 1

Examples of rational, natural, and folk arguments

Questions	Types of arguments	Examples
Determine the value of the internal angle x , given the values of the angles α (inside the triangle) and d (outside the triangle), at:	Folk	x = 90°, because its sides seem to meet at a corner. x = 90°, because in all exercises in this class, triangles have an angle of 90°.
	Rational (also a demonstration)	**x = 90°,** as = (i **a = 50°** is an internal angle of the triangle; ii **)** **x** is also an **internal** angle of the triangle; (iii) the third internal angle of the triangle is complementary to angle **d (140°)**, and thus, **d + i = 180°,** thus **i = 40°**; iv) the sum of the internal angles of a triangle is 180°. Thus, **50° + 40° + x = 180°** and, therefore, **x = 90°**.
Is it true that the sum of two even numbers is always an even number?	Natural	2 + 4 = 6; 8 + 12 = 20; 14+ 24 = 38; As in all the examples which we can think the result is an even number, we conclude that the statement is true.
	Rational	2a + 2b = 2 (a + b) Even numbers are always multiples of two.

Source: Adapted from Sales (2010).

THE QUESTIONS APPLIED TO THE STUDENTS

Six questions were applied to the students; 4 of them were contextualized, consisting of narratives of everyday situations while two were out of context, consisting in the presentation of expressions that were ready to be solved. For the sake of space, we will discuss here only three of them, two contextualized and one out of context. The questions are present below.

The first question required that the students solve a problem by performing addition, subtraction, and multiplication operations, without determining that they had to do so by means of an arithmetic expression. Thus, the students could perform the necessary operations in the order they found it convenient, provided they left their calculations recorded on paper. After formulating, presenting, and justifying the requested answer, the students had to indicate, among the options for arithmetic expressions, the ones that they considered correct and the one that best represented the way that they solved the question. Later, they also justified their choice to the audience.

We consider that the demands of the question involved a justifiable argument, because the students were required to make their reasoning clear and convincing to an audience composed of teachers and classmates. Initially, they had to show how they achieved the results, and they were free to make use of different languages (natural, symbolic, schematic). Next, they had to use a semi-formal language, respecting methods and rules that more clearly express the legitimate agreements of the scientific community (mathematics), through an arithmetic expression, which they discussed subsequently. At this second moment, the question implicitly demanded that students should reflect on their own reasoning, taking into account mathematical language. This is the 1st question:

Question 1: A tank was filled with 120 liters of water. Water was equally poured into six 10-liter buckets and six 4-liter containers. How many liters of water were left in the tank? (Question 1a, 2021).

Which of the following expressions do you consider correct and which one best expresses your reasoning regarding the resolution of Question 1?

a)*120 – 6 X 4 + 10*

b)*120 – 6 X 4 – 10*

c) *120 – 6 X (10 + 4);*

d)*120 – [(6 X 10) + (6 X 4)]*

e) *[120 – (6 X 10)] – (6 X 4)*

f) *120 – [(6 X 10) – (6 X 4)]* (Question 1 b, 2021).

The students were asked to present their answers to the second question through an arithmetic expression. Compared to the first question, the second question involves a greater cognitive demand. However, it also requests that the students should explain their reasoning while solving it, as with the other questions.

Question 2[3]*: Grandma Rosa decided to buy Christmas gifts for her little grandchildren. She bought three balls for R$14.00 each, two dolls for R$25.00 each, and four toy cars for R$19.00 each. If she paid her purchases with a bill of R$ 50.00 reais, and then paid the remaining balance into two installments, what is the amount to be paid for each installment?*

Solve the above question by means of an arithmetic expression and then try to explain your reasoning to the class. If you cannot solve by means of an expression, do it the way that you consider best, but make your solution clear. (Question 2, 2021).

According to the narrative of the question, students must deal with the four basic arithmetic operations - addition, subtraction, multiplication, and division - and organize such operations into an equation, making use of the three elements of association - round brackets, square brackets, and braces - which involves ordering rules. Moreover, the question requires inversion -to some extent - of the order of the actions performed by the character in the narrative and the order of the respective mathematical operations in the expression, when compared to question 1. The information that Grandma Rosa paid the purchase with a R$50.00 bill is something that, chronologically, occurs after the sum of the prices of the toys; thus, the operation referring to such action must come after this sum in the expression. The students should also consider that the remainder of the purchase

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payment will be divided into two installments, and this operation should operationally come in the final part of the equation. So, we have: {[(3x14) + (2x25) + (4x19) - 50]} ÷ 2.

The third question involved the presentation of two arithmetic expressions ready for resolution. The first consists of a more elementary expression, which requires only the addition and multiplication operations. Students, first of all, should reflect on which operation must be applied first. The structure of the second expression is similar to that of questions 1 and 2. This is the question:

Question 3 – Solve the following expressions:

a)10 + 4 x 5

b){[75 + (10 x 4) + (12 x 2) + (8 x 12)] – (20 + 15)}: 2 (Question 3, 2021).

We consider that when the students are asked to solve the proposed questions by means of expressions, and manage to do so, it means that they have mastery of the required mathematical relations, and relate them to contextual aspects. When this does not happen and they can only solve expressions that are ready, as in Question 3, it means that they were taught formalization before understanding the use of the symbols and conventions of this disciplinary field, i.e., they did not experience contextualized learning that involves justifying argumentation. The literature points out that this weakens the understanding of the resolution itself of a given numerical expression (Ferreira, 2014). Thus, it is not unusual to find students who cannot not solve expressions even though they were able to work with the operations involved and solve the problems without making use of the expressions.

