ACTIO
Docência em Ciências

ACTIO: Docência em Ciências
ISSN: 2525-8923

http://periodicos.utfpr.edu.br/actio

Combinando: a material for teaching combinatorial analysis to blind students

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ABSTRACT

This work aimed to use a 3D printer to develop material that contributes to the teaching process of Combinatorial Analysis (specifically the Fundamental Counting Principle) to blind students. Even though Combinatorial Analysis is a content with varied practical applications, there is a lack of manipulative pedagogical materials suitable for teaching this content to blind students. The *Combinando* material was modeled and printed considering an understanding of the use of manipulative materials in teaching blind students, associated with 3D modeling and printing. Based on this material, a task proposal is presented to work the fundamental counting principle with blind students. The task will be guided by discussions about Combinatorial Analysis, presentation and explanation of the material, delivery of the activity and material, setting up groups, developing and discussing the task. The pieces of material have reliefs that allow them to be identified by touch. It is also possible to differentiate the parts visually, since the material has overlapping colors. Thus, blind and sighted students can explore the same material together in an inclusive manner.

KEYWORDS: 3D technology. Mathematics. Fundamental Counting Principle.

INTRODUCTION

The Curriculum Guidelines for Basic Mathematics Education point to the importance of Combinatorial Analysis in education as “a means to solve problems that require analysis and interpretation” (PARANÁ, 2008, p. 61). In the National Curricular Parameters (PCNs), the objective of the Combinatorial Analysis content “is to lead the student to deal with problem situations that involve Combinations, Arrangements, Permutations and, especially, the Multiplicative Principle of counting” (BRASIL, 1998, p. 36). This knowledge can be useful in the daily lives of students to solve problems related to counting, such as situations involving the possibilities of a certain event to occur.

Although Combinatorial Analysis has a practical utility, there is a lack of manipulative pedagogical materials suitable for teaching it to blind students. This lack motivated the development of this work, which aimed to develop, in the 3D printer, materials that contribute to the teaching of Combinatorial Analysis to visually impaired students (specifically the Fundamental Counting Principle). For this, we are guided by theoretical studies on the inclusion policy of Special Education students in the school environment, the difficulties of teachers and the use of didactic materials in teaching Mathematics to these students. We address these studies briefly in the first two sections of the text. Next, we present the methodological context, 3D printing, in which we detail the printing process, the characteristics of the available printers and the materials that can be used to manufacture three-dimensional objects. The following section details the process of creating the didactic material for blind students, called *Combinando*, followed by a suggestion of approach for the material in the classroom.

THE INCLUSION OF BLIND STUDENTS IN SCHOOLS

According to Law No. 9,394/96, students who have disabilities and high abilities/giftedness are guaranteed the right to education in the regular education network (BRASIL, 1996). These students can be enrolled in public schools in Brazil with the right to free quality education. The law emphasizes that the target audience of Special Education can receive specialized support when needed, such as an interpreter or assistant teacher.

The National Guidelines for Special Education in Basic Education (BRASIL, 2001) emphasize that, in a classroom, the student must be included in the activities that the others carry out — often in an adapted way, but they cannot be apart from the rest from the team. In this sense, the Brazilian Inclusion Law (BRASIL, 2015) advises that equal conditions should be promoted for people with disabilities.

According to Joslin (2012), in schools there are factors that hinder the maturation of the inclusion process, such as the lack of infrastructure to serve the target audience of Special Education and the lack of didactic-pedagogical support and training for teachers. For Joslin (2012, p. 92), “the absence of continuing education for common class teachers weakens inclusive school practice.”

Teachers are not prepared for the diversity of these students, so they need to learn, during classes, how to teach these students, which can generate negative results in the inclusion process. In addition, the responsibility rests with the teacher, who "should seek methodologies and materials that help them in their pedagogical practice to work in a way where there can really be inclusion in the classroom and so disabled students can learn as well as others" (KOEPSEL, 2016 p. 2).

In this context, in many situations, the teacher needs to adapt materials, given that some Special Education students may need manipulative materials to facilitate learning, as is the case with blind students, who need to resort to their remaining senses to learn. According to Koepsel (2016), one of the most acute senses in these students is touch, and it is through it that the student with this special need explores the pedagogical materials in an attempt to assimilate mathematical concepts. This process is detailed in the next section, where we discuss how these resources can favor the learning of Mathematics by blind students.

TEACHING MATHEMATICS TO BLIND STUDENTS AND THE USE OF MANIPULATIVE MATERIALS

In Brazil, blind students have a guaranteed right to quality education, just like other students enrolled in any school. The teaching of the target public of Special Education starts in Basic Education, being based on the National Guidelines for Special Education, with an

> [...] educational process defined by a pedagogical proposal that ensures special educational resources and services, institutionally organized to support, complement, supplement and, in some cases, replace common educational services, in order to guarantee school education and promote development. development of the potential of students with special educational needs, in all stages and modalities of basic education (BRASIL, 2001, p.1).