PRODUCTION, PROCESSING, AND ANALYSIS OF DATA

The 6 questions were applied to ten 7th grade elementary school students of a government-funded school in the countryside of the state of Bahia. Students were identified as A1, A2, A3 ... A10; the researcher was identified as Rsr. The 3 sessions took place in the classes taught by the school's mathematics teacher. It is noteworthy that, in the face of the pandemic, students were not required to attend virtual classes, since not everyone could have access to the Internet. However, it was strongly recommended that everyone should solve the activities proposed by the teachers, which were made available both on the Internet and by the school itself, in hard copy. Thus, the number of students who participated in the study did not correspond to the total number of students in class. All participating students delivered the Informed Consent Form signed by their respective parents.

In each session, after listening to the researcher's instructions, the students answered the proposed questions and submitted their answers online. Next, the students held a discussion in the virtual classroom to orally present the reasoning they used while solving the questions. Throughout this discussion, the students revisited concepts that they had previously formulated in school, as well as those experienced on a daily basis, and ultimately framed new concepts. Data treatment involved transcription of the discussions recorded on video and comparison with written data on the resolution of the problems by the students. After that, we applied the categories described in the previous sections. First, we sought to check how the answers, as argumentation, could be understood - in relation to the level of rationality - as folk, natural, or rational. We continued the analysis using Toulmin's Argument Pattern.

RESULTS AND DISCUSSION

When applying the first question, there were only 6 students in the virtual classroom. Although everyone had been able to solve it through separate mathematical operations, only 4 students could actually argue because there were Internet connectivity issues. Therefore, we will present the analysis of the arguments of 4 students (A1, A2, A4, and A6) who remained until the end of the class and explained their reasoning behind the resolution of the question. The answers given by **students A3** and **A5** were correct, but they did not present their arguments in the discussion. Out of the 4 students who argued and managed to provide the correct answer, 2 of them (A1 and A6) did not do this at first, owing to some minor misconceptions regarding the arithmetic operations and the interpretation of the question. Since this fact was recurrent with different students and relevant for us, below we will present the discourse of students **A1** and **A6** and that of the researcher:

Rsr: *Look, a tank was filled with 120 liters of water. 6 buckets of 10 liters were taken out, right? And 6 containers had a 4-liter capacity each. So, the question was: Considering that all the buckets and containers were filled to the brim, how many liters of water were left in the tank? Then A1, you can answer it now.*

A1: *44 liters were still left.*

Rsr: *44 liters were left? How did you come up with this result?*

A1: *As I said, right? I had checked it there, I multiplied the number of buckets that were filled up and also the small containers, added them all, and then I subtracted it from the original result, which was 120, then I got 44.*

Rsr: *120 minus how much?*

A1: *Minus 84.*

Rsr: *Minus 84, and the result was 44?*

A1: *Mm-hmm... [...]*

A1: *Ms. Rsr.*

Rsr: *Yes?*

A1: *It's because I was doing it another way here too, I recalculated it, and I realized that I had mistaken the small number four in the middle of the calculation. So, what should I do? Do I have to resend the thing (the question) or should I leave it like that?* (Dialog between researcher and students, 2021).

Student A1 clearly reported making a mistake while performing the subtraction operation. Before that **, A1** had also misinterpreted the text. Let us look at some of her discourse when she initially presented her result to the researcher:

Rsr.: *Right. And what result did you get, A1?*

A1: *There were 4 buckets and 1 container, which were missing to take out all the water.*

***Rsr.**: Hmm. And the end result, what was it?*

***A1**: 4 buckets and 1 container, that was what I said, that were still left to finish the thing.*

***Rsr.**: Hmm. Let's see...*

***A2:** (((interrupting the teacher))) Hang on. Wasn't it the number of liters left?*

***A1**: Did I have to guess how many liters were left? Oh! I thought it was to guess how much I still needed to take out!!*

***Rsr:** Nooooo! It's much simpler, isn't it? ((Laughter))*

***A1:** WOW! I can't believe it!* (Dialog between researcher and students, 2021).

As can be seen, the student assumed that she should "guess" (calculate) the number of buckets and containers corresponding to the volume of water left in the tank. Thus, she would have to inform how many buckets and containers could still be filled for the tank to be empty. The interaction between A1 and the researcher allowed the student to express her ideas, which made it clear that she was inclined to express a rational argument, dealing with mathematical knowledge, although with small misconceptions/difficulties in interpreting the narrative of the question and in making the calculations. The difficulty in interpreting the question was clearer in student A6's discourse. The excerpt is transcribed below:

***Rsr.**: Why did you subtract 120 from 6? Did you subtract 6 from 120 ((self-correcting))?*

***A6**: Because the question said that I had a tank with 120 liters, then it said that 6 buckets were taken out, so I subtracted it. [...]*

***Rsr.**: Right, I got it, OK. Think a little bit more about it. Are you taking out water or are you taking out buckets from the tank? Taking out water or taking out buckets?*

***A6**: Water.*

***Rsr.**: Water by using what?*

***A6**: Buckets.* (Dialog between researcher and students, 2021).

After explaining their results, A1 and A6, with the researcher's mediation, realized that they had interpreted the question incorrectly or were mistaken about the operations. After the discussion, they reflected and solved the question again, and presented new arguments, now with correct answers. Considering the arguments presented by the four students, we can realize that all of them use mathematical reasoning, although sometimes inaccurate at first; some had misconceptions, owing to difficulties in interpreting the question or miscalculations. The researcher offered brief information or posed questions that made them reflect, without immediately informing whether their answers were correct or incorrect. Therefore, we classify 3 of the arguments as rational and one of them as natural. Below are the arguments that each student presented throughout the discussion, after they had submitted their solutions in writing.

***A1**: Well, I started it in a simple way, I just got the result and I had the result in mind. Then I realized that as there were six 10-liter buckets, I multiplied there 6 times 10, 60. Then, I saw the other one that was about 4-liter containers, I forgot the thing now, then I multiplied the two of them, used multiplication, then combined the two results, then I subtracted it, then I got thinking again, right, the bucket has a capacity for 10 liters and* *that small container fits 4, then I just made a simple division, right out of my head, which was am easy, rounded number, and that was it, I got the result, 4 buckets and one container.* (Student A1's argument, 2021).

A1 presents a rational argument for the question, although she had misinterpreted the instructions of the question at first. Throughout her attempts, **A1** developed a reasoning that, according to Sales (2011), can be considered systematic, achieving the correct result, in a second moment.

A2 also shows a rational argument. The student explains his resolution and finds the correct result for the question, which is 36. The excerpt is transcribed below:

***A2:** So look, as there were 6 buckets with 10 liters each, I already figured out that it was 60, then there were six 4-liter containers, so I was going to make an addition, then I remembered that it was simpler to count: four times five was 20; 5, 10, 15, 20 ((finger-counting), then I only had to add 4, then I calculated 60 plus 24, then I took that number of the result and subtracted from 120.*

***Rsr.**: And how much was it?*

***A2**: It was 36.* (Student A2's argument, 2021).

We consider **A2**'s argument also rational, because he used the mathematical operations that he had learned formally, considering a better or faster way to perform the calculations. We infer that the "resources" that he uses for this calculation are originated in school, in the use of a multiplication table, something that is part of the school culture, but not in their experience outside this environment. When it comes to considering the volume of water taken out using the containers, the student reported that he would make an addition (possibly 4+4 4+4+4+4), which suggests that they perceive multiplication as an addition of equal numbers, but they decided to resort to the table of 5, adding a unit to each appearance of number 5 - *then I remembered that it was simpler to count: four times five was 20; 5, 10, 15, 20 ((finger-counting), then I only had to add 4 [...]*.

A4 also provides the correct answer, as did **A2**, and her argument can be characterized as natural, because the student did not clearly explain the reason behind her procedures. The student justifies her answer more briefly and does not make her reasoning clear, despite the researcher's prompts. The excerpt is transcribed below:

***A4**: Well, first, I started to reason about how I was going to make the calculation, so I made a calculation of 120 minus, hang on, hang on, I have to do something quickly. There's a mistake here.*

***A4:** Well, as I was saying, I calculated 120 minus 84, which is 36.*

***A4**: I started, like, by the number of buckets and the amount of water, then I did the calculation and saw that it turned out to be 36 and I concluded that the answer is 36.* (Student A4's argument, 2021).

For Sales (2010, p. 6), in the natural argument "(...) one formulates reasoning, a chain of ideas, an integration between the parts of reasoning, but there is no systematization" [our translation]. The author also admits that, despite proposing three categories for justifying argumentation, there may be several levels of complexity and, therefore, several levels of argumentation. Thus, from natural to logical-deductive reasoning, there is a range - although not very wide - of possibilities. For the author, there may also be

> [...] a level of reasoning similar to the natural one, but already permeated with some formal knowledge and lacking only the inclusion of a specific terminology. In the same way, there can be a pre-logical-deductive reasoning, which lacks specialized terminology, but allows us to glimpse the origin of a formal logic [our translation] (Sales, 2010, p. 6).

A6 also made mistakes, but her argument on the matter was also rational.

A6*: Well, I took 120, then I multiplied 6 times 10 which is 60, and subtracted it, which is 60 ((referring to the subtraction 120-60)), then I multiplied 6 times 4, which is 24, then I subtracted 60 minus 24 which is 46.* (Student A6's argument, 2021)

The reasoning employed by the four students, shows that **A1**, **A2** and **A4** performed the multiplication first (6x10 and 6x4) , added the products (60+24 120) and then performed the subtraction (120-84). **A6,** in turn, expressed a somewhat different rational procedure: After each multiplication, she performed a subtraction. So, we have 6x10, resulting in 60, then we have 120-60. Next, they calculated 6x4, resulting in 24, to perform the new subtraction, 60-24, which resulted in 36 rather than 46, as informed by the student. Thus, based on the discussion and calculations presented in writing, we found that **students A1**, **A2** and **A4** solved question 1 in the form of the expression "120 – [(6 X 10) + (6 X 4)]", which corresponds to option **d** presented in part 2 of question 1; while **A6** expressed something similar to "[120 – (6 X 10)] – (6x4)", which corresponds to option **e**.