The National Guidelines for Special Education guide teachers so that teaching contributes to the construction of knowledge of blind students, respecting the strict relationship with space and objects. This relationship occurs through the remaining senses of these students, and may occur through hearing (vestibular system), touch (kinesthetic awareness) and smell (BRASIL, 2003).

In this context, Braille is an "essential language for the development of students' autonomy" (DIAS, 2017, p. 35), as it enables the whole interpretation of the content that the teacher is working on in the classroom. Regarding the mastery of this language by blind students, Souza and Fratari (2011, p. 5) defend the importance of pre-braille, so that blind children are "stimulated with different resources, involving games and using materials adapted to their needs."

In this phase, the visually impaired student develops motor skills, a sense of direction and the ability to differentiate objects. With pre-braille, blind people can stimulate their senses through tactile discrimination of objects. In the process, they begin to distinguish large objects; and as it evolves, they become able to discriminate smaller and smaller objects with different reliefs, sizes, shapes and textures. Progress enables them to read Braille, since a very refined sense of touch is needed to recognize the different reliefs of the symbols of this writing system. At the end of pre-Braille learning, blind students should be prepared for the next phase, literacy in the Braille system, which should still occur in Basic Education, in order to enable them to express and read symbols on a sheet of paper.

Sometimes, writing in Braille may be insufficient for mathematical understanding, which implies that the teacher must seek other ways to teach mathematical concepts — which are usually exposed visually and orally (DIAS, 2017). One of the means to facilitate the learning of blind students "is the use of manipulative materials, which will help the student to better understand and fix some concepts that are being taught" (DIAS, 2017, p. 40).

Lorenzato (2006, p. 30) states that the use of Didactic Materials (DM) in the classroom "facilitates learning, whatever the subject, course or age." In addition, the material "enables students to learn at their own pace" (LORENZATO, 2006, p. 30), which allows them to build knowledge in their own time. The author defines manipulative materials as "any instrument useful to the teaching-learning process" (LORENZATO, 2006, p. 18). When dealing with tangible objects, materials such as geometric solids, games, calculators, materials developed for students from the target audience of Special Education, among others, are included.

Manipulative materials can bring positive results when used. However, attention must be paid to the context, as the material has the power to influence the students, and "this power depends on the state of each student and also on the way the DM is used by the teacher" (LORENZATO, 2006, p. 27). The different approaches employed by the teachers when working with DMs lead to different pedagogical results. If the teacher addresses a subject orally and exemplifies with the DM, it will have a certain result, which will be different if they allow the students to explore the DM, test hypotheses and reach their conclusions with the teacher's mediation (LORENZATO, 2006). Therefore, according to the author, DMs do not, by themselves, entail significant learning: their use, along with manipulative materials, must favor mental activity in order to present satisfactory results.

The teacher's knowledge when using DMs in the classroom is another determining factor for student learning. The teacher must know the material very well and know when to use it; "otherwise, the DM may be ineffective or even harmful to learning" (LORENZATO, 2006 p. 34). As the responsibility for teaching falls on the teacher, it is possible that the teacher ends up frustrated if they do not have a good experience in the classroom. To avoid frustrations, planning is paramount. In this regard, Lorenzato (2006) recommends that mathematics teachers ask themselves: Why would it be convenient, at this moment, to use DMs in this class? How should this DM be used? Although these questions do not guarantee student learning, they can help teachers during planning and, consequently, to improve learning.

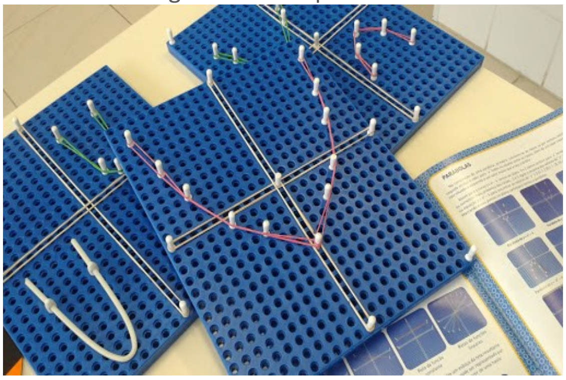
When it comes to students with special educational needs, such as blind students, the use of manipulative DMs is even more important. This is because, many times, it will be the only support that the student will have as an aid to understanding the content, allowing them to explore their remaining senses.

Today there are several manipulative DMs available on the market that can be used to teach Mathematics. We can mention, as an example, the Golden Material, Soroban, logic blocks, geometric solids, among others. In the same way that these materials can help in teaching sighted people, blind students can be contemplated with these resources. However, some care must be taken when choosing the material, such as:

> [...] the relief must be easily perceived by touch and, whenever possible, be made up of different textures to better highlight the component parts of the whole. Smooth/rough, fine/thick type contrasts allow proper distinctions. The material should not cause rejection when handled and be resistant, not easily damaged and resistant to tactile exploration and constant handling (SÁ; CAMPOS; SILVA, 2007, p. 27).