A1, A2, and A4 selected only letter **d**, while A6 marked the options **d** and **f**, presented in the second part of the question. They were unable to realize, initially, that the options **c** and **e** were also correct. However, by justifying why they had indicated such an option, **A1** and **A2** made a correct and explicit correlation between the path of operations indicated in the expression and the rational path taken by them throughout the resolution. It was very gratifying to understand how the students expressed the relations between the arithmetic expressions and the way they reasoned and resolved the question, which demonstrates their awareness about the reasoning they employed.

Rsr.*: A2, can you explain why it was "d"?*

A2*: Because it looked more like my question, because as you normally solve what is in the brackets first, you have 6 times 10 is 60, 6 times 4 is 24, so it was all sorted except 120, but there's also the multiplication, that is, adding the two numbers in brackets, it was sort of what I did.*

Rsr.*: Right, so you already have this idea that you first solve what is inside the brackets, right?*

A2*: But, isn't it like that? ((intrigued))*

Rsr.*: Yes, yes, I just want it to be clear.*

[...]

A1: *Right, coming back to these things in brackets. I did exactly as it was there, I first solved this thing in brackets, 6 times 10 and 6 times 4, then I added these two, then I subtracted it from 120 as it is written there, right, neat, that's why.*

Rsr.*: Perfect.* (Dialog between researcher and students, 2021).

Interestingly, student **A6** did not indicate the option **e**, on the contrary, she indicated the options **d** and **f**. The way the student refers to option **f** makes us consider how similar that option is to option **e**, which would be compatible with the way the student solved the question. At the time **A6** explains it, however, she described (by means of discourse that has some ambiguities) the expression of option **e** rather than **f**. We can also consider the hypothesis that the student cited option **f** in addition to option **d**, because of the persuasive effect of the answers given by **A1** and **A2**, which were presented before that of A6's. In short, **A6** did not establish a good relationship between the expression and the reasoning that she had explained earlier, as occurred with **A1** and **A2**. **A4**, in turn, did not speak at this time of the discussion, owing to Internet connectivity issues.

The students had already studied (with some limitations imposed by the pandemic) the content of **arithmetic expressions**, but from what we gradually realized, this had not been based on contextualized questions. This could be inferred by their difficulty in interpreting the text and writing the expression, as well as in associating the expression with the resolution they used. Still regarding the second part of question 1, only after the researcher's explanation did the students realize that the expressions **c** and **e** produced the same result as option **d**. In the discussion about option **c**, the students did not realize that 6 x (10 + 4) would produce the same result as (6 x10) + (6 x4), which corresponds to the distributive property of multiplication. The discursive movement that develops between the researcher and students **A1** and **A2** shows how they begin to understand that. **A1**'s discourse ("Wow, awesome!!") shows her enthusiasm for learning new conceptions. The excerpt is transcribed below:

***Rsr.**: Did anyone choose option "c"? Do you think option "c" could be correct too?*

***A1**: I don't think so.*

***A2**: Neither do I.*

***Rsr.**: Why?*

***A1**: Because there, instead of being... because it's like that, 10 plus 4*

***A2**: It would be 14 times 6, 14 six times minus 120.*

***A1**: Yeah, then it wouldn't work out that even the letter "d".*

[...]

***Rsr.**: So multiply 14 by 6, and check what it is!!*

***A6**: It's 84.*

***Rsr.**: This, here ((shows it on the computer screen while sharing the screen)) or we could be doing what is in the brackets first...*

***A2**: 84 minus 120*

***Rsr.**: That's it. It will be 120 minus 84 that would give the same result as letter "d", isn't it?*

***A1**: Wow, amazing!!* (Dialog between the researcher and the students, 2021).

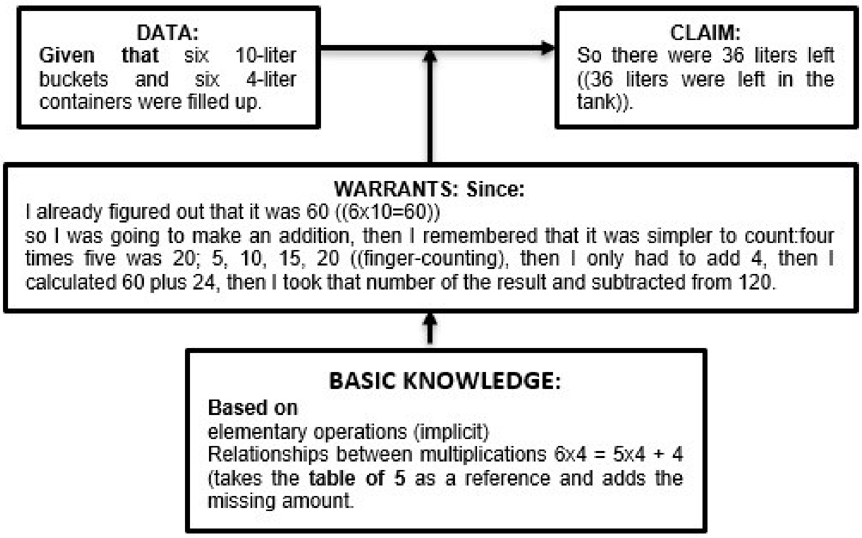
In the discussion about option **e**, the students are encouraged to perceive the role of brackets in the comparison between this option and option **f**, since option **f** is wrong and only differs from option **e** - which is the correct one - by the position of that element.

The above analysis highlights the idea that students argue from a logical, rational perspective, making use of three operations in problem solving, recognizing, and overcoming difficulties. With regard to the knowledge about arithmetic expressions, it becomes clearer that they know some fundamental rules, but they do not have the habit of using expressions to represent the resolution of problems involving narratives of situations, that is, they had not learned this content by means of contextualized questions, but using rules, conventions, and mathematical formalizations. The excerpts from the discussions between the researcher and the students, indicate, above all, that the interventions of the former favor the students' interaction and argumentation, promoting a reflection about their ideas and an advance toward the correct conceptions in the field of mathematics.

Below, we present the arguments of the students through Toulmin's model (2006). We started by student A2, whose argument is systematized in Figure 2, below. Student A2's argument presents the basic elements of Toulmin's model. The warrants and basic knowledge explain the reasoning and knowledge involved in the argument.

Figure 2

Toulmin's layout for student A2's argument for question 1



Source: Prepared by the authors (2022).

The other arguments formulated by the students, according to Toulmin's model, present similar structures to those of A2; they were also composed of basic elements of the model: data, claim, warrants, and basic knowledge, while the latter is always implicit. For students A4 and A6, data is also in such a condition.

Table 2

Toulmin's layout for the arguments put forward by students A1, A4 and A6 for question 1

Student	Data	Warrant	Backing	Claim
A1	**Given that** *(...) there were six 10-liter buckets... Then I checked the other one that was empty there, which had 4 liters.*	**Since** *I multiplied 6x10=60 then I multiplied the two of them. I used multiplication, then I added the results. Then I used subtraction.*	**Based on** *knowledge of multiplication, addition, and subtraction ((implicit)).*	**Then** *((there were)) 4 buckets and 1 container ((left)).*
A4	The statement of the question ((implicit)))	**Since** *I started, like this, by the number of buckets and the amount of water, then I made the calculation(...) I calculated 120 minus 84 (...)*	**Based on** *knowledge of multiplication, addition, and subtraction ((implicit)).*	**Then**, *36 was the answer.*
A6	The statement of the question ((implicit)))	**Since** *I used 120, I multiplied 6 times 10, which is 60, and subtracted it, which is 60 ((referring to the subtraction 120-60)), I multiplied it, 6 times 4, which is 24, I subtracted 60 minus 24.*	**Based on** *knowledge of multiplication, addition, and subtraction ((implicit)).*	**Then**, *it was 46.*

Source: Prepared by the authors (2022).

When the second question was applied in the 2nd session, there were 10 students in the virtual classroom. The following codes were assigned to the new students: ***A7, A8, A9*** *and* ***A10***. This question asked students to offer the resolution by using an arithmetic expression. If they could not do so, they should still attempt to solve the question and clearly present their reasoning to the researcher and their classmates. Nine students (A1, A2, A3, A5, A6, A7, A8, A9 and A10) answered the question using separate mathematical operations rather than an arithmetic expression. However, only five students (A2, A3, A5, A7, A8) were able to find the correct result of the question, at first, without the researcher's help. Five students argued, in different ways, in favor of how they solved the question (A1, A2, A3, A5 and A6). Finally, only three students, after encouragement, were able to formulate the expression correctly (A1, A2, A6).

The brief report above already indicates that the difficulties encountered in session 1, that is, interpreting the statement of the question and its relations with mathematical operations and among mathematical operations themselves, happened again in session 2. However, three students went further and managed to formulate the arithmetic expression along their dialog with the researcher. Below is the analysis of such dialog.

Student **A1** solved the question by making small calculations. She was unable, at first, to produce the arithmetic expression. She had not been able to understand the statement of the question. A1 was in doubt about what "installment" is when it comes to a purchase. Later, they were able to come up with the expression correctly. The excerpt is transcribed below:

***Rsr**: Right, and after you made these multiplications, what did you do about the result?*

***A1**: I added all this.*

***Rsr**: Perfect. When you added all this, what result did you get?*

***A1**: 168.*

***Rsr**: So, 168. What does this amount correspond to?*

***A1**: The absolute amount of the purchases made by Mrs. Rosa.*

***Rsr**: Right, then Grandma Rosa, she made a purchase. How much would she have to pay for that purchase?*

***A1**: 168.*

***Rsr**: R$ 168.00, great, so she had to pay 168 reais, but how much did she pay straight away?*

***A1**: She made a down payment of R$ 50 reais.*

***Rsr.**: Right, now explain your reasoning. How did you go on from there?*

***A1**: Then I only got the result of 168 because ....wait... I've only realized it now , Jesus, my God!*

***Rsr.**: What did you realize? ((laughs))*

***A1**: ((laughs)) now that I've realized, that it went wrong, I should've divided it into two installments, right? Two.*