These characteristics are essential for blind students to learn, as handling and tactile exploration are sources of information for them, as they need to use their remaining senses (such as touch) to obtain knowledge. On the market, there are materials developed exclusively for blind students with the characteristics necessary for good learning, among which we can mention the Multiplano (Figure 1).

Figure 1 - Multiplano Set



Source: Multiplano (2020).

According to Multiplano (2020, s. p.), the material favors “the understanding of Mathematics content such as: Construction of numbers; multiplication table; Operations; Fractions; Regular and irregular geometric figures; Symmetry; Trigonometry; Plane and spatial geometry: Statistics and many others.” The product helps visually impaired students learn, and can also be used with sighted students, a characteristic that allows the inclusion of blind students in the school environment. However, this material has a high financial cost and is often not available in schools, as it is common for teachers who have students with visual impairment to face the lack of manipulative materials at school and the difficulty in acquiring them (KOEPSEL, 2016), because often the school does not have the resources to purchase.

As an alternative, teachers make materials with homemade objects to meet the student's needs (KOEPSEL, 2016). In this production, materials are used that are often discarded, such as bottle caps, egg cartons, pots, among others. The problem with this improvisation is the demand for extra-class time from the teacher to make it, in addition to the fact that these materials may not have good resistance and may deteriorate after use, contrary to one of the characteristics presented by Sá, Campos and Silva (2007).

A case similar to those reported by Koepsel (2016) was recorded by us, based on a conversation with a teacher who taught a blind student. This teacher mentioned the need for manipulative didactic materials to teach the content of Combinatorial Analysis to blind students. In this case, he needed to employ effort and creativity to build adequate materials, which can be seen in the images provided by the teacher (Figure 2).

Figure 2 - Material developed by the teacher.

Source: Photos provided by the teacher (2020).

The teacher’s proposal to teach Combinatorial Analysis was to combine pieces of clothing made with textured EVA and MDF boards, in which the students could choose a piece of clothing and fix it on the base; by altering the other piece, they would be able to compose the tree of possibilities. The contact with this teacher aroused our interest in creating a similar material, which could help blind students to understand Combinatorial Analysis and facilitate the work of teachers who teaches Mathematics to these students. This is because, although the material has proven adequate, we believe that it could be improved in some aspects, such as usability, storage, number of possibilities for combinations and production.

In this context, the production of teaching materials with 3D printing

> [...] can be very useful in printing teaching materials that help teach different areas. More than that, 3D printing technology enables the construction of concrete materials that were previously only accessible virtually, enabling the student to better understand the content worked with these models through the handling and exploration of these materials (BASNIAK; LIZIERO, 2007, p. 4).

One of the benefits of this technology is that 3D printing can be used indirectly, since “it is a resource that allows the construction of materials for teaching, and thus, its use in the classroom does not need to be direct” (BASNIAK; LIZIERO, 2017, p. 7). In this way, the teacher does not need to specialize in 3D printing, being only necessary to plan the lesson based on the printed material. The prototypes printed using this technology have good resistance, fast construction, low cost and high level of detail.

Thus, we believe that 3D printing technology can provide favorable experiences for the teaching and learning of visually impaired students, helping to print manipulative didactic material that enables blind students to understand the Fundamental Counting Principle. In the next section we discuss a little about the context of 3D printing, its benefits, its evolution and the process that permeates printed objects.

METHODOLOGICAL CONTEXT: 3D PRINTING TECHNOLOGY

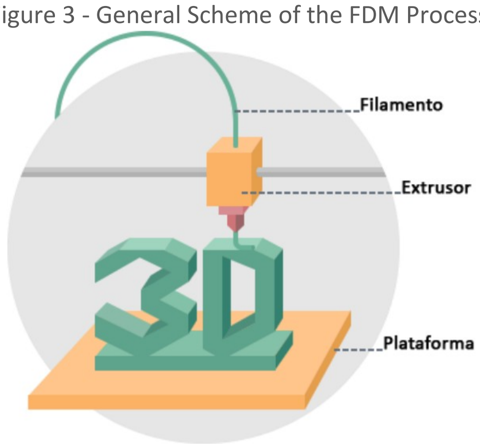
On a daily basis, we see in the media the revolutions caused by 3D printing in the most diverse fields, such as civil construction, the printing of building walls, dental clinics, for the manufacture of dental prostheses, and even the most advanced medicine in which “3D bioprinting is being applied to regenerative medicine in order to produce tissues and organs suitable for transplantation” (OLIVEIRA et al., 2017, p. 1036).

The first 3D printer prototype was created in 1980 by the Japanese designer Hideo Kodama. He was one of the first to invent a way of curing (drying) resin with a single beam laser (BESKO; BILYK; SIEBEN, 2017). The discovery gave rise to the stereolithography (STL) process. In 1986, Chuck Hull introduced stereolithography using acrylic-based photopolymer materials to the market. This technology instantly solidifies the photopolymer material (which is initially liquid) when exposed to a beam of ultraviolet light. To market his invention, he called it the 3D System. Two years later, Scott Crump created a printing technology, Fused Deposition Modeling (FDM). As the name suggests, this technology involves melting plastic and then casting it to print the piece.