***Rsr.**: That's it.*

***A1**:Then I got 168 and divided it by 2, then it was after I got the result that I subtracted R$ 50.00.*

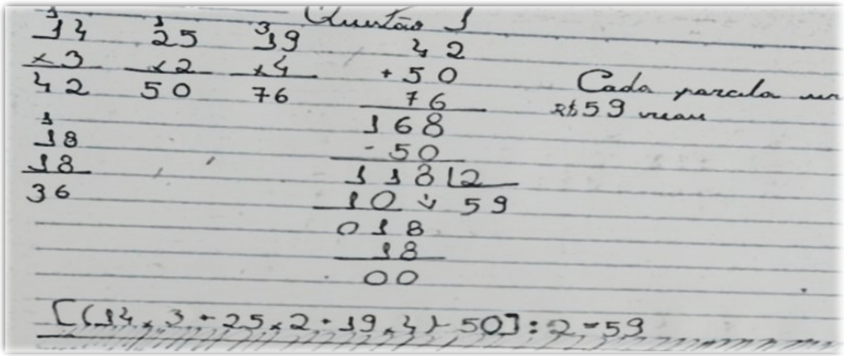
***Rsr.**: And how do you think you should have done it?*

***A1**: I think I should have subtracted 50 right after I had the result and before, like, I divided it by 2.* (Dialog between researcher and students, 2021).

Student **A1** reflected on her ideas throughout the discussion with the researcher, who interacted with **A1** to encourage such reflection on how the student had solved the question. Thus, **A1** can realize where she was wrong, made the adjustments to her resolution and managed to produce her expression correctly. Student **A6** also found the correct result for the question by solving the operations by counting. Then, this student managed to formulate the arithmetic expression and was mistaken by decreasing the installment of R$ 50.00, which had been placed at the beginning of the expression. The excerpt is transcribed below:

Correctly written expression: *{[(3x14) + (2x25) + (4x19)] – 50} ÷ 2 =59*

Figure 3

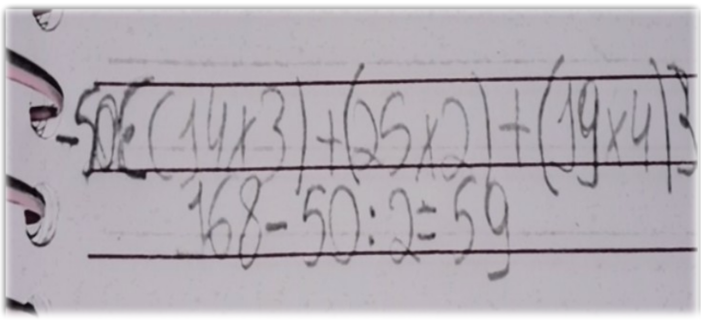


Expression written by A1 for question 2

Source: Research files (2022).

Figure 4

Expression written by A6 for question 2



Source: Research files (2022).

This article does not intend to discuss the relations between metacognition and the researcher's interventions in her dialogs with the students, but it is worth considering the students' reasoning about the arithmetic expressions, which is developed throughout their interactions. Notably, with previous knowledge of the rules that they had already learned and somehow resumed in the study, the students began to attribute new meaning to such knowledge when they attempted to apply it for solving the contextualized questions. The arithmetic expressions began to make sense to them. Thus, their discourse expresses the construction of rational arguments, or arguments that involve some natural arguments that are similar to rational ones. Hence, errors occur; mistakes that are overcome by some students at a certain "speed". Below is the transcription of the dialog between the researcher and student A2.

Rsr.: *You can speak, A2*

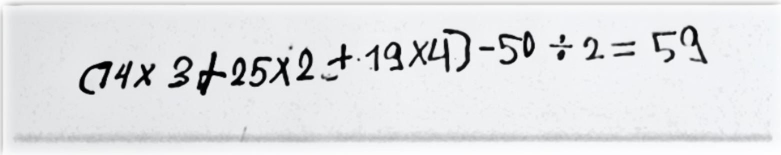
***A2**: First, as she had bought 3 balls, 2 dolls and 4 toy cars, I multiplied these numbers by the amounts; I multiplied by the prices of each of them, the 3 toy cars by 14, the 2 dolls by 25, then I combined them all, all the values to know how much everything cost, then it was 168, then as she had already paid R$ 50.00, 118 reais was left,, then she divided in into two installments, so I only had to divide it in half, which is 59.*

***Rsr.**: Well, did you manage to write the expression, A2?*

***A2**: Yes* (Dialog between the researcher and the student, 2021).

Figure 5

Expression written by A2 for question 2



Source: Research files (2022).

Also, in this meeting there was a student using a natural argument, in the way that we have already defined. The excerpt is transcribed below:

***Rsr.**: And what was the total value that you found?*

***A3**: 168.*

***Rsr.**: 168, how did you get this result of 168?*

***A3**:I made the calculations.*

***Rsr.**: Very well, what calculations?*

***A3**: For the dolls and the things she bought.*

***Rsr.**: And what calculations are these, what is the name of the operation you did?*

***A3**: I don't even know the way I did it ((laughs))*

***Rsr.**: You know, multiplication, addition, subtraction, things like that...*

***A3**: So I added the down payment, the number of installments (x) and added (+) and then I got the value of 59.* (Dialog between the researcher and the student, 2021).

We know that for many students, it is not simple to express how they developed their ideas. In fact, for many people, depending on what they have accomplished, the task of explaining their reasoning is not simple. **Student A7**, for example, solved the question quickly, demonstrating his ability to interpret the context and express it mathematically. However, because he was **silent,** we could not look further into how this student actually dealt with the relationship between context and formal mathematics. A3, in turn, made it clear that she has difficulty in explaining the reasoning employed in solving the question, although she had solved it correctly.

Out of the 5 students who argued in this session, only 3 of them wrote the expressions while solving the questions. The following table shows the classifications regarding rationality and the elements of the TAP present by the students.

Table 3

Types and elements of the arguments put forward by students A1 to A6 for question 2

Student	Type of argument			Elements of Toulmin's model			
	Rational	Natural	Folk	Data	Warrant	Backing	Claim
A1					Implicit		
A2						Implicit	
A3			I	Implicit		Implicit	
A5						Implicit	
A6				Implicit		Implicit	

Source: Prepared by the authors (2022).

After this second question, we moved on to the third, which involved two items: a and b. In item **a**, we present a simple arithmetic expression, which the students were able to solve, but they were not able to explain why the multiplication was made before the addition, for example (10 + 4 x 5). They only reported that they knew they had to solve the multiplication first, but they could not present a justification. Then, they were asked to solve option **b** as homework and send a photograph of the resolution.

Below is an excerpt of the discussion about option **a**:

Rsr.: *Why do you perform multiplication before addition and subtraction? Do you know why?*

A2: *I don't know the reason, but I know there is an order: first work out the round brackets, the square brackets, the braces; then comes potentiation, radication, multiplication, division, addition, and subtraction. Since there's no potentiation and no radication, it starts by multiplication.*

Rsr.: *Do you think it is a rule only that should be followed, I mean, performing multiplication before addition?*

A1: *There is a reason, but I can't explain why, because for example, in this calculation, even if we, for example, make the sum before the multiplication, it'll go wrong, but I don't know how to explain what the rule is because the right thing to do is multiplication first.* (Dialog between the researcher and the students, 2021).

Clearly, the students know the rules and mathematical conventions, but they do not know the reason behind them. The excerpt from the researcher's interaction with A2 and A1 also shows two ways to deal with this situation. A2 presents folk reasoning while A1 presents reasoning that is similar to the natural type, as discussed by Sales (2010). The former follows a tradition, based on procedures performed according to the teacher's guidance, but not questioned. The latter seeks to justify the rule while being guided by the perception that not following such rule would lead to an incorrect result. There is, however, systematization of a rational argument. Thus, mathematics teaching, still marked by memorization and repetition of algorithms, leads to the construction of arguments that are distanced from the rational one, thus strengthening a naive logic in the approach to questions. However, the use of contextualized questions stimulates the construction of rational arguments associated with a metacognitive movement, as could be seen.

Regarding option b, students A1, A2, A3, A6 and A7 submitted their answers, which were correct. In conversation with the students through WhatsApp, the following question was asked: "Did you feel any difficulty in solving this expression?" (The researcher asked the students who sent their answers to the question). Everyone replied "no". They said that it was easier, because they did not need to think about the arithmetic expression, since it was ready.

Taking into account the whole process of data collection, that is, the three sessions, we could observe the students' difficulties in writing an expression based on contextualized questions. On the other hand, we could understand how the investment in argumentation provided the development of rational arguments, which is much less requested when a question involves only the application of mathematical rules and conventions.

We now consider the results found in this study in view of the context of its development. As mentioned earlier, this study was developed through online classes held during the COVID 19 pandemic. For the reasons explained in the previous section, the classes in this format had a very small number of students. There were 10 students in total, but not all students attended all sessions or even a complete session, because some students did not stay until the end of class.

However, the *online* interactions maintained with this group of students were quite intense. The questions that the researcher asked the whole group were followed by answers from more than one student. These questions also took into account the answers presented by the classmates, thus favoring the interanimation of ideas, as can be seen in the transcripts. When addressed to a specific student, most of the time, the researcher's questions were answered and encouraged argumentation through chains of interactions, which were in line with the contextualized nature of the questions. Also, the students were engaged in questions assigned as homework. This all contributed to the students' movement of metacognition and the evolution of the quality of their arguments. In this perspective, the data allowed us to realize the advance in the students' abilities when dealing with the arithmetic expressions, based on the arguments that they presented.

The results discussed in this paper confirm the conception that the investment in argumentation around contextualized questions favors the learning of concepts and, more specifically, the learning of numerical expressions, considering the specificities of this content.

Although most students engaged in activities and felt quite confident to present their views, little could be done to encourage engagement of those who refused to express themselves, despite their agreement to participate in the study. This was, for example, the case of A7, who barely opened his webcam during the session.

The dynamics of the online classes consisted in the researcher always interacting with all the students or in being in the presence of them all. Considering face-to-face classes with a larger class, as is the case with most schools, small-group discussions can also occur, which helps shy students to gradually express themselves and build ideas in front of a smaller number of colleagues before presenting them to the whole class. Also, the researcher could interact with the students more particularly in small groups, giving greater support to those who felt more difficulty, which would clearly show how they deal with the demands of the proposed questions. These possibilities would certainly highlight a greater diversity of the process of drawing up ideas and building arguments. Therefore, on the one hand, online classes can offer benefits regarding the interactions between teacher and students and the convenience of using video and audio records; on the other hand, this format of classes somehow limits the interventions of the teacher/researcher, which cannot be disregarded in understanding the results of the study.