3D printers only became popular after 2009, the year in which Crump's FDM printer patent ended and several startup companies emerged with projects to make the technology accessible to the public — a fact similar to what happened with Apple and the computer. Ten years later, we can find simple models of 3D printers on the market, with values close to that of entry-level computers.

Currently, there are several 3D printing technologies; however, here we will only discuss the most widely used printing technology: FDM. It is a very versatile and accessible printing method, with which it is possible to print with several different materials, the most used being ABS (Acrylonitrile Butadiene Styrene), PetG (Polyethylene Terephthalate modified with Cyclohexanedimethanol), Acrylic and PLA (lactic polyacid). The printing process is simple: the filament used is heated by a resistance until it is in a liquid-paste state; the material is lightly pressurized to pass through an extruder nozzle; then it is deposited on a platform, as shown in Figure 3 (MONTEIRO, 2015).

Figure 3 - General Scheme of the FDM Process.



Source: THRE3D (2014, apud MONTEIRO, 2015).

Printers with FDM technology are easily found in the market, due to the low cost of printing. Printing quality and speed depend on the thickness of the extruder nozzle installed on the machine: the thicker it is, the higher the speed and, consequently, the lower the quality. Generally, the accuracy of such a machine is 0.1mm.

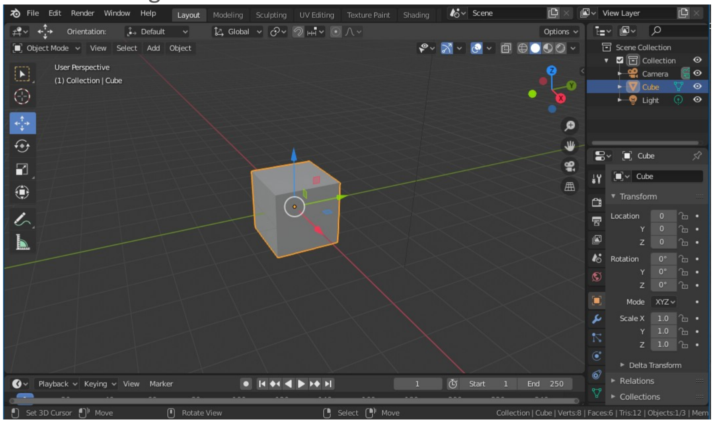
As an advantage, there is a wide range of materials to print according to the project's demand, such as thermoplastics (which have greater mechanical resistance and greater durability in exposure to the weather), malleable and even biodegradable plastics. In addition to the possibility of printing on different materials, we can program printing stops to change the filament, being able to print objects in different colors, or to add aggregates to the printed parts, such as nuts or inserts, which is impossible with the STL method.

Printing objects on a 3D printer takes place in three stages: a) modeling the object in CAD software; b) configuration of print properties to generate print routes; and c) printing the object on the 3D printer.

The modeling of the object is done in Computer Aided Design (CAD) software. There are several software available on the market for acquisition or subscription, such as AutoCAD, Solidworks, Inventor, SketchUp, Solid Edge, 3Ds

Max, among others. There are also free software that were created by communities and made available for free use, such as Blender (Figure 4), 3D Crafter, 3D Builder, LibreCAD, among others. These software are widely used in areas such as Engineering, Geography, Architecture, Design, Medicine, Fashion, among many others. Each software has a specific application, such as AutoCAD, which is a software aimed at civil construction and engineering, and SolidWorks, focused on the construction of mechanical parts.

Figure 4 - Blender CAD software interface.



Source: The authors (2020).

Generally, object modeling starts with simple figures, and the designer uses the tools available in the software to combine different figures into more complex shapes (MONTEIRO, 2015). Modifications can occur manually, using the mouse to make changes, or parametrically, entering values on the keyboard so that the transformation occurs precisely.

After completing the modeling of the object, it is necessary to export the model in an image file so that another software can generate the print routes. The most used extension for 3D objects is .stl. With the exported file, it is necessary to generate the print routes for the printer; for this, some 3D printer manufacturers provide specific software for their products. This is the case, for example, of Cliever, which made available the virtual assistant for the Cliever Studio printer, with which we can calibrate, change filament, generate print routes and perform other tasks. There are also software developed by groups of researchers, as is the case of Slic3r, which can also generate the printing routes of an object.

The correct configuration of the wizard is of paramount importance for a good impression. The values established in each configuration parameter directly influence the quality of the object, resistance, time spent, among other factors. In this step, the printing speed, the thickness of the walls of the object, the types of filling, the amount of material deposited, the temperature of the extruder nozzle, the temperature of the table, among other parameters, are established.

Once parameters are established, print routes can be generated; it is at this moment that the assistant analyzes the .stl object and generates the printing routes in the form of coordinates. Routes establish the path that the extruder nozzle will follow, the speed and amount of material to be deposited in each location, in addition to the selected temperature, calibration parameters and possible pauses.

In 3D printers, there are several ways to print. The least efficient uses a connection to a computer via a USB cable to send the coordinates to the printer. As a disadvantage, it is necessary for the computer to be turned on while the object is being printed, which generates a higher energy cost. Another way is to insert the file with the coordinates into the printer using a memory card, which excludes the use of a computer during printing and generates less energy expenditure. In newer printers, it is possible to import print routes directly from the computer connected to the Internet, which makes the process simpler and safer, avoiding interruptions at the time of printing.

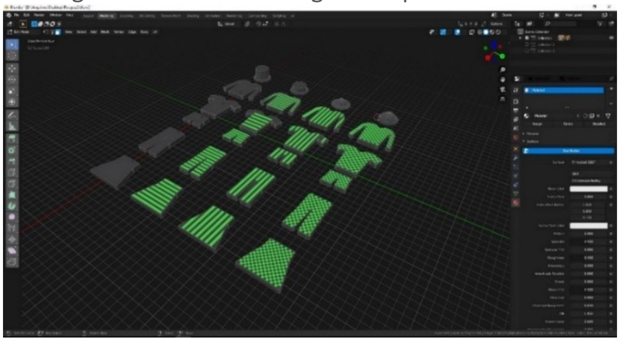
In our project, we chose to use the Blender software to model the objects. We also used the Simplify3D software to generate the print routes that were later printed on the Cliever CL1 - Black Edition printer. The material we used was PLA, as it has biodegradable characteristics, which generates a lower environmental impact. The entire manufacturing process and settings are detailed in the next section.

COMBINANDO: THE MATERIAL PRODUCED

The modeling of the material took place using Open Source software, named Blender, which belongs to the non-profit organization Blender Foundation. The software was chosen for the vast arsenal of tools available and for its simplicity of use. In this software, it is possible to model different objects in different ways, and the modeling can be done through commands typed on the keyboard that generate precise modifications or even by tools that use human interface devices (such as the mouse), which allow a lot of versatility in object modeling.

For *Combinando*, the most widely used tools were the parametric ones, which consist of entering values on the computer keyboard to carry out precise modifications. The material modeling process began when we designed clothing sketches. We used the tools available in the software to model primitive figures (such as a cube or a sphere) in clothing representations. During the process, we looked for the ideal size of the pieces (Figure 5) so that handling and identification would not cause blinds students to reject them, as pointed out by Sá, Campos and Silva (2007).

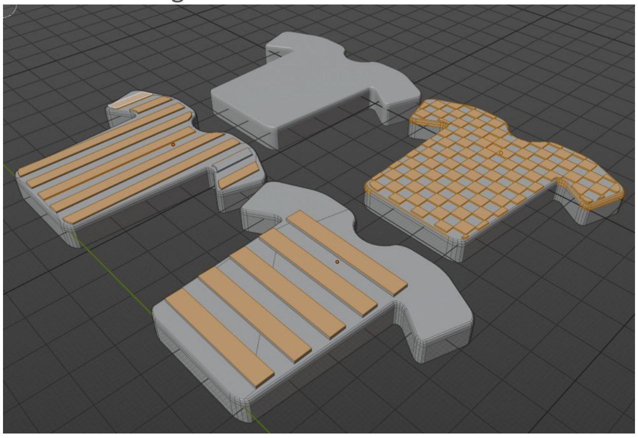
Figure 5 – Material being developed in Blender.



Source: The authors (2020).

We created six types of clothing: hats, t-shirts, blouses, shorts, pants and skirts. For each of them, with the exception of the hats (which we built four different models), we established different textures, as mentioned by Sá, Campos and Silva (2007). Such textures were: plain, vertically striped, horizontally striped and checkered (Figure 6).

Figure 6 - Different textures.



Source: The authors (2020).

Combinando consists of 24 pieces made up of different clothes (4 hats, 4 blouses, 4 shirts, 4 pants, 4 shorts and 4 skirts), a base where they can be fitted and a box with a drawer to store them (Figure 7).

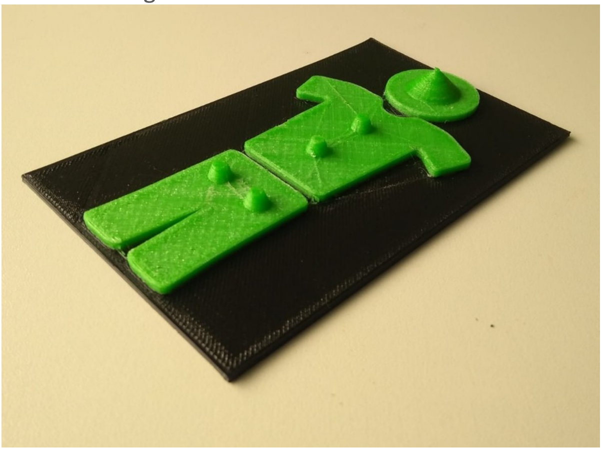
Figure 7 – *Combinando.*

Source: The authors (2020).