FINAL REMARKS

In view of the discussions, the students' resolutions of the proposed problems and their oral and written explanations, show the types of arguments they used, based on the categories proposed by Sales (2010), as well as characteristic elements of Toulmin's Argument Pattern (2006). The arguments put forward by the students tended to rational, based on the systematized knowledge of mathematics, but we also found natural arguments when dealing with the contextualized questions. Regarding Toulmin's model, most arguments presented the basic elements, such as data, claim, and warrants, with underlying knowledge being implicit. The data, in some situations, were also implicit. The natural arguments presented fragile warrants. This association could be noted when relating the categories proposed by Sales with Toulmin's Argument Model.

The students' arguments confirmed that they had knowledge of rules and conventions that supported the procedures for the resolution of expressions, but they did not know the reasons behind them. Thus, the results indicated that most students had the ability to solve the mathematical operations required in the questions and they could understand, even in the face of some obstacles, the context of each question. However, the students had difficulty in expressing such questions through expressions, despite their ability to solve them when they were decontextualized. In this situation, folk and traditional arguments surfaced, in addition to natural ones, when the students were asked about the reasons why they adopted specific procedures.

The contextualized questions proposed in this study promoted a discussion that enabled students to understand certain mathematical rules, which they knew but did not understand; helped students understand the context of the question more easily; eased the transition from natural language to mathematics and, therefore, enabled the perception of the relationship between these two forms of **register**.

Therefore, this study can provide teachers and researchers with more elements to reflect on didactic strategies and methodologies that favor students' learning of the content addressed here.

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NOTES

1. Theory proposed by Yves Chevallard in the 1980. According to Sales (2010, p.27) the theory is called anthropological because "it assumes the processes of knowledge as a social product, something that happens within social institutions" and is a theory of didactic "because it is a study of the object of didactics" [our translation].

2. It is the study of the arguments presented in the common language, in contrast to the presentation of arguments in an artificial, formal, or technical language.

3. Questions 1 and 2 were adapted from A.R. Ferreira. Teaching numerical expressions through illustrated games and stories. Available at: www.ensinandomatematica.com/expressoes-numericas-historias/

4. Translated by DUO Translations. Email: contato@duotranslations.com

REFERENCES

Base Nacional Comum Curricular. (2018) BRASIL. Ministério da Educação, Brasília, DF. 2018. http://basenacionalcomum.mec.gov.br/images/BNCC_EI_EF_110518_versaofinal_site.pdf

Costa, M. B. L. (2022). *A argumentação de alunos do ensino fundamental na solução de problemas envolvendo expressões aritméticas*. (Dissertação de Mestrado em Ensino de Ciências e Matemática), Universidade de Sergipe, São Cristóvão.

Freitas, T. S. (2005). *A lógica da argumentação Jurídica no pensamento de Chaim Perelman: reflexos na interpretação do direito*. São Paulo, SP: Malheiros.

Jimènez-Aleixandre, M. P., & Erduran, S. (2007). Argumentation in science education: an overview. In: Erduran, S., Jiménez-Aleixandre, M. P. (Org.), *Argumentation in science education: Perspectives from classroom-Based Research*. Dordrecht, Holanda: Springer.

Oliveira, E. C. (2012). Persuasão: o componente pragmático da argumentação. *Caderno CRH*, 25, 97-104. https://periodicos.ufba.br/index.php/crh/article/view/19444

Oliveira, H. S. J. de., & Oliveira, R. J. de. (2018). Retórica e argumentação: contribuições para a educação escolar. *Educar em Revista*, 34(70), 197-212. DOI: 10.1590/0104-4060.52510

Oliveira, M. G. de. (2017). *A questão do relativismo na teoria da argumentação de Stephen Toulmin*. (Dissertação de Mestrado em Filosofia), Universidade de Coimbra, Coimbra.

Pais, L. C. (2006). *Ensinar e aprender Matemática*. Belo Horizonte, MG: Autêntica Editora.

Rosa, C. A. S. (2020). *Expressões Aritméticas: Dificuldades encontradas entre alunos ingressantes no primeiro ano do ensino médio da Escola Técnica Guaracy Silveira*. (Trabalho de Conclusão de Curso Especialização em Práticas Educacionais em Ciências e Pluralidade), Universidade Tecnológica Federal do Paraná, Dois Vizinhos.

Sales, A. (2011). *Argumentação e Raciocínio: uma revisão teórica*. Nova Andradina, MS: Universidade Estadual de Mato Grosso do Sul.

Sales, A. (2010). *Práticas Argumentativas no Estudo da Geometria por Acadêmicos de Licenciatura em Matemática.* (Tese de Doutorado em Educação), Universidade Federal de Mato Grosso do Sul, Campo Grande.

Silva, E. R. da. (2003). O desenvolvimento do senso crítico no exercício de identificação e escolha de argumentos. *Rev. Brasileira de Linguística Aplicada,* 3(1), 57-184.

https://www.scielo.br/j/rbla/a/NPWMcpnNzDfdh6TgLV6rcMz/?format=pdf&lang=pt

Toulmin, S. (2006). *Os usos do argumento*. 2. ed. São Paulo, SP: Martins Fontes.

Van Eemeren, F. H., & Grootendorst, R. (2004). *Developments in Argumentation Theory*. Amsterdam, NL: University of Amsterdam.

Velasco, P. D N. (2009). Sobre a Crítica Toulminiana ao Padrão Analítica – dedutivo de Argumento. *Cognitio*, 10(2), 281-292. https://revistas.pucsp.br/cognitiofilosofia/article/download/13443/9967

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