We developed this material so that blind students can make combinations with clothing items. By performing different combinations and recording the results, we hope they can understand concepts of Combinatorial Analysis, more specifically the Fundamental Counting Principle.

The process of matching pieces of clothing can be easy for sighted students; for blind students, however, it may be a little more difficult to organize the parts. With that in mind, we developed a base with a kind of guides using the clothing templates (Figure 8), so that the student can fit the pieces in an organized way and make their combinations.

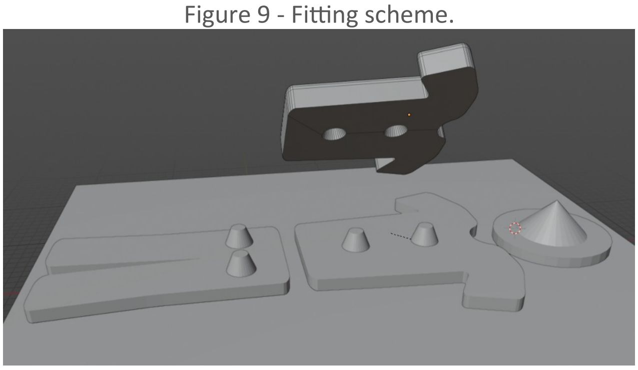
Figure 8 - Basis for combinations



Source: The authors (2020).

In order to facilitate the fitting of the pieces, we place trunks of cones on top of the clothing molds and create the corresponding fittings on the clothes and hats. We concluded that this would be the most efficient way to work with the material, as the cone trunks would provide robustness and ease of fitting. In Figure 9, we can see, on the bottom of a T-shirt, the fittings to attach the cone trunks to the base guides.

Figure 9 - Fitting scheme.



Source: The authors (2020).

The truncated cone shape facilitates the fitting of the pieces because the smallest base has a smaller diameter compared to the largest base. In this way, the perfect alignment of the garment with the trunks of cones is not necessary: the student only needs to fit the smaller base of the cone into the larger base of the hole, and gravity takes care of aligning the object.

The student can be guided by the model of the garments shown in the base and, thus, fit the chosen garment in its proper place. When positioning the garment on the pattern, the fitting occurs in an easier way, allowing the construction of the tree of possibilities when exchanging the pieces, as shown in Figure 10.

Figure 10 - Combination made in *Combinando*.

Source: The authors (2020).

After use, some parts could be misplaced. In order to avoid loss and unusability of the material, we built a drawer that was sized to store all the parts, occupying the smallest possible volume. In this way, the material can be stored anywhere, without the danger of losing parts (Figure 11).

Figure 11 - Top view of the opened material

Source: The authors (2020).

The drawer also has an opening limiter that makes it impossible to remove it. To facilitate the opening of the drawer without compromising the robustness of the material, we developed a handle at the bottom of the drawer. Once the material is built, we discuss a possibility of its use, whose process is detailed in the section that follows.

AN APPROACH TO THE FUNDAMENTAL COUNTING PRINCIPLE USING *COMBINANDO*

Combinatorial Analysis is important for solving and understanding counting problems in students' lives. As an example, we can use concepts from Combinatorial Analysis to find out the number of games needed in a football championship for a number n of registered teams, or the possibility of hitting the Mega-Sena draw with a certain number of bets.

Combinatorial Analysis involves contents such as Combinations, Permutations, Arrangements and the Fundamental Counting Principle. There are countless applications that can be related to concepts of Mechanical Engineering, Electrical Engineering, Fashion and, mainly, Statistics. Questions related to Combinatorial Analysis are loosely associated with an event, more specifically with how many different ways a certain event can occur.

Widely used for solving counting problems, the Fundamental Counting Principle is almost always related to situations such as: If each object in one group A is combined with all elements in another group B, how many clusters will be formed? (BRASIL, 1998). To address the Fundamental Counting Principle, NCPs emphasize: "the exploration of counting problems will lead the student to understand the Multiplicative Principle" (BRASIL, 1998, p. 137). It is in this sense that we have developed this proposal, which can be used, for example, to start the study of Combinatorial Analysis in High School, approaching the Fundamental Counting Principle.

The Fundamental Counting Principle is enunciated by Lima, Carvalho, Wagner and Morgado (2006, p. 125) as: "If a decision D_1 can be made in p ways and, whatever that choice is, decision D_2 can be made in q ways, then the number of ways in which decisions D_1 and D_2 can be made consecutively equals $p \cdot q$.". This principle can be extended to n decisions, such as D_3 taking r ways and D_4 taking s ways, and so on. The number of possibilities related to D_1, D_2, D_3 and D_4 is expressed by the sentence: $p \cdot q \cdot r \cdot s$.

We can take as an example the possible combinations of 3 shirts and 4 pants. As we have 3 shirt options and 4 pants options, the sentence that expresses the result is defined as $3 \cdot 4 = 12$, thus obtaining 12 different ways of dressing with 3 shirts and 4 pants.

In this sense, taking all the possibilities of combinations between the groups of parts of the material — being Group A: 4 hats; Group B: 8 upper garments and Group C: 12 lower garments—the sentence expressing the number of combinations is as follows: $4 \cdot 8 \cdot 12 = 384$. That is, we can perform 384 different combinations using all the pieces of material.

All parts of *Combinando* can be used to work on the Fundamental Counting Principle, because, in this case, the groups do not need to contain the same number of elements, being possible to vary the quantities of elements in each group or restrict the use of a group. Thus, it is expected that the use of the material will favor students to understand the Fundamental Counting Principle, exploring the material from the number of combinations that it is possible to make in each group (A, B, C) and considering the different clothes in each one of them.

As a suggestion, the teacher can start working with a reduced number of pieces in each group and gradually increase the number of pieces, until students can identify a certain pattern. Next, it is important that a discussion takes place on the subject for better validation, as we will address in the next section, in which we present a task proposal to work on the Fundamental Counting

Principle, subsidized by tables of anticipation of student actions in the development of the task that favor the teacher's mediation.

This proposal consists of using *Combinando* to start studying the Fundamental Counting Principle, with a view to including blind students. We established a class organization framework (Chart 1) in order to facilitate the teacher's pedagogical practice.

Chart 1 - Class organization chart

Phases	Actions
1st Stage	Discussions about Combinatorial Analysis
2nd stage	Presentation and explanation of the material
3rd stage	Delivery of activity and material. Formation of groups
4th stage	Task development
5th stage	Discussions about the task

Source: The authors (2020).

The time required for each stage may change, depending on the subject, the pace of the students and other issues that may arise. The teacher can make the time required for each action more flexible with planning, aiming at better use of their time in the classroom.

We suggest that the teacher start a conversation with the students asking if they have heard of Combinatorial Analysis and if they have any idea what it is about. It is important that the teacher encourages students to respond in order to facilitate teacher/student interaction within the classroom.

Before the attention of all students, the teacher can present the material that will be used and explain its operation, that is, the correct way to handle it, remove the parts, make the combinations and store the material after use.

For the development of the task, students can form groups of up to 4 members, with the purpose of making the discussions more fruitful. Each group will receive a set of *Combinando* and a sheet with the task (Chart 2).

The sighted classmates of blind students should be instructed to help them handle the material. It is important that this student makes their own records, conjectures and dialogues with their classmate in the group.

The proposed task (Chart 2) was developed so that students can solve it autonomously, based on the teacher's mediation and discussion with colleagues.

Table 2 - Task – Let's combine.

Task – Let's combine.
1. Using the material, answer the following questions. a. Using 2 pants and 3 shirts, how many combinations can be made? b. How many combinations can we make when considering 3 pants and 3 shirts? c. Considering 4 pants and 4 shirts, how many combinations are possible? d. With 2 hats, 4 pants and 4 shirts, how many combinations are possible? 2. Based on the answers obtained in items a, b, c and d of Question 1, look for a strategy that allows you to calculate all possible combinations using all parts of *Combinando*, without performing all the combinations in the material and answer how many combinations can we make with 4 hats, 8 shirts, 8 blouses, 12 pants, 12 skirts and 12 shorts? 3. From your answer in Question 2, explain how we can calculate the number of combinations with any number of garments, without having to use *Combinando*.

Source: The authors (2020).

The teacher should pay special attention to the group(s) that have blind student(s). It is important that the teacher encourages dialogue between the group members, since hearing and touching are the senses most used by blind students to acquire knowledge. Once most students have completed the task, the teacher can move on to the next step, the discussion, which should start from the students' resolutions.

The discussion should take place from the solutions delivered by the students. The teacher can choose to photograph the resolutions and project them on slides or hand out the sheet and ask the group to explain their reasoning, using the available DMs to give examples. Ideally, the students' resolutions should be transcribed into Braille and given to blind students, which will allow them to better follow the discussions.

Students can organize themselves again in the groups previously formed to facilitate interaction between members and the inclusion of blind students. This student can be helped by other classmates, including regarding the use of the *Combinando* material, which can be used by students to validate or exemplify their classmates' solutions.

Lorenzato (2006) believes that, for learning to be meaningful, mental activity must be present. The discussion is a propitious moment for learning, since the students need to rethink what they did to explain it to their colleagues. This practice encourages mental activity in the students; then learning. It is also a crucial moment for the teacher. Lorenzato (2006, p. 27) states that it is "at that moment that the teacher will be able to assess how the students learned" and, thus, rethink their teaching practice and the evaluation of their students.

It is possible that several different resolutions will appear. We suggest that the teacher select the most interesting resolutions to be presented by students. There can also be a comparison between different solutions to favor the students' reasoning. We suggest that solutions be chosen according to complexity: from least to most complex.

From this, the teacher should systematize the content, explaining that this is the Fundamental Counting Principle and that it is applied in two or more successive steps that are independent. In this case, the number of combinations will be expressed by the product of the possibilities of each set.

Next, we present considerations regarding the experience we obtained in developing suitable material for teaching combinations to visually impaired students. We discuss the positives and negatives of using 3D printing technology to make manipulative materials and the feasibility of using these materials in the classroom.

CONSIDERATIONS

As a result of reports made by teachers about the difficulty of finding suitable materials to work with blind students, more specifically regarding the content of Combinatorial Analysis, we developed a material called *Combinando* to help teachers who teach Mathematics to these students. This material enables blind and sighted students to explore the Fundamental Counting Principle in an inclusive manner.

Faced with these difficulties, we understand that the use of teaching materials combined with good planning can facilitate the learning of students with special educational needs. The responsibility for teaching lies with the teacher, who must look for elements that contribute to the teaching and learning of blind students, facilitating the understanding of mathematical concepts and allowing interaction and sharing of knowledge and discoveries. However, they need support for this task.

3D printing technology can help teachers in this regard, more specifically in building materials in line with their needs in the classroom. The construction process can be made possible without the teacher operating the 3D printer, that is, the teacher does not need to specialize in 3D printing but can only use the already printed material. Prints can be made with different materials and colors, according to the needs of each project. The printing cost is usually low as the objects are printed in a semi-hollow way.

However, so that the teacher can transform an idea into a material, it is necessary for a specialized person to model and print the objects. In addition, it is necessary to have a 3D printer available for use. Printing objects on a 3D printer is not so simple: generally, the modeling phase is time consuming, and, during the prints, some unforeseen events can happen that compromise the work, such as filament breakage, clogging, power outages and a defective printer. In newer printers, the chance of unforeseen events is very low, as they are equipped with sensors that pause printing if the filament breaks or the nozzle becomes clogged; in addition, they support resuming printing after a power outage.

Printing the *Combinando* material on the 3D printer took 48 hours (non-continuous) of printing. The impressions were given to a blind high school student to assess the quality of the material. The student's reports reveal that the material has the necessary characteristics for good tactile exploration by a blind person. According to the student, it is possible to differentiate the shape and texture of the parts printed on the 3D printer, in addition to the parts being pleasant to the touch, which shows that 3D printing can be used to help blind students. Because this research took place during the Covid-19 pandemic, it was not possible to develop the teaching proposal with the student or to analyze the potential of the material for teaching Combinatorial Analysis to blind students, which is the objective of future work.

However, through the analysis carried out by the student, we found evidence that 3D printing technology can favor the teaching and learning of blind students. The pieces we print have reliefs that allow the pieces to be identified by touch. It is also possible to differentiate the parts visually, which have overlapping colors for a better visualization. In this way, blind and sighted students can explore the same material together in an inclusive way; from this premise, we developed the teaching proposal of this work in order to help teachers to use *Combinando*.

Thus, the proposal has an inclusive character with the use of manipulative didactic materials, called *Combinando*. Blind students can work together with sighted students exploring the concepts of the Multiplicative Principle, building the tree of possibilities, and completing the generalization to the Fundamental Counting Principle. Essentially, the proposal was developed for high school students, but the material can be used with students of any age — including, it may be suitable for elementary school students.

Combinando: um material para ensino de análise combinatória a estudantes cegos

ABSTRACT

Este trabalho objetivou desenvolver, na impressora 3D, um material que contribua para o ensino de Análise Combinatória a estudantes cegos (especificamente o Princípio Fundamental da Contagem). Embora a Análise Combinatória seja um conteúdo com grande aplicação prática, verifica-se carência de materiais pedagógicos manipuláveis adequados para o ensino desse conteúdo a estudantes cegos. Pautado na compreensão sobre o uso de materiais manipuláveis no ensino de estudantes cegos e associado à modelagem e à impressão 3D, o material Combinando foi modelado e impresso. A partir desse material, apresenta-se uma proposta de tarefa para trabalhar, com estudantes cegos, o princípio fundamental da contagem. A tarefa será pautada em discussões a respeito da Análise Combinatória, apresentação e explicação do material, entrega da atividade e do material, formação dos grupos, desenvolvimento e discussões sobre a tarefa. As peças do material têm relevos que permitem identificá-las por meio do tato; também é possível diferenciar as partes visualmente, as quais têm sobreposição de cores. Dessa forma, estudantes cegos e videntes podem explorar juntos o mesmo material de maneira inclusiva.

PALAVRAS-CHAVE: Tecnologia 3D. Educação Matemática. Princípio Fundamental da Contagem.

ACKNOWLEDGMENTS

We would like to thank PRPGEM and Capes for the financial support received.

NOTES

1. Translate: Universo Traduções, e-mail: contact@universotraducoes.com

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Received: 3rd april 2022
Approved: 19th march 2023
DOI: 10.3895/actio.v8n1.15346
How to quote:
BASNIAK, Maria Ivete; DOMBROWSKI, André Felipe. Combinando: a material for teaching combinatorial analysis to blind students. **ACTIO**, Curitiba, v. 8, n. 1, p. 1-23, jan./apr. 2023. Available at: <https://periodicos.utfpr.edu.br/actio>. Accessed on: XXX
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